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OPTIMISATION OF TUNED MASS DAMPER DEVICES TOWARDS STRUCTURAL VIBRATION REDUCTION: THEORETICAL SETTINGS AND NUMERICAL ANALYSES

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Tesi di Dottorato in Meccatronica, Informazione, Tecnologie Innovative e Metodi Matematici

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Abstract

The present doctoral thesis concerns theoretical concepts and numerical studies on the optimisation of Tuned Mass Damper (TMD) devices towards the control and reduction of structural dynamic responses, with specific reference to the context of civil and seismic engineering. The fundamental background of the optimisation of the TMD parameters, also called tuning, is presented first in its main features, based on reference to the mainstream literature, and in the assessing the relevance of the structural properties and of the role of the characteristics of the external excitation.

A comprehensive analysis on the tuning of passive TMDs as applied to a single-degree-of-freedom primary structure subjected to benchmark ideal excitations is carried out, focusing on a range of structural parameters typical of real engineering applications. Then, the outcomes have been interpolated through nonlinear least squares and optimum tuning formulas of the TMD parameters have been outlined for each excitation case, based on *ad hoc* polynomial fitting models. Comparisons with main references from the literature are provided.

The optimisation of TMD devices has been also investigated for the mitigation of the transient response, with main focus on the impulse excitation. Initially, a throughout optimisation of the passive device is derived, with consideration of different excitation cases, objective functions and structural parameters. Then, the control device has been upgraded to a hybrid TMD by means of the addition of an active controller, with the main task of reducing as well the peak response. Different feedback control strategies have been evaluated, from the points of view of: stability, device performance and amount of supplied control force.

An important part of this research deals with the concept of optimum seismic tuning of TMDs, with real earthquake input directly involved within the tuning process. This feature represents an innovative way of investigating the TMD performance, since the control device is theoretically optimised on each specific structure and seismic event. The proposed tuning procedure is presented in detail and applied to a significant selection of structures and earthquake input signals. The so obtained optimum TMD parameters are first depicted and compared to those obtainable from reference tuning formulas from the literature. A wide set of results concerning the performance of the TMD is presented, considering different kinematic and energy response indexes, in order to trace down general trends on the effectiveness of the TMD in reducing the seismic response. A further and important stage of this study deals with a crossed comparison involving the TMD performance and relevant indexes such as the modal parameters, the frequency amplitude of the seismic signal and the response spectra, so that to inspect possible connections between the efficiency of the control device and the characteristics of the structural and the dynamic context.

The studies and related outcomes presented in this thesis shall represent a contribution to the development and improvement of Tuned Mass Damper devices in terms of optimum performance, towards the control of a wide range of structures. Therefore, the presented thesis work, though connoted by a main theoretical character, may display different crucial implications in practical engineering applications.

Keywords: Tuned Mass Damper (TMD), Optimisation, Harmonic Loading, White Noise Loading, Impulse Loading, Transient Response, Seismic Tuning, Earthquake Excitation.

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Nomenclature

Latin letters	Description	Units
$\mathbf{C}$	Structural system damping matrix	Ns/m
c_S, $\mathbf{C}_S$	Primary structure damping coefficient, damping matrix	Ns/m
c_T	TMD damping coefficient	Ns/m
D	Dissipation power	W
f	Frequency	Hz
f	TMD frequency ratio	-
f_c	Controller force	N
F	Excitation amplitude	N
E	Young's modulus	Pa
E	Elastic energy	J
g	Excitation frequency ratio	-
g_a	Acceleration control gain	kg
g_d	Displacement control gain	N/m
g_v	Velocity control gain	Ns/m
H, $\mathbf{H}$	Receptance coefficient, receptance matrix	m/N
I	Inertia moment	m^4
J	Objective function	-
$\mathbf{K}$	Structural system stiffness matrix	N/m
k_S, $\mathbf{K}_S$	Primary structure stiffness coefficient, stiffness matrix	N/m
k_T	TMD stiffness coefficient	N/m
$\mathbf{M}$	Structural system mass matrix	kg
m_S, $\mathbf{M}_S$	Primary structure mass coefficient, mass matrix	kg
m_T	TMD mass coefficient	kg
s	Laplace variable	-
t	Time	s
T	Period	s

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Latin letters	Description	Units
T	Kinetic energy	J
x_g	Ground displacement coordinate	m
x_S	Primary structure displacement coordinate	m
x_T	TMD displacement coordinate	m
Z, $\mathbf{Z}$	Impedance coefficient, impedance matrix	N/m

Greek letters	Description	Units
β	Rayleigh damping model coefficient	-
δ	Dirac's delta function	-
$\mathbf{\Phi}$	Eigenvector	-
ζ_S	Primary structure damping ratio	-
ζ_T	TMD damping ratio	-
μ	Mass ratio	-
ω	Excitation angular frequency	rad/s
ω_S	Primary structure angular frequency	rad/s
ω_T	TMD angular frequency	rad/s

Abbreviations	Description	Units
MDOF	Multi-Degree-Of-Freedom	-
PGA	Peak Ground Acceleration	-
SDOF	Single-Degree-of-Freedom	-
S_d	Displacement Response Spectrum	m
S_{pa}	Pseudo-acceleration Response Spectrum	m/s^2
S_{pv}	Pseudo-velocity Response Spectrum	m/s
TMD	Tuned Mass Damper	-

Chapter 1

Introduction

1.1 Motivations and contributions of the doctoral thesis

The present doctoral dissertation mainly presents and outlines appropriate procedures and methods devoted to the optimum tuning of the parameters of Tuned Mass Damper (TMD) devices for the reduction of the dynamic response of structural systems, with consideration of different loading conditions, representing suitable models or records of real dynamic excitations.

The proposed optimum device is tested, through numerical trials, by investigating the actual TMD performance in reducing the dynamic response of benchmark structures. The outcomes of this research work finally aim at providing useful indications towards the best design of TMDs for practical applications, especially in the field of structural, civil and earthquake engineering.

In particular, the present thesis outlines the following main contributions within the related research field of TMDs optimisation:

- A unified approach for the optimum design of Tuned Mass Dampers for different loading conditions, which is a context that may easily occur in real structures and systems.

- A comprehensive investigation on the potential role and relevance of the control device in mitigating the dynamic response.

- A strong attempt in clarifying the actual effectiveness of TMDs in the field of seismic engineering, through an *ad hoc* formulated optimisation procedure.

1.2 Structure of the thesis

The thesis is composed of several main parts, related to the specific treated research topics, as briefly presented as follows.

Present **Chapter 1** assembles a brief introduction on the main general contents of the thesis and their meaning within the present research field.

Chapter 2 provides a general part where the main features of the different typologies of TMDs are presented, with reference to the mainstream literature and a brief recall to their engineering applications. Moreover, the specific device components and their related issues of optimal design are outlined in the main characters.

Chapter 3 concerns the optimum tuning of TMDs for the case of benchmark ideal excitations, with the task of achieving a unified tuning of TMDs based on a common polynomial model. In particular, a structural system composed of a damped SDOF primary structure and a TMD added on top is supposed to be subjected to either harmonic or white noise loading, acting either as point force on the primary structure or as base acceleration. The optimum TMD parameters have been evaluated for a range of structural parameters consistent with potential real engineering applications.

The ensemble of the so obtained results has been fitted through nonlinear least squares interpolation, in order to outline suitable and effective analytical tuning formulas, which could be close to optimum conditions and sufficiently simple in view of practical design of TMDs. In particular, the tuning formulas developed through the adoption of a proposed original fitting model has been compared to those based on literature tuning formulas referred to a special case, i.e. for undamped primary structure.

Finally, an investigation on the TMD optimum parameters and global performance has been outlined, with a comparison to the outcomes of main reference tuning formulas.

Chapter 4 deals with the optimisation of passive and hybrid TMDs for the reduction of the transient response, specifically in the case of impulsive excitation, which appears to be a framework not deeply explored yet for these control devices. First, the passive TMD added on top of a SDOF primary structure is tuned, considering as dynamic excitation involved within the optimisation process a unit impulse acting as base displacement, which could be a suitable model for real sudden dynamic loading.

Then, the control device is upgraded as a hybrid TMD, following the addition of an active feedback closed loop controller, which is supposed to be able to improve the performance of the device, through a properly designed control force. In this sense, a specific investigation on different possible control laws has been carried out, with particular focus on the stability analysis and the features concerning the optimisation process of the control gains.

A final comparison in terms of efficiency of the control device and required supplied force is presented and the related implications are discussed.

Chapter 5 is dedicated to a comprehensive analysis on an innovative optimisation procedure, that is the seismic tuning of TMD, which involves the seismic input signal as directly included into the optimisation process. Hence, the optimised TMD is expected to represent the optimum tuning, at least for the selected index of dynamic response assumed as objective function. In this way, it is possible to establish a sort of reference for the considered case, and to outline a study on the highest potential effectiveness of TMDs in reducing the seismic response.

Due to several uncertainties related to some features of such a framework, most of all the uniqueness of the seismic event, in order to develop an exhaustive analysis, a wide range of buildings and of earthquakes has been considered within the analyses. Specifically, 16 shear-type frame buildings, with number of floors ranging from 1 to 40, assumed as primary structures and 18 earthquakes, for a total of 288 cases, have been studied, with an assumed value of the primary structure damping ratio $\zeta_S = 0.05$, which is quite high (in terms of challenging the TMD effectiveness) and truly representative of possible buildings and civil structures. In particular, the primary structures are characterised by different number of floors and floor masses, in order to explore the influence of the variability of the structural parameters in terms of optimum TMD parameters and related efficiency of the TMD device.

On the other hand, the various real earthquake events exhibit different characteristics in terms of duration, magnitude, frequency content, spectrum, which are factors that could easily produce important consequences on the dynamic behaviour of the structural system.

The so conceived study gave rise to many set of results, at different stages. First, the optimum TMD parameters obtained with the proposed tuning method are presented for different values of the mass ratio μ, discussed and compared to those obtained from reference tuning formulas from the literature. Then, comprehensive outcomes concerning the TMD performance in reducing the structural seismic response are presented, with consideration of several response indexes, so that to provide a comprehensive representation of the effect of the addition of the TMD for the different cases. A complementary analysis has then outlined, concerning the crossed comparison between the response reduction and several indexes related to modal parameters, frequency content of the seismic signal and response spectra, in order to investigate possible connections that link the TMD performance and the structural and dynamic context, with the task of outlining preliminary guidelines towards potential design and massive use of these device in earthquake engineering.

Chapter 6 summaries the salient contributions and remarks related to the present research work and outlines possible future research scenarios.

Chapter 2

General framework on the tuning of TMDs

2.1 TMD devices for structural control

Tuned Mass Dampers are doubtlessly one of the most studied and adopted control devices, used in different engineering fields, since these devices allow for efficiently counteract the dynamic effect of a wide range of possible dynamic excitations, such as wind, earthquake, vibrations due to service conditions (e.g. human-induced vibrations [27] or machinery activities).

For these reasons, with reference to the civil engineering context, in many cases TMDs have been implemented as optimal solutions for the reduction and the control of the dynamic response of towers [18, 51, 62, 75, 108], skyscrapers [29, 61, 103, 105], chimneys [59], bridges [97], pedestrian footbridges [24, 88, 113, 118], offshore platforms [66] (see e.g. Figs. 2.1–2.3).

One of the first structures where a TMD was installed is the Sydney Tower (also called Centerpoint Tower, Sidney, Australia, 1971), which is a 309 m-high office building, topped with a water tank with a volume of 162000 liters that, being provided with hydraulic shock absorbers, serves not only as building water and fire protection supply, but also as a Tuned Liquid Damper (TLD); moreover, a secondary added mass of 40 t allows for the increase of structural damping [51, 102]. Two blocks of 300 t of weight represent the TMD added to the John Hancock Tower

(Boston, USA, 1975), which exhibit a first modal mass of 47000 t, moving in phase for the lateral response control and out of phase for the torsional control [102]. The Canadian National Tower (Toronto, Canada, 1976) displays a total height of 553 m, whose 102 m are the extension of the antenna located on top of the tower, where two ring-shaped TMDs of 9 t/m weight were inserted at 488 m and 503 m of height, respectively [21, 51]. The TMD installed at the 63rd floor of the Citicorp Center (New York, USA, 1977) is represented by a mass of 373 t, which represents approximately the 2% of the mass of the first vibration mode. Besides the mass, the TMD device is composed of linear gas springs, pressure balance supporting system, control actuator, power supply and electronic control. Several tests displayed that the addition of the TMD allowed for an increase of the structural damping ratio from 1% to 4%, leading to an abatement of the building acceleration of about 50% [102].

Figure 2.1: *Vertical TMDs installed under the footpath of the Millennium Bridge, London, UK, 2001 (Source: Internet, http://www.gerbusa.com).*

The TMD placed into the 340 m-high Nanjing Tower (China, 1998), composed of 1% of the first modal mass, which corresponds to 60 t, with a damping ratio of 7%, was proved to reduce the displacement due to wind loading of about 30% [18]. One of the most recent examples of this control device is represented by the well known pendulum-type TMD installed into the 508 m-high skyscraper Taipei 101 (Taipei, Taiwan, 2003), placed between the 88th and the 92th floor and mainly composed of 41 layered steel plates (see Fig. 2.3), for a total mass of 660 t; the so conceived TMD allows for an abatement of the wind oscillations of about 30%–40%.

Figure 2.2: *TMD placed into the steel arms of the Burj Al Arab hotel in Dubai, UAE, 1999 (Source: Internet, http://www.gerbusa.com).*

Also, in the context of mechanical engineering, the Tuned Mass Damper found numerous applications for the control not only of automotive systems (vehicles, shock absorbers), but also of a broad range of various systems and contexts, such as machineries [89], turbines, telescopes [99], etc.

Although the concept of the original TMD seems to have been introduced more than one century ago by the patent of Frahm [32], a large number of studies on Tuned Mass Dampers are currently under development, concerning the investigation of different points of view and features related to this device, including an but not limited to the methodology of the optimal tuning, the proper adjustment to the different contexts of application, the development of new typologies of TMD. In this sense, in the following, just the main versions of this devices will be briefly introduced, with consideration of their salient features. In general, the present dissertation will mainly focus on the passive and the hybrid TMD.

Figure 2.3: *The 660 t Tuned Mass Damper placed between the 87th and the 92th floor of the Taipei 101 skyscraper, Taipei, Taiwan, 2003 (Source: Internet, https://buildcivil.wordpress.com).*

2.2 Passive TMDs

2.2.1 Tuning for ideal dynamic loading

The passive Tuned Mass Damper (TMD) is a device usually composed of an additional (or secondary) mass, a spring and a damper (also briefly defined mass-spring-dashpot system) attached to a primary system in order to reduce its structural dynamic response [102]. The most important principle of the TMD device is represented by the tuning of its mechanical free parameters so that to match the characteristics of the vibration mode of the primary structure to be controlled.

The original Tuned Mass Damper concept could likely be dated back to the patent of Frahm [32], concerning an undamped TMD applied to mechanical systems in order to reduce the vibrations induced by external actions, e.g. for the rolling of ships produced by sea waves.

Ormondroyd and Den Hartog [76], Brock [15] and Den Hartog [26] further deepened and codified the study on this device, by providing a systematic framework and introducing the so called Fixed-Point Theory, based on the case of a damped TMD added to an undamped SDOF primary structure subjected to harmonic force. Such theory leads to a tuning based on the same height of the frequency response peaks of the structural system, where both TMD parameters have been determined analytically, but while the so called frequency ratio can be evaluated exactly [76]; the TMD damping ratio is obtained approximately as the average value between those achieved for the two peaks [15]. This method immediately became the fundamental way of TMD tuning, on which almost all subsequent theories have been based.

Afterwards, many contributions have considered the optimum tuning for damped main structures [12, 33, 35, 60, 65, 127], despite higher difficulties in treating the governing dynamical equations [7, 11]. The studies of Warburton and Ayorinde [10, 124–126] have reported progress in connection to the tuning proposed by Den Hartog with respect to harmonic and stationary random excitations. A comprehensive comparison has revealed at that stage that even for moderately damped structures ($\zeta_s < 5\%$) Den Hartog tuning formulas guarantee an adequate structural response decrease. In a large number of works which focused on this topic, the adoption of numerical optimisation approaches has been proposed as a suitable way to achieve optimum tuning. Among those, one of the first examples is probably represented by that of Ioi and Ikeda [41], where optimum TMD tuning formulas have been pointed out, as a result of an optimisation process developed with a Newton's method. Within subsequent studies which shared a similar *modus operandi*, of great interest appear those of Randall, Halsted and Taylor [84], which obtained the optimum

TMD parameters in the form of graphical representations, of Tsai and Lin [116,117], where design formulas for the loading cases of harmonic force and base acceleration are provided, and of Rana and Soong [83], based on a Minimax algorithm, where the resulting optimum TMD parameters have been condensed in design abaci. Furthermore, Asami et al. [7] provided tuning formulas for several loading cases, by analytical approaches, while Leung and Zhang [53] have presented a numerical tuning procedure based on a Particle Swarm Optimisation method.

A different stream of research considered instead a TMD tuning independent of the dynamic excitation, based on control techniques, such as the optimum pole placement, which method was introduced for TMDs by Thompson [110, 111] and recently rediscovered by Bisegna and Caruso [14].

2.2.2 Passive TMDs for seismic applications

A context of study of the passive Tuned Mass Damper, of great relevance within civil engineering applications, is the optimal tuning in the case of seismic excitation.

In this sense, one of most important stream of research is mainly represented by the work of Villaverde and Newmark [120], Villaverde [119] and Villaverde and Koyama [121] which considered a TMD tuning independent of the dynamic excitation, but based instead on the modal analysis of the primary structure, i.e. focused on inherent properties of the primary structure only. Such theory has been further deepened by Sadek et al. [90], and later adopted by Miranda [67, 68].

On the other hand, a large number of studies focused instead on the TMD tuning with consideration of the seismic event [36], even if in different ways, as it will be depicted in the following for the most representative works. Wong and Chee [129] assessed a remarkable capability of TMDs in the seismic energy dissipation, mostly in the case of long-period structures; Marano et al. [63] approached the TMD tuning problem by means of a stochastic optimisation procedure; Paredes et al. [79] performed several tests on six-storey buildings with two earthquake records, whereby the TMD tuning was based on Villaverde's equations [121], by finding out that the TMD effectiveness, which appears to be related to the earthquake frequency content, is not that apparent. However, despite the bulk of studies on the best tuning of

TMD parameters, the seismic assessment still appears to be an open research topic. Above all, there also emerge different conflicting opinions on the real effectiveness of TMD devices in the case of seismic excitation, see e.g. [45, 90, 98, 107].

Furthermore, a series of recent works considered the given earthquake input as directly embedded within the tuning process. The main differences among these studies concerned the adopted optimisation algorithm and the modeling of the seismic input. Some significant works, where the seismic analysis has been developed in the frequency domain and the earthquake record has been modeled through the Kanai-Tajimi formula [44], are that of Hoang et al. [38, 39], where the TMD tuning was carried out within a numerical optimisation based on the Davidon-Fletcher-Powell algorithm, that of Lee et al. [52], where the Golden Section method was used in the optimisation process, and that of Leung et al. [54], where the seismic input was modeled as a non-stationary process and TMD tuning was carried out within a Particle Swarm Optimisation algorithm.

Farshidianfar et al. [28, 100] considered a forty-storey frame building subjected to a given earthquake, with seismic analysis carried out in the time domain and TMD parameters optimised through an Ant Colony Optimisation method [100] or a Bee Colony Optimisation algorithm [28]. Adam et al. [2, 114, 115] have dealt with an investigation on two different tuning approaches, one based on literature tuning formulas [39,126] and on the assumption of simulating the earthquake as a stationary white noise excitation, and the other which considers the actual earthquake record in the frequency domain and recovers, as optimal TMD parameters, the median of those obtained for each seismic input; the so obtained results pointed out: (a) a negligible difference between the two adopted approaches, for the considered structures and seismic records; (b) a remarkable effectiveness of TMDs in earthquake applications. Bekdas and Nigdeli [13] proposed a tuning in the frequency domain based on a Harmony Search algorithm, assuming a harmonic load in the optimisation process, which has been further tested with a seismic loading. Mohebbi and Joghataie [69] considered an eight-storey frame building subjected to an earthquake modeled as white noise excitation in the time domain, with a TMD tuned by a Distributed Genetic Algorithm (DGA) optimisation.

2.3 Active TMDs

During the last decades, active Tuned Mass Damper devices, also known as active Mass Drivers [4, 73], have arisen in the field of structural control as complementary or alternative with respect to passive TMDs. Active TMDs are mainly composed of an active controller and a small resonant mass, this latter usually quite lower than 1% of the primary structure mass.

The required amount of control force in active devices is usually generated by electrohydraulic or electromechanical actuators ruled by control strategies based on feedback information from the structural response and feedforward data from the dynamic excitation [40, 101, 131]. The recorded measurements from response and excitation are managed *ad hoc* and monitored by a controller which, by means of a selected control algorithm, determines the control signal almost in real time (besides time delays usually occurring in real systems), then providing it to the actuator.

The operativity of active Tuned Mass Dampers is mainly based on the supplied control force, and therefore such devices require a considerable amount of power, i.e. on the order of tens of kilowatts for small structures and even of several megawatts for large structures. The main effect obtained from active TMDs seems to be a significant increase of damping, with a less remarkable change in the structural stiffness.

The most important research issue related to active control devices concerns the investigation on the optimal control strategy [17, 48, 71]. In this sense, the base for the development of a theoretical control law is usually composed of the entire kinematic response of the primary structure, i.e. displacement, velocity and acceleration [40, 51, 101]. Then, such design choice is realised through the appropriate placement of sensors and transducers. In other studies, less response indexes have been assumed for the definition of the control law, namely either displacement or velocity or acceleration feedback closed loop controllers are alternatively investigated.

In general, for active TMDs, a sort of multitasking research may take place, since the goal is not only the abatement of the dynamic response of the primary structure, but also the limitation of the supplied control force plays a central role within the optimisation process. Such position of the problem allows for a handling management of the multi-objective needs within the optimisation process described above.

2.4 Hybrid TMDs

The hybrid Tuned Mass Damper is ideally a combination of the passive TMD and the active TMD presented before. In particular, the main capability of this control device is basically committed to the passive TMD, with the further contribution of an additional active controller [9]. As a consequence of this configuration, the hybrid TMD remains substantially a narrowband control device, but its performance results to be improved by the active controller, which is able to immediately supply a control force, in order to respond to possible sudden changes within the dynamic context [96].

Since the main part of the device performance is delegated to the passive component, the amount of supplied control force turns out to be far less from that required by a fully active TMD [103, 105]. This feature, together with the intrinsic robustness of the passive device, which is operative also in case of failure of the external power source and therefore is able to guarantee a basic level of response reduction, leads to prefer the assumption of this typology of control device in many full-scale civil engineering applications, with respect to either the passive or the active TMD alone [105].

The optimal design of this control device is mainly composed of a two-stage optimisation problem, where the first stage concerns the optimal tuning of the passive TMD, while the second phase involves the optimisation of the behaviour of the active controller. In this sense, as explained previously for the active TMD, the investigation on the most appropriate control law doubtlessly covers the main part of the study on the optimum configuration of the hybrid TMD.

2.5 Semi-active TMDs

The semi-active TMD is mainly composed of a passive TMD and an added active controller, which is able to change the mechanical parameters of the global control device, so as to extend their operating range and adapt the performance of the TMD to possible modifications of the structural conditions and dynamic response [1,46,82].

The semi-active TMDs are usually conceived as either stiffness–variable or damping–variable devices. The former group is involved in a proper change of the operative frequency, while the latter deals with special damping components, such as electrorheological or magnetorheological dampers, whose behaviour is ruled by an electric or a magnetic field, respectively [46].

The main positive feature of semi-active TMDs lies into the very small amount of supplied energy for the variation of the parameters, which characteristic has contributed to the noticeable diffusion of this control solution in the last decades, and in many engineering application cases where it led to prefer this device with respect to the active TMDs. Moreover, the small contribution of the control force usually does not jeopardise the stability of the system (at least intended in the Bounded Input-Bounded Output sense) [40].

Chapter 3

Passive TMDs for benchmark ideal excitations

3.1 Introduction

The optimum tuning of TMD parameters is often developed by basing the procedure on a specific excitation, which is supposed to represent a suitable model of a real dynamic loading. From the main literature focused on this topic, it appears that the tuning parameters may change according to the applied dynamic loading. Most of all, in the presence of structural damping ($\zeta_S \neq 0$), tuning formulas and relevant estimates may take quite elaborate forms (see e.g. [7,8,72]), leading to the assumption of numerical methods for the accomplishment of the TMD tuning process.

In this sense, the present chapter deals with a comprehensive TMD tuning in the case of ideal dynamic loading and damped primary structure. The study outlined in the following has been developed in two main phases.

First, a wide range numerical tuning based on a nonlinear gradient-based optimisation algorithm has been pursued. In particular, the tuning has been developed for a TMD added to a SDOF primary structure, subjected to a Harmonic or White Noise loading, applied as both input Force or base Acceleration on the primary structure, for a total of four loading cases (HF, HA, WNF, WNA, as described below). In the present tuning approach, the optimisation variables are taken as the frequency ratio f and the TMD damping ratio ζ_T, as a function of two free given parameters, i.e. mass ratio μ and damping ratio of the primary structure ζ_S, both fixed *a priori* within a wide range of values, including those suitable for engineering applications.

Second, an interpolation process is attempted for all the four cases, where the optimum TMD parameters have been fitted with proper unifying analytical models, calibrated through nonlinear least squares, in order to obtain final compact TMD tuning formulas, in view of possible practical use. Such final output has been compared to that from the relevant literature, in terms of both optimum TMD parameters and achieved dynamic response reduction of the primary structure.

3.2 Structural and dynamic context

The structural system assumed as benchmark in this study is composed of a SDOF primary structure and a TMD added on it (Fig. 3.1), subjected to either point force on the primary structure $F(t)$ or base acceleration $\ddot{x}_g(t)$. The primary structure is characterised by a mass m_S, a constant linear elastic stiffness k_S and a linear viscous damping coefficient c_S. The natural frequency ω_S and damping ratio ζ_S of the primary structure are defined as usual, i.e. respectively:

$$\omega_S = \sqrt{\frac{k_S}{m_S}}\,, \qquad \zeta_S = \frac{c_S}{2\sqrt{k_S\, m_S}}\,. \tag{3.1}$$

Conversely, the parameters of the TMD device are an added secondary mass m_T, a constant stiffness k_T of an added elastic spring and a damping TMD coefficient c_T of an added viscous damper. As above, the TMD natural frequency ω_T and damping ratio ζ_T are respectively:

$$\omega_T = \sqrt{\frac{k_T}{m_T}}\,, \qquad \zeta_T = \frac{c_T}{2\sqrt{k_T\, m_T}}\,. \tag{3.2}$$

The main free TMD parameters, useful to achieve the most appropriate tuning, are defined in terms of mass ratio μ, tuning frequency ratio f and TMD damping ratio ζ_T itself, as:

$$\mu = \frac{m_T}{m_S}\,, \qquad f = \frac{\omega_T}{\omega_S} = \sqrt{\frac{1}{\mu}\frac{k_T}{k_S}}\,. \tag{3.3}$$

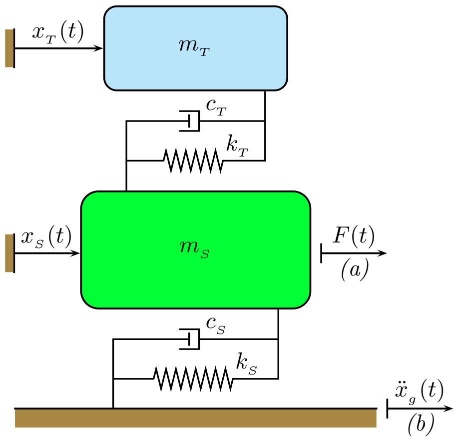


Figure 3.1: *Structural parameters and absolute (relative to the ground) dynamic degrees of freedom of a 2DOF mechanical system composed of a SDOF primary structure (S) equipped with an added TMD (T), subjected to: (a) point force, (b) base acceleration.*

The tuning concept is based on the minimisation of a given dynamic response index, which basically depends on both the structural system and the applied dynamic loading. In this sense, four dynamic loading cases have been considered (Fig. 3.1):

1. Harmonic force on the primary structure (HF);

2. Harmonic base acceleration (HA);

3. White noise force on the primary structure (WNF);

4. White noise base acceleration (WNA).

The corresponding response indexes, in terms of displacement of the main structure x_S, which are taken as objective functions in the optimisation process, are reported in Tables 3.1–3.2 below [23, 26, 126]. Such dimensionless frequency response functions are in the form of dynamic amplification factors R for the case of harmonic loading (with excitation frequency ω, frequency ratio $g = \omega/\omega_S$, force amplitude F or acceleration magnitude $\ddot{x}_g$) [126] and in the form of mean square response indices N for the case of stationary gaussian white noise loading (with constant power spectral density of the loading S_0 and variance of the displacement structural response σ_{x_S}) [126].

Table 3.1: *Objective functions for Harmonic loading in terms of displacement of the primary structure [126].*

Harmonic Force (HF)	$R_F = \left\lvert \frac{x_S}{F/k_S} \right\rvert = \sqrt{\frac{A_F^2 + B_F^2}{C^2 + D^2}}$
Harmonic Acceleration (HA)	$R_A = \left\lvert \frac{x_S}{\ddot{x}_g/\omega_S^2} \right\rvert = \sqrt{\frac{A_A^2 + B_A^2}{C^2 + D^2}}$

$$A_F = f^2 - g^2\,, \qquad B_F = 2\,g\,\zeta_T\,f\,, \qquad A_A = f^2(1+\mu) - g^2\,, \qquad B_A = 2\,g\,\zeta_T\,f(1+\mu)$$

$$C = (f^2 - g^2)(1-g^2) - \mu f^2 g^2 - 4\,\zeta_S\,\zeta_T\,f g^2\,, \qquad D = 2\,\zeta_T\,f g[1 - g^2(1+\mu)] + 2\,\zeta_S g(f^2 - g^2)$$

Table 3.2: *Objective functions for White Noise loading in terms of displacement of the primary structure [126].*

White Noise Force (WNF)	$N_F = \frac{\sigma^2_{x_S}}{2\pi S_{0,F}\,\omega_S/k_S^2} = \frac{1}{4}\frac{I_F}{L}$
White Noise Acceleration (WNA)	$N_A = \frac{\sigma^2_{x_S}}{2\pi S_{0,A}/\omega_S^3} = \frac{1}{4}\frac{I_A}{L}$

$$\begin{aligned} I_F = {} & f^4[\zeta_T(1+\mu)^2] + f^3[\zeta_S\mu + 4\zeta_S\zeta_T^2(1+\mu)] + f^2[-\zeta_T(2+\mu) + 4\zeta_S^2\zeta_T + 4\zeta_T^3(1+\mu)] \\ & + f(4\zeta_S\zeta_T^2) + \zeta_T \end{aligned}$$

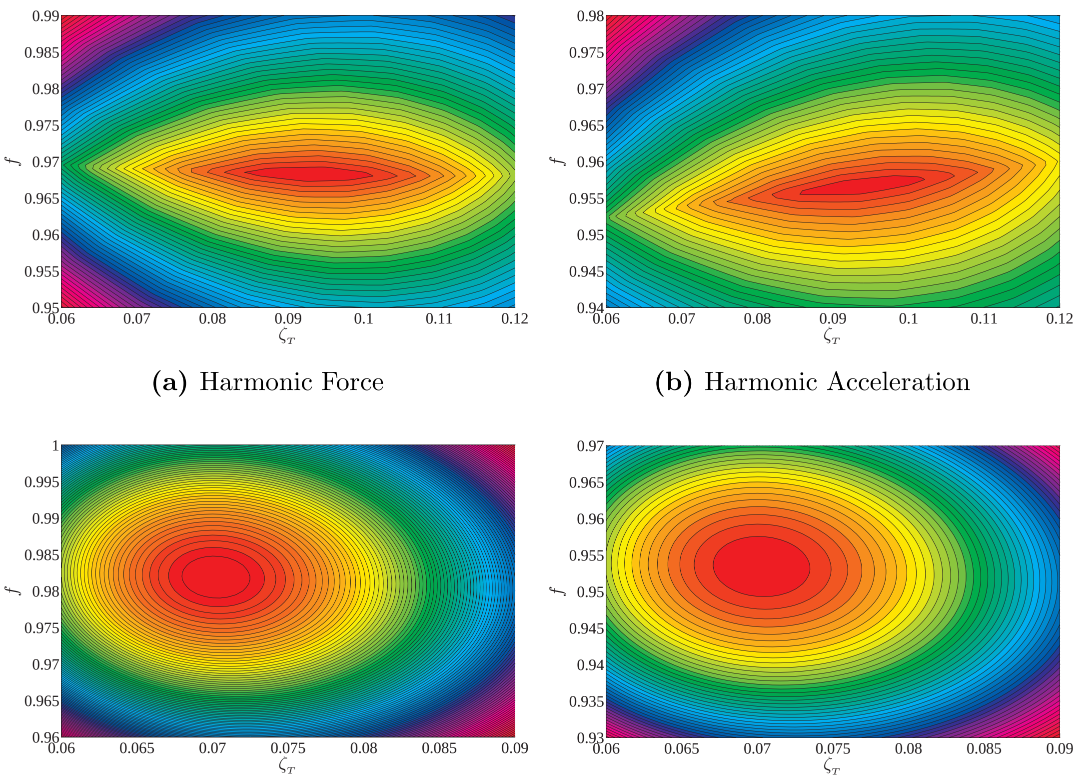
$$\begin{aligned} I_A = {} & f^4[\zeta_T(1+\mu)^4] + f^3[\zeta_S\mu(1+\mu)^2 + 4\zeta_S\zeta_T^2(1+\mu)^3] + f^2[-\zeta_T(2-\mu)(1+\mu)^2 + \\ & + 4\zeta_S^2\zeta_T(1+\mu)^2 + 4\zeta_T^3(1+\mu)^3] + f[\zeta_S\mu^2 + 4\zeta_S\zeta_T^2(1+\mu)^2] + \zeta_T \end{aligned}$$

$$\begin{aligned} L = {} & f^4[\zeta_S\zeta_T(1+\mu)^2] + f^3[\zeta_S^2\mu + 4\zeta_S^2\zeta_T^2(1+\mu)] + f^2[-2\zeta_S\zeta_T + 4\zeta_S^3\zeta_T + 4\zeta_S\zeta_T^3(1+\mu)] \\ & + f(\zeta_T^2\mu + 4\zeta_S^2\zeta_T^2) + \zeta_S\zeta_T \end{aligned}$$

3.3 Tuning process

3.3.1 Preliminary analysis on the objective functions

Before that the optimisation process could take place, it is suitable to develop a preliminary investigation on the characteristics of the selected objective functions, for the different loading cases. In particular, the response index assumed as objective function has been evaluated nearby the attended optimum region, i.e. where it is expected to take the smallest values. An extract of the outcomes of this study is reported in Fig. 4.8, where the response indexes reported in Tables 3.1–3.2 have been evaluated in the case of $\mu = 0.02$, $\zeta_S = 0.05$, leading to the following considerations, which however hold as well for generic values of the structural parameters.



(a) Harmonic Force **(b)** Harmonic Acceleration

(c) White Noise Force **(d)** White Noise Acceleration

Figure 3.2: *Optimum region of the objective function for the considered four loading cases, for $\mu = 0.02$, $\zeta_S = 0.05$.*

The main feature, quite positive in view of the TMD tuning, is the presence of a clear region with a global minimum of the considered function, that allows, in principle, for a robust optimisation process. Indeed, these regions of minimum denote a quite convex shape of the objective function, and therefore it is expected that the optimisation algorithm could easily find the optimum values of the TMD parameters, corresponding to the smallest amplitude of the response index.

In general, as expected the location of the global minimum is close to the coordinate $f = 1$, i.e. to resonance conditions, and with a TMD damping ratio about $\zeta_T = 0.05$, but with some differences between the loading cases. In this sense, the objective function related to harmonic excitations (HF, HA) exhibits a quite narrow shape as a function of f and lengthened along ζ_T, which gives more relevance to the precision of the detection of the optimum value of the frequency ratio. On the other hand, the region of minimum of the response indexes for the white noise loading cases (WNF, WNA) display an almost equal width in all the directions, i.e. it looks quite convex with respect to both TMD parameters.

Besides these specific considerations, from this investigation one could point out that the tuning process for the considered loading cases turns out to be well posed, and therefore it should be possible to provide suitable optimum TMD parameters. This is indeed the case, as shown in the sequel.

3.3.2 Main features on the optimisation methodology

The optimisation process has been carried out for each dynamic loading case, through a nonlinear gradient-based algorithm, and developed within a MATLAB environment [109]. In this sense, different nonlinear numerical methods have been preliminary tested, so that to assess their ability in finding the global minimum of the objective function and therefore to assume the most suitable algorithm for the tuning purposes. In particular, Interior Point, Trust Region and Sequential Quadratic Programming methods have been analysed in their performance, finding that all of them could easily detect the optimum region, for all the considered objective functions, proving once again the well posedness of the present tuning problem. The final choice of a Minimax optimisation method is mainly due to its wide and successful use in the TMD tuning literature [83, 84, 116, 117] and to its proved effectiveness in the present previous experiences [87, 91, 92].

The goal of the Minimax algorithm is that of minimising the worst case, in terms of maximum values, of a set of multivariable functions, given an initial estimate, possibly limited by lower and upper bounds on the optimisation variables. Within the present context, the Minimax problem may be stated as follows:

$$\min_{\mathbf{p}} \max_{\mathrm{J}} \mathrm{J}(\mathbf{p}) \, , \qquad \mathbf{l}_b \leq \mathbf{p} \leq \mathbf{u}_b \, , \tag{3.4}$$

where $\mathbf{p}$ is the vector of the tuning variables, $\mathrm{J}(\mathbf{p})$ is the objective function, $\mathbf{l}_b$ and $\mathbf{u}_b$ are the lower and upper bound vectors of the tuning variables.

Here, the task of the numerical algorithm consists in the minimisation of the maximum value of the previously reported response functions (Tables 3.1–3.2), which obviously depend, given the fixed primary structure parameters, on the free TMD parameters. In this sense, it is worth noting that the Minimax principle turns out quite reliable to state and to solve the optimisation problem related to the minimisation of the frequency peak response for the cases of harmonic loading, as displayed in Figs. 3.3–3.4.

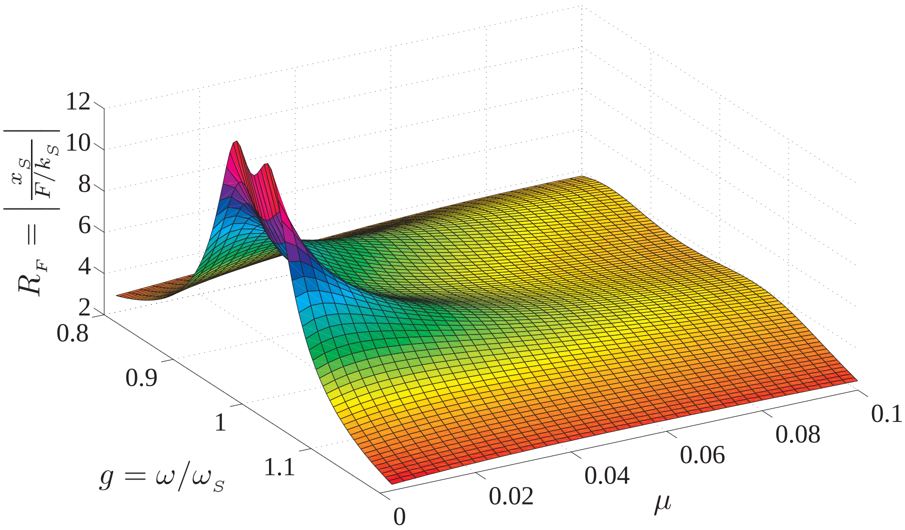


Figure 3.3: *Frequency response of the primary structure as a function of the excitation frequency ratio g and of the mass ratio μ (here reported for the harmonic force on the primary structure).*

Although in principle the method would allow for the optimisation of all three TMD parameters μ, f, ζ_T, the following typical *modus operandi* has been adopted, i.e. for a given fixed mass ratio μ and a primary structure damping ratio ζ_S, the algorithm seeks the optimal frequency ratio f^{opt} and TMD damping ratio ζ_T^{opt}, lead-

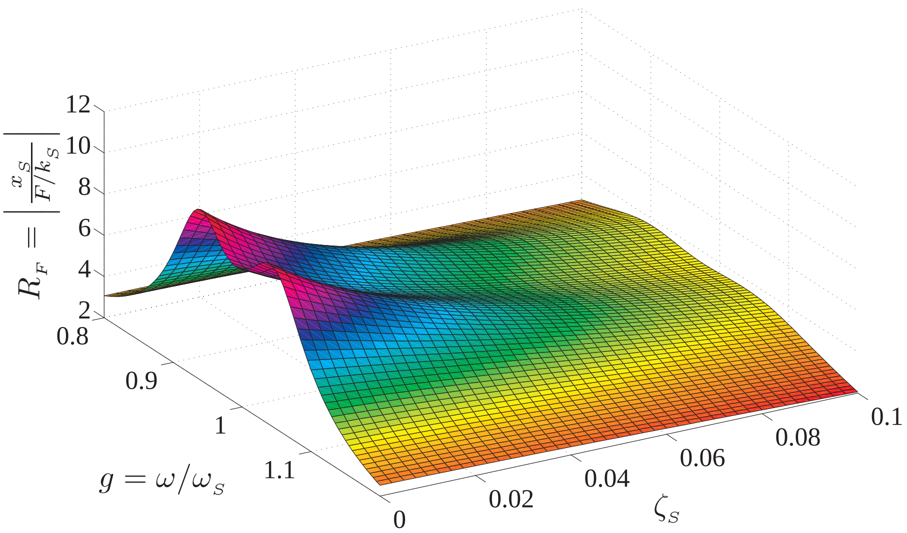


Figure 3.4: *Frequency response of the primary structure as a function of the excitation frequency ratio g and of the primary structure damping ratio ζ_S (here reported for the harmonic force on the primary structure).*

ing to best tuning. Thus, f and ζ_T are here the two assumed free variables of the optimisation process, listed in (2×1) vector $\mathbf{p}$.

To start the optimisation process, it is necessary to initialise the values of the two variable parameters f and ζ_T. This has been done, for each loading case, by means of well known tuning formulas from the literature [26, 126], referring to the case of undamped primary structures ($\zeta_S = 0$), gathered in Table 3.3 below.

Table 3.3: *Optimum tuning formulas from the literature for undamped primary structures ($\zeta_S = 0$).*

Loading	Author [ref.]	f^{opt}	ζ_T^{opt}
Harmonic Force	Den Hartog [26]	$\dfrac{1}{1+\mu}$	$\sqrt{\dfrac{3}{8}\dfrac{\mu}{1+\mu}}$
Harmonic Acceleration	Warburton [126]	$\dfrac{1}{1+\mu}\sqrt{\dfrac{2-\mu}{2}}$	$\sqrt{\dfrac{3\mu}{4(1+\mu)(2-\mu)}}$
White Noise Force	Warburton [126]	$\dfrac{1}{1+\mu}\sqrt{\dfrac{2+\mu}{2}}$	$\sqrt{\dfrac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}$
White Noise Acceleration	Warburton [126]	$\dfrac{1}{1+\mu}\sqrt{\dfrac{2-\mu}{2}}$	$\sqrt{\dfrac{\mu(4-\mu)}{8(1+\mu)(2-\mu)}}$

All the characteristic parameters of the optimisation process have been listed in Table 3.4 (in MATLAB vector notation). These values turn out to assure a good compromise between convergence characteristics and achieved accuracy. Other given external constraints are the maximum number of iterations and the maximum number of function evaluations, both fixed at 300. Actually, from the numerical tests it has been noticed that the optimisation process converges promptly and smoothly, much earlier than reaching such bounds.

Table 3.4: *Main characteristics of the optimisation procedure.*

Tuning variables	$\mathbf{p} = [f;\, \zeta_T]$
Lower bounds	$\mathbf{l}_b = [0.85; 10^{-3}]$
Upper bounds	$\mathbf{u}_b = [1.05; 0.3]$
Mass ratio	$\boldsymbol{\mu} = [0.0025 : 0.0025 : 0.1]$
Primary structure damping ratio	$\boldsymbol{\zeta}_S = [0 : 0.0025 : 0.05, 0.055 : 0.005 : 0.1]$
Harmonic loading frequency ratio	$\boldsymbol{g} = [0 : 0.0005 : 2]$
Tolerance on variable parameter	10^{-6}
Tolerance on constraint violation	10^{-6}
Tolerance on objective function	10^{-6}

The tuning results obtained from the numerical optimisation process for the four considered loading cases (HF, HA, WNF, WNA) are displayed by surface plots in Figs. 3.5–3.6, respectively in terms of optimal frequency ratio f^{opt} and TMD damping ratio ζ_T^{opt}. Line sections drawn out from the surface maps in Figs. 3.5–3.6 will be presented later for further quantitative analysis (Section 3.5).

From the obtained tuning plots in Figs. 3.5–3.6, the following basic considerations arise (to be noted for the subsequent interpolation process). First, from Fig. 3.5 on f^{opt}, three main trends of f^{opt} may be mainly observed, out of the four loading cases. The higher values of f^{opt} belong to the case of White Noise Force (WNF), while lower values of f^{opt} are recovered in the case of Harmonic Force (HF) and lowest tight values are obtained for Harmonic and White Noise Accelerations (HA, WNA), which display an almost similar trend along both μ and ζ_S directions. At the same time, from WNF to WNA cases, it appears that f^{opt} becomes more variable with respect

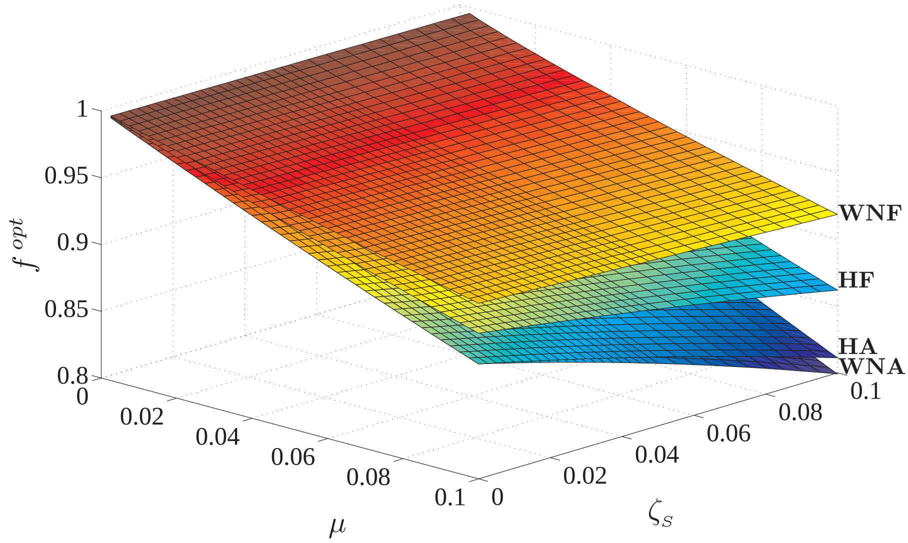


Figure 3.5: *Optimum frequency ratio f^{opt} from the numerical optimisation process for the four loadings.*

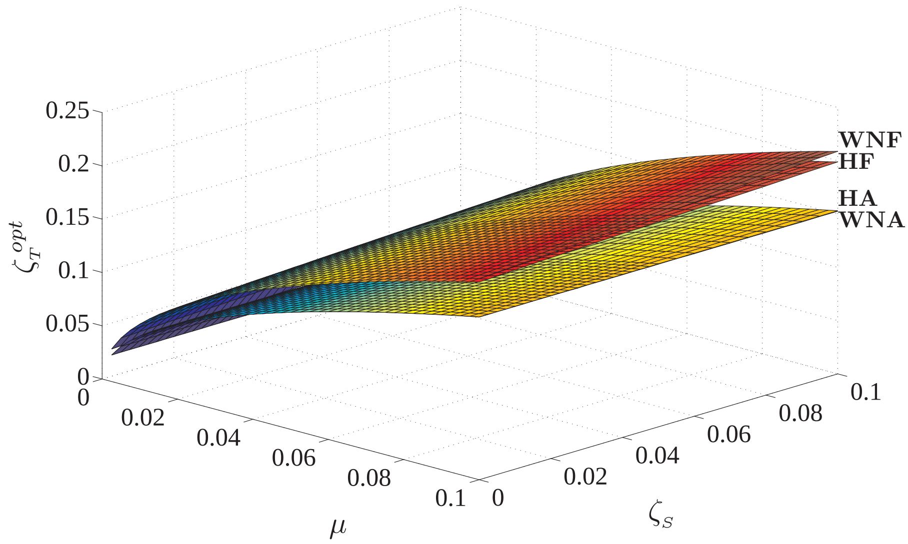


Figure 3.6: *Optimum TMD damping ratio ζ_T^{opt} from the numerical optimisation process for the four loadings.*

to structural damping ratio ζ_S. Particularly, for the WNF case, the variability on ζ_S appears almost negligible; for the HF case just a bit more visible and almost bilinear; for HA and WNA cases more apparent and with increasing nonlinearity. Also, the outcomes for HF (reference case in Den Hartog's analysis [26] of undamped structures, $\zeta_S = 0$) are almost halfway to those from WNF and HA/WNA.

The context represented by the optimal TMD damping ratio ζ_T^{opt} in Fig. 3.6 is quite different, as clearly pointed out in the surface plot. Two main trends are actually displayed, which separate the cases of Harmonic and White Noise loadings (both for point Force and base Acceleration). The common and important feature of these two trends is the quite negligible variation of ζ_T^{opt} as a function of structural damping ratio ζ_S. This holds true especially for White Noise loading.

In short, from a visual comparison between the two series of plots in Figs. 3.5–3.6, it should be said that trends sort out as follows: by point of load application (Force vs. Acceleration) for f^{opt}; by type of loading (Harmonic vs. White Noise) for ζ_T^{opt}. Results on both f^{opt} and ζ_T^{opt} recall strongly those obtainable for the case of undamped primary structures ($\zeta_S = 0$), as reported by the tuning formulas listed in Table 3.3.

3.4 Tuning formulas

3.4.1 Fitting process

The obtained optimal TMD parameters f^{opt}, ζ_T^{opt} have been post-processed through a proper interpolation method, seeking the best match of the achieved results to possible unifying analytical fitting proposals that may be elaborated as described in the following. Direct 2D surface fittings on both variables μ and ζ_S have been attempted. The numerical interpolation method relies on nonlinear least squares estimates based on the minimisation of the sum of the residuals between optimal and fitting values [109]. The fitting model coefficients have been evaluated iteratively.

First, a starting estimate of the model coefficients is attempted. The so obtained fitting is assessed, and its Jacobian evaluated. Then, the model coefficients are adjusted by an optimisation algorithm based on a Trust Region method, by improving the achieved interpolation until appropriate convergence criteria are met.

Once a best fitting is obtained with a proposed model, its reliability may be assessed by various error indices, such as the *Summed Square of Error* (SSE) or the *Root Mean Square of Error* (RMSE), and accuracy indices, such as the *R-square* correlation between optimal and fitting values [109]. This last index has been reported in the results that follow. In general, the closer the R-square index to 1 is, the smaller and nearer to zero the error indices are, and the better the fitting estimate is considered.

3.4.2 Fitting models

The following contents stand as selected outcomes of a wider study, where different analytical fitting models have been considered and assessed, within the task of seeking a model apt to match the best compromise among appropriate representation of optimum results and simpleness of the tuning formulas. Specifically, two fitting models are presented here, for each optimised TMD parameter f^{opt} and ζ_T^{opt}.

The first model, which outlines the present main proposal (validated also in following Section 3.5), will be denoted as Proposed Fitting Model (PFM). This model does not explicitly refer to the tuning formulas for the case of undamped primary structures (Table 3.3). It just refers to polynomial expressions in the variables μ, ζ_S. Conversely, the second model, denoted as Literature-based Fitting Model (LFM), follows a similar approach but basically relies on the tuning formulas in Table 3.3, which may be recovered exactly as particular cases for $\zeta_S = 0$.

3.4.3 Frequency ratio

The fitting model for frequency ratio $\tilde{f}^{opt}$ and related tuning formulas are introduced and discussed in detail in this section. Based on the direct inspection of the surface plots in Fig. 3.5, it considers first a fitting model expressed by a polynomial expansion in the variables μ, ζ_S, generalising a bilinear dependency, endowed with proper exponents e, f, g, h accounting for possible nonlinearity and with additional coefficients a, b, c, d ruling the importance of each term:

$$\tilde{f}^{opt} = a - b\,\mu^e - c\,\mu^f\,\zeta_S^g - d\,\zeta_S^h\,. \tag{3.5}$$

The signs appearing in Eq. (3.5) have been assigned by taking into account results obtained from the tuning procedure itself. Indeed, it may be noticed from Fig. 3.5 that the values assumed by f^{opt} are largest for small values of mass ratio μ, and decrease at increasing values of both free variables μ, ζ_S. A first estimate of the model coefficients, for the four different loading cases (HF, HA, WNF, WNA), has been reported in Table 3.5, whose results lead to the following considerations. For all the loading cases, parameter a takes, as expected, values near 1 (meaning that a Tuned Mass Damper characterised by a small mass and attached to a lightly-damped main structure should be resonant with the structure itself). Coefficients e and g are also near 1, while coefficient f approaches 1/2. The remaining coefficients are quite different for each loading case but, as a general observation, it may be pointed out that very similar values have been obtained for the cases of Harmonic Acceleration and White Noise Acceleration, while Harmonic Force and White Noise Force cases follow different trends.

Table 3.5: *Optimum coefficients for the fitting model of frequency ratio f^{opt} in Eq.* (3.5).

Loading	a	b	c	d	e	f	g	h	R-square
HF	1.003	0.7365	0.9475	0.8791	0.8959	0.4214	0.9794	1.936	1.0000
HA	1.003	0.9272	1.696	1.229	0.8977	0.4223	1.004	2.014	1.0000
WNF	1.002	0.5610	0.3969	0.01145	0.9018	0.5057	1.033	0.7986	1.0000
WNA	1.004	0.9219	1.787	1.659	0.8948	0.4307	0.9872	1.926	1.0000

These outcomes lead, after further refinements, to the final tuning formulas for f^{opt} of the Proposed Fitting Model reported in Table 3.6, based on square-root dependencies, which display an important feature: with quite a simple model and slight changes of the coefficients among the different loading cases, it is possible to achieve a good general fitting in terms of f^{opt}. Also, the fittings for the two acceleration loading cases are unified and actually set the same.

Table 3.6: *Tuning formulas for the Proposed Fitting Model of frequency ratio f^{opt}.*

Loading	$\tilde{f}^{opt}$	R-square
Harmonic Force	$1-\sqrt{3\,\mu}\left(\frac{1}{2}\sqrt{\mu}+\zeta_S\right)$	0.9916
Harmonic Acceleration	$1-\sqrt{3\,\mu}\left(\frac{2}{3}\sqrt{\mu}+\frac{3}{2}\zeta_S\right)$	0.9947
White Noise Force	$1-\sqrt{3\,\mu}\left(\frac{2}{5}\sqrt{\mu}+\frac{1}{4}\zeta_S\right)$	0.9974
White Noise Acceleration	$1-\sqrt{3\,\mu}\left(\frac{2}{3}\sqrt{\mu}+\frac{3}{2}\zeta_S\right)$	0.9924

Based on such an experience, fitting formulas may be further refined by taking into account, as basis, the tuning formulas for undamped structures ($\zeta_S = 0$) listed in Table 3.3. This allows to recover them exactly, when structural damping ratio ζ_S is set to zero. Namely, by combining formulas in Table 3.3 and fittings originated from the analytical model in Eq. (3.5), one may re-state the fitting as:

$$\tilde{f}^{opt} = f^{opt}_{ref(\zeta_S=0)} \cdot \left(a - b\,\mu^e - c\,\mu^f\,\zeta_S^g - d\,\zeta_S^h\right) . \tag{3.6}$$

Best fitting on this further interpolation model leads then to results presented in Tables 3.7–3.8 below, which are homologous to those derived earlier (Tables 3.5–3.6). The final Literature-based Fitting Model tuning proposal in Table 3.8 unifies again cases HA and WNA.

Table 3.7: *Optimum coefficients for the fitting model of frequency ratio f^{opt} in Eq.* (3.6).

Loading	a	b	c	d	e	f	g	h	R-square
HF	1.0000	0.003704	1.229	1.181	9.662	0.4632	0.9916	2.106	1.0000
HA	1.0000	0.2065	2.262	1.260	3.516	0.4989	1.003	1.987	1.0000
WNF	1.0000	0.006310	0.3795	0.1529	9.208	0.4439	0.9875	4.370	1.0000
WNA	1.0000	10.0000	2.456	1.780	5.687	0.5041	0.9943	1.947	1.0000

Table 3.8: *Tuning formulas for the Literature-based Fitting Model of frequency ratio f^{opt}.*

Loading	$\tilde{f}^{opt}$	R-square
Harmonic Force	$\frac{1}{1+\mu}\left(1-\sqrt{3\,\mu}\,\zeta_S\right)$	0.9969
Harmonic Acceleration	$\frac{1}{1+\mu}\sqrt{\frac{2-\mu}{2}}\left(1-\frac{3}{2}\sqrt{3\,\mu}\,\zeta_S\right)$	0.9988
White Noise Force	$\frac{1}{1+\mu}\sqrt{\frac{2+\mu}{2}}\left(1-\frac{1}{4}\sqrt{3\,\mu}\,\zeta_S\right)$	0.9998
White Noise Acceleration	$\frac{1}{1+\mu}\sqrt{\frac{2-\mu}{2}}\left(1-\frac{3}{2}\sqrt{3\,\mu}\,\zeta_S\right)$	0.9943

3.4.4 TMD damping ratio

Easier interpretation is achieved for possible analytical fittings $\tilde{\zeta}_T^{opt}$ of the optimal TMD damping ratio ζ_T^{opt}, which are approached again in two ways. Following what stated above for $\tilde{f}^{opt}$ in Eq. (3.5), a similar fitting model is attempted:

$$\tilde{\zeta}_T^{opt} = a + b\,\mu^e + c\,\mu^f\,\zeta_S^g + d\,\zeta_S^h\,, \tag{3.7}$$

where the signs take into account the trends in Fig. 3.6. Table 3.9 reports the results of a first estimate of the model coefficients, which leads to the following observations. First, for all the loading cases, parameter a takes values near 0 (meaning that an optimal TMD should be lightly damped for small μ, ζ_S), while coefficient e approaches 1/2; parameter b results slightly lower than 3/5 and 1/2, respectively for Harmonic and White Noise loadings; the values assumed by coefficients c, d, f, g, h seem to point out that the contribution of ζ_S may be negligible overall.

Table 3.9: *Optimum coefficients for the fitting model of TMD damping ratio ζ_T^{opt} in Eq. (3.7).*

Loading	a	b	c	d	e	f	g	h	R-square
HF	−0.01166	0.539	0.01705	0.1024	0.4371	0.202	1.045	0.9029	1.0000
HA	−0.001	0.5734	0.2994	0.001005	0.4795	0.1832	0.9973	10	1.0000
WNF	−0.005521	0.4534	0.03684	0.08062	0.4572	4.999	5.818	8.749	1.0000
WNA	−0.005614	0.4548	0.1584	0.001293	0.4579	1.245	1.485	9.587	1.0000

Table 3.10 reports the final obtained PFM tuning formulas for TMD damping ratio ζ_T^{opt}, based on square-root dependencies on μ similar to those for $\tilde{f}^{opt}$, according to the following considerations. First, ζ_T^{opt} is weakly influenced by structural damping ratio ζ_S, basically just by mass ratio μ, especially for the case of White Noise loading. Instead, a slight dependence on ζ_S is displayed in the case of Harmonic loading. Second, the reported tuning formulas are quite simple and display at the same time high accuracy in the considered ranges. Third, the four loading cases are unified by two common formulas, differing just by a single coefficient (besides for an additional term related to ζ_S) and coupled in two by the type of acting loading (Harmonic vs. White Noise).

Table 3.10: *Tuning formulas for the Proposed Fitting Model of TMD damping ratio ζ_T^{opt}.*

Loading	$\tilde{\zeta}_T^{opt}$	R-square
Harmonic Force	$\frac{3}{5}\sqrt{\mu}+\frac{1}{6}\zeta_S$	0.9947
Harmonic Acceleration	$\frac{3}{5}\sqrt{\mu}+\frac{1}{6}\zeta_S$	0.9987
White Noise Force	$\frac{1}{2}\sqrt{\mu}$	0.9920
White Noise Acceleration	$\frac{1}{2}\sqrt{\mu}$	0.9928

Also for TMD damping ratio ζ_T^{opt}, a literature-based model may be further provided, as reported in Eq. (3.8) below, whose optimal coefficients and final obtained tuning formulas are respectively gathered in Tables 3.11–3.12, respectively:

$$\tilde{\zeta}_T^{opt} = \zeta_{T,\,ref(\zeta_S=0)}^{opt} \cdot \left(a + b\,\mu^e + c\,\mu^f\,\zeta_S^g + d\,\zeta_S^h\right). \tag{3.8}$$

Such LFM tuning formulas in Table 3.12 confirm the quite independence of ζ_T^{opt} on ζ_S (in fact, just a slight contribution occurs in the case of Harmonic loading). Therefore, the formulas valid for the undamped case have proved to provide good predictions also for the case of damped primary structures.

Table 3.11: *Optimum coefficients for the fitting model of TMD damping ratio ζ_T^{opt} in Eq. (3.8).*

Loading	a	b	c	d	e	f	g	h	R-square
HF	1.002	0.2549	0.1614	0.6359	10	8.196	8.12	0.8518	0.9988
HA	0.9972	0.2419	0.3477	1.079	9.344	5.298	7.73	0.9602	0.9987
WNF	1.000	0.006248	0.02605	0.1006	1.808	7.893	7.901	8.833	1.0000
WNA	1.000	0.001	1.140	1.339	1.757	1.457	1.238	4.560	1.0000

Table 3.12: *Tuning formulas for the Literature-based Fitting Model of TMD damping ratio ζ_T^{opt}.*

Loading	$\tilde{\zeta}_T^{opt}$	R-square
Harmonic Force	$\sqrt{\dfrac{3}{8}\dfrac{\mu}{1+\mu}}\,(1+\zeta_S)$	0.9985
Harmonic Acceleration	$\sqrt{\dfrac{3\mu}{4(1+\mu)(2-\mu)}}\,(1+\zeta_S)$	0.9981
White Noise Force	$\sqrt{\dfrac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}$	1.0000
White Noise Acceleration	$\sqrt{\dfrac{\mu(4-\mu)}{8(1+\mu)(2-\mu)}}$	1.0000

3.4.5 Considerations on the fitting results

Globally, for both PFM and LFM fitting models the tuning proposals are characterised by a good matching of the results from numerical optimisation, together with quite a low level of complexity. Also, a unified way of tuning is foreseen, by switching from the various loading cases through changes of few coefficients. Main results are condensed in Tables 3.6 ($\tilde{f}^{opt}$) and 3.10 ($\tilde{\zeta}_T^{opt}$) for the PFM proposal and in Tables 3.8 ($\tilde{f}^{opt}$) and 3.12 ($\tilde{\zeta}_T^{opt}$) for the LFM proposal.

Particularly, the PFM proposal clearly shows that the optimal TMD parameters are matched by quite simple relations. On the other hand, the LFM proposal allows to match, with a simple additional term, the optimum results with good accuracy, referring also to the case of damped primary structures. The remarks pointed out above are valid specifically for TMD damping ratio ζ_T^{opt}, with optimum values that can be obtained by very simple formulas. Moreover, an interesting consideration arises for $\tilde{\zeta}_T^{opt}$ from the PFM tuning formulas for the cases of White Noise loadings (Table 3.10), since they confirm those obtained, with a different approach, by Krenk and Høgsberg [50].

Specifically, identical fittings are proposed, in couples, for $\tilde{f}^{opt}$ in HA and WNA cases, and for $\tilde{\zeta}_T^{opt}$ in HF and HA cases and in WNF and WNA cases. Thus, while for $\tilde{f}^{opt}$, formulas rather group for the point of application of the loading action (Force vs. Acceleration), for $\tilde{\zeta}_T^{opt}$ they rather group for the type of loading (Harmonic vs. White Noise), see trends in Figs. 3.5–3.6.

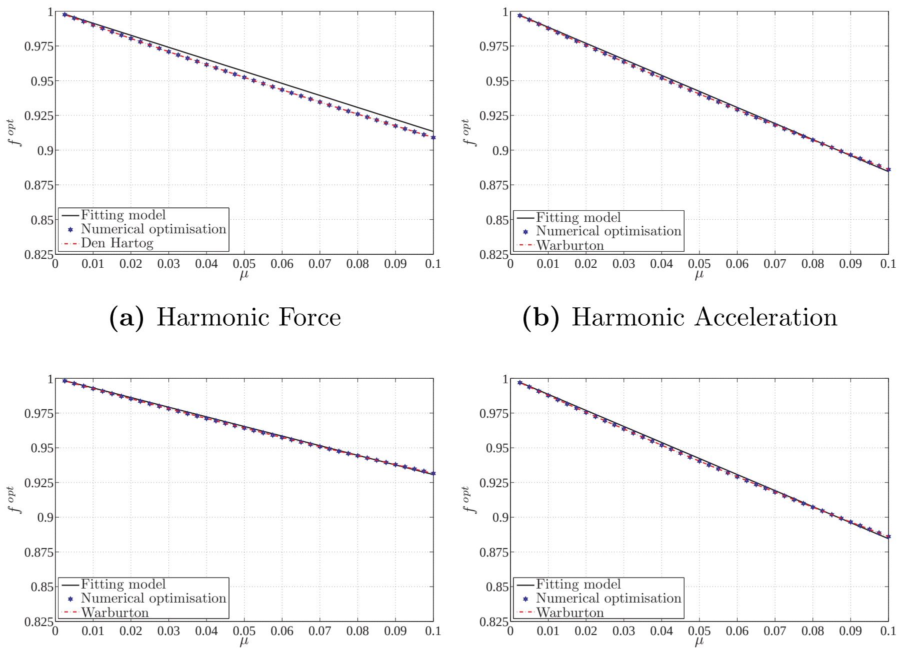
Obviously, validity and accuracy of the various tunings are attached to the assumed range of free variables ($0 < \mu \leq 0.1$, $0 \leq \zeta_S \leq 0.1$). However, for different ranges of μ and ζ_S, it would be possible to adjust the calibration coefficients, within the same proposed fitting models, by keeping reasonable levels of accuracy in the achieved predictions.

3.5 Comparisons to the tuning literature

In this section, the Proposed Fitting Model (Table 3.6 for $\tilde{f}^{opt}$, Table 3.10 for $\tilde{\zeta}_T^{opt}$), has been inspected and validated through a series of line plots, reported in the following. They concern both the optimum TMD parameters f^{opt}, ζ_T^{opt} and the optimised response functions of the primary structure (Tables 3.1–3.2), as a function of mass ratio μ. Two cases have been reported here, i.e. those of undamped ($\zeta_S = 0$) and damped ($\zeta_S = 0.05$) primary structures. The literature formulas adopted for comparison purpose are those in Table 3.3, for the case of undamped primary structures, or come from additional literature works [7, 11, 41, 50, 53, 90, 116, 117], for the case of damped main structures. Results are reported in following Figs. 3.7–3.12.

First, the case of undamped primary structures ($\zeta_S = 0$) is considered and represented in Figs. 3.7, 3.8 and 3.11, respectively for optimum frequency ratio f^{opt}, TMD damping ratio ζ_T^{opt} and corresponding structural response indices R, N. Despite that the tuning formulas from the literature display clear nonlinear trends on f^{opt}, Fig. 3.7 shows that the trends of frequency ratio f^{opt} may be considered almost linear, at least in the considered range of μ. This remark supports the validity of the proposed fitting formulas (Table 3.6), which reduce to linear functions in the case of undamped main structures. Except for the fitting case of HF loading, which shows little discrepancy with respect to classical Den Hartog's formula $f^{opt} = 1/(1 + \mu)$, all the other fitting cases point out a good agreement between the proposed and the corresponding literature formulas (Fig. 3.7). For TMD damping ratio ζ_T^{opt} (Fig. 3.8), a similar situation may be noticed. A general correspondence with output from literature formulas is achieved, particularly for the case of HA, where an accurate matching is recovered. Also, Fig. 3.11 shows a very good agreement among all represented trends and supports a high effectiveness of the proposed TMD in reducing the primary structure response.

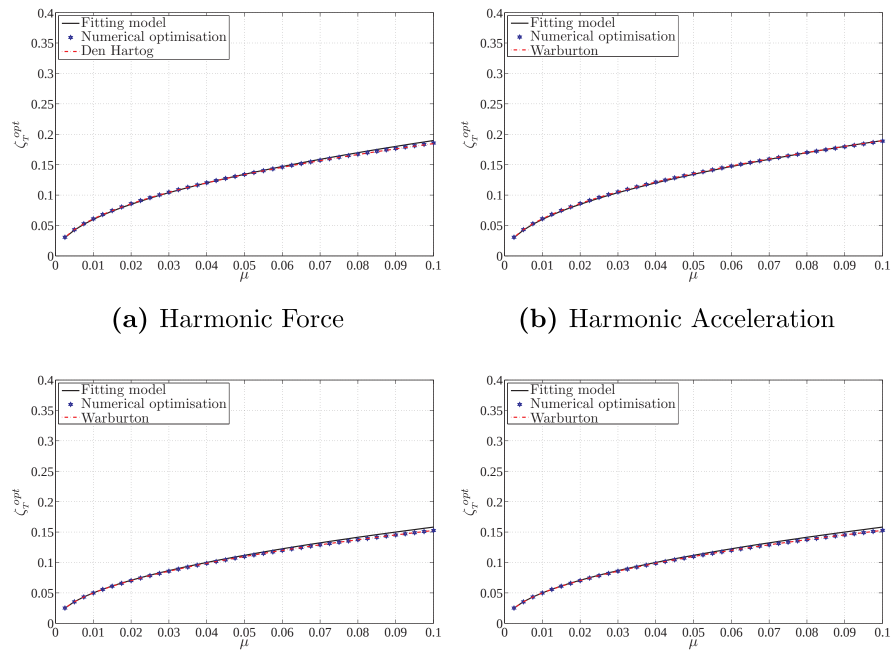
From Figs. 3.9, 3.10 and 3.12, representative of the case of a damped primary structure ($\zeta_S = 0.05$), important considerations may be noted. First, a higher spread of the various trends is generally obtained, with respect to those of the undamped case, most of all for frequency ratio f^{opt}. In this sense, the proposed tuning seems to imply lower trends, except for the WNF case. On the other hand, the proposed TMD damping ratio ζ_T^{opt} appears to take almost the same values as those from most of the literature formulas, except for the case of Sadek et al. [90] formula, which leads (intentionally) to quite higher values of ζ_T^{opt}. Finally, important observations arise from frequency response functions R, N in Fig. 3.12, depicting the achieved optimum response of the damped structure. Indeed, it may be noted that the proposed tuning formulas enable to achieve the most effective Tuned Mass Damper, for all the four considered loading cases.



(a) Harmonic Force **(b)** Harmonic Acceleration

(c) White Noise Force **(d)** White Noise Acceleration

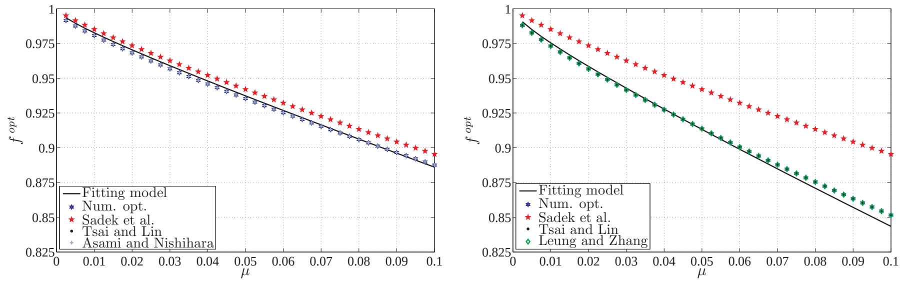
Figure 3.7: *Optimum frequency ratio f^{opt} in the case of undamped primary structure ($\zeta_S = 0$) for the considered four loading cases, compared to results from the PFM proposal (Table 3.6) and from tuning formulas in the literature.*



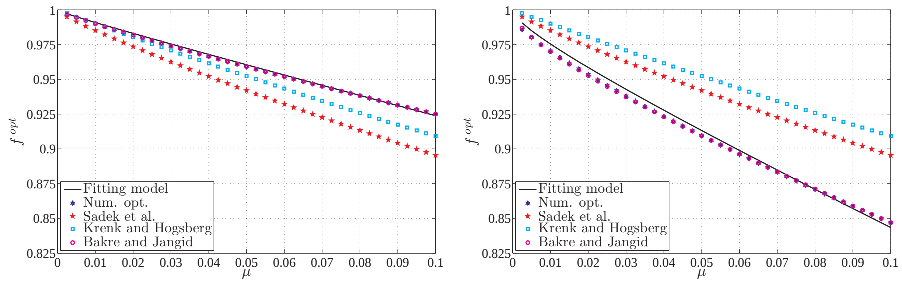
(a) Harmonic Force **(b)** Harmonic Acceleration

(c) White Noise Force **(d)** White Noise Acceleration

Figure 3.8: *Optimum TMD damping ratio ζ_T^{opt} in the case of undamped primary structure ($\zeta_S = 0$) for the considered four loading cases, compared to results from the PFM proposal (Table 3.10) and from tuning formulas in the literature.*

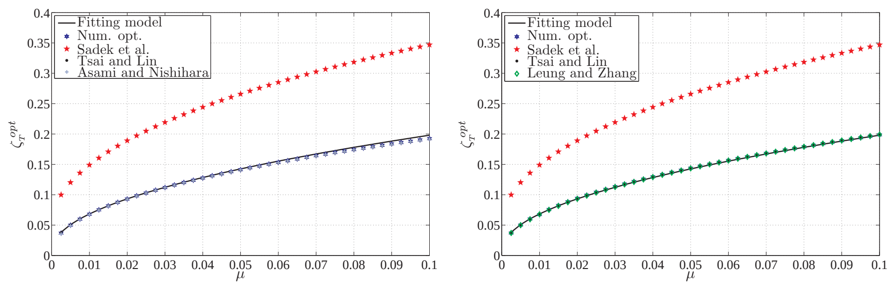


(a) Harmonic Force **(b)** Harmonic Acceleration

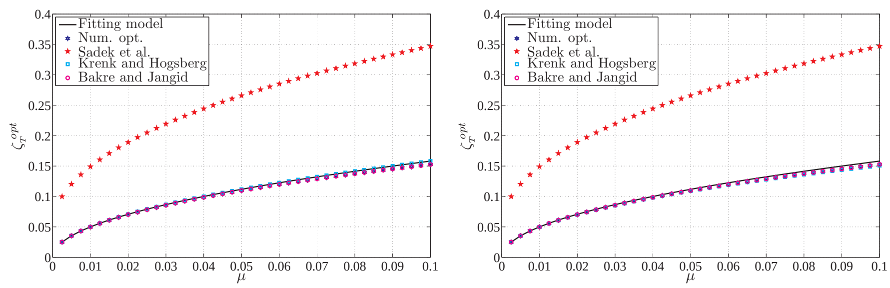


(c) White Noise Force **(d)** White Noise Acceleration

Figure 3.9: *Optimum frequency ratio f^{opt} in the case of damped primary structure ($\zeta_S = 0.05$) for the considered four loading cases, compared to results from the PFM proposal (Table 3.6) and from tuning formulas in the literature.*

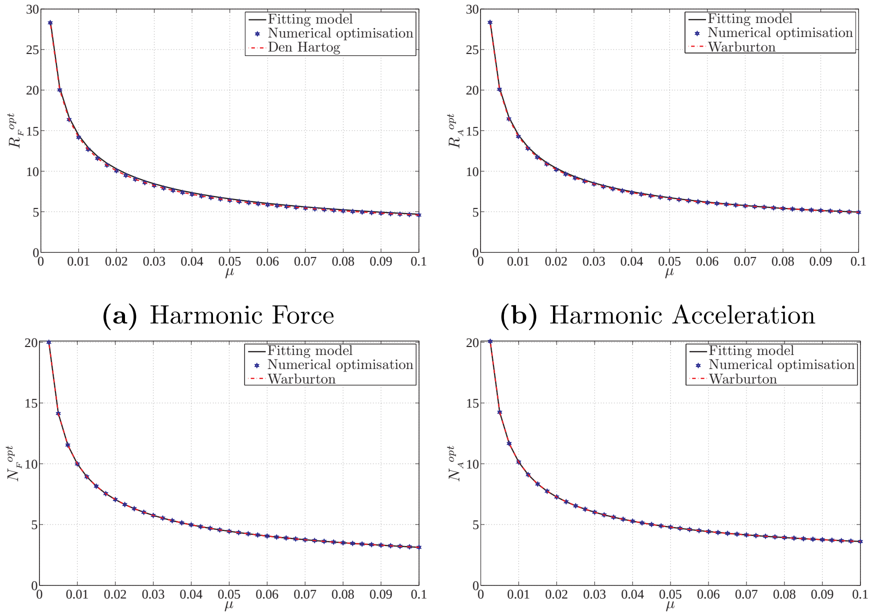


(a) Harmonic Force **(b)** Harmonic Acceleration



(c) White Noise Force **(d)** White Noise Acceleration

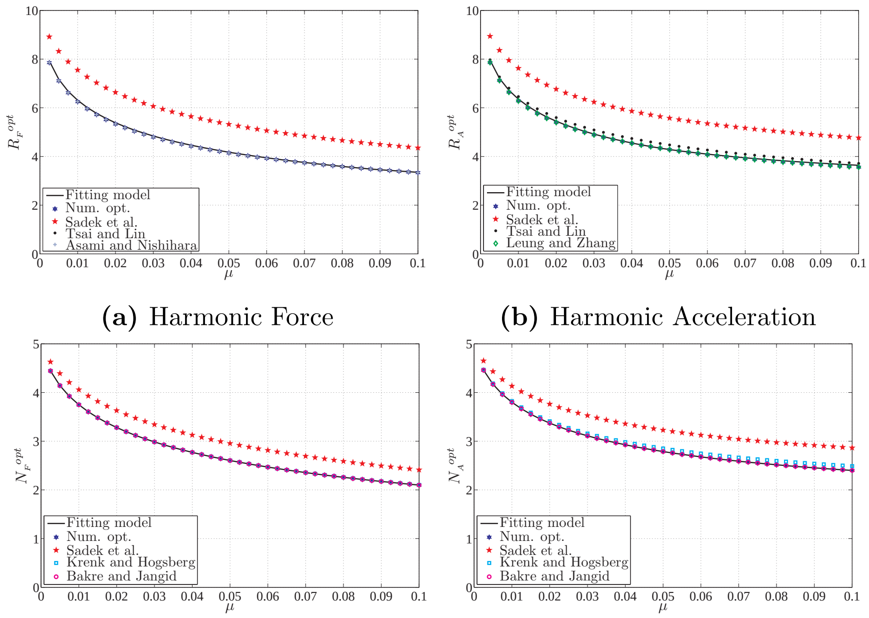
Figure 3.10: *Optimum TMD damping ratio ζ_T^{opt} in the case of damped primary structure ($\zeta_S = 0.05$) for the considered four loading cases, compared to results from the PFM proposal (Table 3.10) and from tuning formulas in the literature.*



(a) Harmonic Force **(b)** Harmonic Acceleration

(c) White Noise Force **(d)** White Noise Acceleration

Figure 3.11: *Maximum response displacement of the primary structure in the case of undamped primary structure ($\zeta_S = 0$) for the considered four loading cases, compared to results from the PFM proposal (Tables 3.6 and 3.10) and from tuning formulas in the literature.*



(a) Harmonic Force **(b)** Harmonic Acceleration

(c) White Noise Force **(d)** White Noise Acceleration

Figure 3.12: *Maximum response displacement of the primary structure in the case of damped primary structure ($\zeta_S = 0.05$) for the considered four loading cases, compared to results from the PFM proposal (Tables 3.6 and 3.10) and from tuning formulas in the literature.*

Chapter 4

Optimum passive and hybrid TMDs for impulse loading

4.1 Introduction

Structural systems can be easily subjected to a wide range of harmful dynamical actions of a different nature, especially from the point of view of duration and intensity. Within this context, the reduction and the control of the dynamic response due to impulse loading is doubtlessly an important research topic, mostly for its potential contribution in several engineering applications, such as those in earthquake engineering and in the automotive field.

In this sense, the passive Tuned Mass Damper is generally considered as not significantly effective in reducing the structural response [1]. However, it appears from the literature that this field has not been thoroughly investigated yet.

The contents presented in this chapter deepen this framework and deal with the study of the optimal tuning of a Tuned Mass Damper when the structural system is subjected to shock excitation. A structural system composed of a damped SDOF primary structure and a TMD added on it, and subjected to unit impulse acting as base displacement has been considered.

First, the structural context and the related dynamic response are explained in detail, then the numerical optimisation of the passive Tuned Mass Damper is developed, showing the potential application benefits. The hybrid configuration of the

TMD is further considered, by the introduction of a feedback closed loop controller between the primary structure and the TMD, in order to investigate the possible improvement in terms of efficiency in reducing the dynamic response of the primary structure with respect to the case of a passive TMD. The optimisation process has been studied in detail, and a comprehensive overview of the obtained results, in terms of optimum parameters of the Tuned Mass Damper and the corresponding obtained reduction of the dynamic response, have been presented, together with first significant remarks for the design of TMDs within the considered framework.

4.2 Passive TMD

4.2.1 SDOF primary structure

A SDOF system is considered, initially at rest and subjected to unit impulse excitation in $t = 0$, which may be ideally defined by a Dirac delta function $\delta(t)$ [81, 85]:

$$\delta(t) = \begin{cases} \infty\,, & t = 0 \\ 0\,, & \text{elsewhere} \end{cases}\,, \qquad \int_{-\infty}^{+\infty} \delta(t)\mathrm{d}t = 1\,. \tag{4.1}$$

Particularly, such impulsive excitation has been considered acting here as base displacement $x_g(t)$ (Fig. 4.1):

$$x_g(t) = X_g\delta(t)\,, \tag{4.2}$$

where $X_g = 0.01$ m denotes the assumed constant amplitude of the excitation.

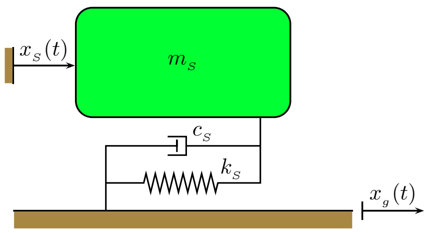


Figure 4.1: *Structural parameters and absolute dynamic degrees of freedom of the SDOF primary structure (S), subjected to generic base displacement $x_g(t)$.*

The mechanical parameters of such system are: the mass m_S, the constant stiffness k_S and the viscous damping coefficient c_S. The natural angular frequency ω_S

and damping ratio ζ_S are classically defined as follows:

$$\omega_S = \sqrt{\frac{k_S}{m_S}}\,, \qquad \zeta_S = \frac{c_S}{c_{S,cr}} = \frac{c_S}{2\sqrt{k_S m_S}} = \frac{c_S}{2\,\omega_S m_S}\,. \tag{4.3}$$

The dynamic behaviour of this structure is ruled by the following equation of motion:

$$m_S \ddot{x}_S(t) + c_S \dot{x}_S(t) + k_S x_S(t) = c_S X_g \dot{\delta}(t) + k_S X_g \delta(t)\,. \tag{4.4}$$

The time response of the primary structure is obtained through a pair of Laplace Transforms [85]. First, Eq. (4.4) is transformed in the Laplace variable s, with homogeneous conditions, consistently with the hypothesis of system initially at rest:

$$[s^2 m_S + s c_S + k_S] X_S(s) = s c_S X_g + k_S X_g \tag{4.5}$$

or, written in symbolic form:

$$Z_S(s) X_S(s) = F_S(s)\,, \tag{4.6}$$

where $Z_S(s)$ is the system impedance, $X_S(s)$ is the degree of freedom and $F_S(s)$ is the transform of dynamic excitation. Thus, the transform of the dynamic response of the primary structure, in terms of displacement as a function of s can be expressed as:

$$X_S(s) = Z_S(s)^{-1} F_S(s) = H_S(s) F_S(s)\,, \tag{4.7}$$

where $H_S(s) = Z_S(s)^{-1}$ is the receptance (or transfer function) of the system. Hence, one obtains:

$$X_S(s) = \frac{s c_S + k_S}{s^2 m_S + s c_S + k_S}\,. \tag{4.8}$$

The inverse Laplace transform of this relation provides, after some algebra, the following analytical expression of the time displacement of the SDOF primary structure:

$$\begin{aligned} x_S(t) =& \frac{1}{m_S\sqrt{4 m_S k_S - c_S^2}} \mathrm{e}^{-\frac{c_S}{2 m_S} t} \cdot \\ & \cdot \left[(2 m_S k_S - c_S^2) \sin\left(\frac{\sqrt{4 m_S k_S - c_S^2}}{2 m_S} t \right) + \right. \\ & \left. + c_S \sqrt{4 m_S k_S - c_S^2} \cos\left(\frac{\sqrt{4 m_S k_S - c_S^2}}{2 m_S} t \right) \right]. \end{aligned} \tag{4.9}$$

The first derivative of Eq. (4.9) takes the form:

$$
\begin{aligned}
\dot{x}_s(t) = & \frac{1}{m_s^2\sqrt{4m_sk_s - c_s^2}} \mathrm{e}^{-\frac{c_s}{2m_s}t} \left[c_s(c_s^2 - 3m_sk_s)\sin\left(\frac{\sqrt{4m_sk_s - c_s^2}}{2m_s}t\right) + \right. \\
& \left. + (m_sk_s - c_s^2)\sqrt{4m_sk_s - c_s^2}\cos\left(\frac{\sqrt{4m_sk_s - c_s^2}}{2m_s}t\right)\right].
\end{aligned}
\tag{4.10}
$$

By setting this derivative equal to zero, such stationary condition marks the peak time $\bar{t}$:

$$
\bar{t} = \frac{2m_s}{\sqrt{4m_sk_s - c_s^2}} \arctan\left(\frac{(c_s^2 - m_sk_s)\sqrt{4m_sk_s - c_s^2}}{c_s^3 - 3m_sc_sk_s}\right). \tag{4.11}
$$

The substitution of Eq. (4.11) into Eq. (4.9) provides the expression of the peak displacement:

$$
\begin{aligned}
\|x_s(t)\|_\infty = & \frac{1}{m_s\sqrt{4m_sk_s - c_s^2}} \mathrm{e}^{-\frac{c_s}{2m_s}\bar{t}} \cdot \\
& \cdot \left[(2m_sk_s - c_s^2)\sin\left(\frac{\sqrt{4m_sk_s - c_s^2}}{2m_s}\bar{t}\right) + \right. \\
& \left. + c_s\sqrt{4m_sk_s - c_s^2}\cos\left(\frac{\sqrt{4m_sk_s - c_s^2}}{2m_s}\bar{t}\right)\right] = \\
= & \sqrt{\frac{k_s}{m_s}} \frac{\sqrt{(c_s^2 - 3m_sk_s)^2}}{3m_sk_s - c_s^2} \cdot \\
& \cdot \mathrm{e}^{-\frac{c_s}{\sqrt{4m_s\,k_s - c_s^2}}\arctan\left(\frac{(c_s^2 - m_sk_s)\sqrt{4m_s\,k_s - c_s^2}}{c_s(c_s^2 - 3m_sk_s)}\right)},
\end{aligned}
\tag{4.12}
$$

which will play an important role in the following tuning process, since the magnitude of the peak displacement is one of the main objectives within the control of the transient system response.

4.2.2 Structural system with TMD

The structural system composed of the SDOF primary structure and of a passive TMD added on it, assumed as benchmark model in the present study, is represented in Fig. 4.2.

Similarly to the primary structure, the TMD parameters are the mass m_T, the constant stiffness k_T and the viscous damping coefficient c_T, while the relevant natural angular frequency ω_T and damping ratio ζ_T are consistently defined as follows:

$$\omega_T = \sqrt{\frac{k_T}{m_T}}\,, \qquad \zeta_T = \frac{c_T}{c_{2,cr}} = \frac{c_T}{2\sqrt{k_T m_T}} = \frac{c_T}{2\,\omega_T m_T}\,. \tag{4.13}$$

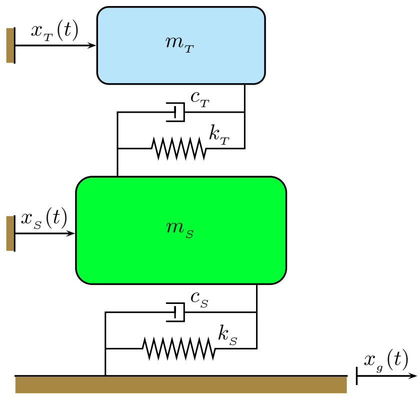


Figure 4.2: *Structural parameters and absolute dynamic degrees of freedom of a 2DOF mechanical system comprised of a SDOF primary structure (S) equipped with an added passive TMD (T), subjected to generic base displacement $x_g(t)$.*

Further two parameters are introduced for the tuning purposes, i.e. the mass ratio μ and the frequency ratio f:

$$\mu = \frac{m_T}{m_S}\,, \qquad f = \frac{\omega_T}{\omega_S}\,. \tag{4.14}$$

The equations of motion of the considered 2DOF linear structural system can be stated in matrix form as follows:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t)\,, \tag{4.15}$$

where:

$$\mathbf{M} = \begin{bmatrix} m_S & 0 \\ 0 & m_T \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_S + c_T & -c_T \\ -c_T & c_T \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_S + k_T & -k_T \\ -k_T & k_T \end{bmatrix} \tag{4.16}$$

denote the structural matrices relevant to mass, viscous damping and elastic stiffness, respectively. The vectors:

$$\mathbf{x}(t) = \begin{bmatrix} x_S(t) \\ x_T(t) \end{bmatrix}, \qquad \dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_S(t) \\ \dot{x}_T(t) \end{bmatrix}, \qquad \ddot{\mathbf{x}}(t) = \begin{bmatrix} \ddot{x}_S(t) \\ \ddot{x}_T(t) \end{bmatrix} \tag{4.17}$$

represent the dynamic response of the structural system, in terms of displacements, velocities and accelerations, respectively.

The vector $\mathbf{F}(t)$, which represents the dynamic excitation, for the case of unit impulse base displacement takes the form:

$$\mathbf{F}(t) = \begin{bmatrix} c_S \dot{x}_g(t) + k_S x_g(t) \\ 0 \end{bmatrix} = \begin{bmatrix} c_S X_g \dot{\delta}(t) + k_S X_g \delta(t) \\ 0 \end{bmatrix} . \tag{4.18}$$

The equations of motion can be rewritten also in terms of relative coordinates $w_i = x_i - x_g$ [85]:

$$\mathbf{M}\ddot{\mathbf{w}}(t) + \mathbf{C}\dot{\mathbf{w}}(t) + \mathbf{K}\mathbf{w}(t) = \mathbf{F}(t) \,, \tag{4.19}$$

where:

$$\mathbf{w}(t) = \begin{bmatrix} w_S(t) \\ w_T(t) \end{bmatrix} , \qquad \dot{\mathbf{w}}(t) = \begin{bmatrix} \dot{w}_S(t) \\ \dot{w}_T(t) \end{bmatrix} , \qquad \ddot{\mathbf{w}}(t) = \begin{bmatrix} \ddot{w}_S(t) \\ \ddot{w}_T(t) \end{bmatrix} , \tag{4.20}$$

and the relative excitation vector takes the following form:

$$\mathbf{F}(t) = - \begin{bmatrix} m_S \\ m_T \end{bmatrix} \ddot{x}_g(t) = - \begin{bmatrix} m_S \\ m_T \end{bmatrix} X_g \ddot{\delta}(t) \,. \tag{4.21}$$

As for the SDOF system, the response of the considered structural system in the time domain is obtained by a pair of Laplace transforms [85]. Firstly, the equations of motion are Laplace transformed:

$$\begin{bmatrix} Z_{SS}(s) & Z_{ST}(s) \\ Z_{TS}(s) & Z_{TT}(s) \end{bmatrix} \begin{bmatrix} X_S(s) \\ X_T(s) \end{bmatrix} = \begin{bmatrix} F_S(s) \\ F_T(s) \end{bmatrix} , \tag{4.22}$$

or, in compact form:

$$\mathbf{Z}(s)\mathbf{X}(s) = \mathbf{F}(s) \,, \tag{4.23}$$

where $\mathbf{Z}(s)$ is the impedance matrix, $\mathbf{X}(s)$ is the degrees of freedom vector and $\mathbf{F}(s)$ is the excitation vector. In particular, the impedance matrix $\mathbf{Z}(s)$ takes the following form:

$$\begin{aligned} \mathbf{Z}(s) &= \begin{bmatrix} Z_{SS}(s) & Z_{ST}(s) \\ Z_{TS}(s) & Z_{TT}(s) \end{bmatrix} = \\ &= \begin{bmatrix} s^2 m_S + s(c_S + c_T) + (k_S + k_T) & -s\, c_T - k_T \\ -s\, c_T - k_T & s^2 m_T + s\, c_T + k_T \end{bmatrix} , \end{aligned} \tag{4.24}$$

and the force vector:

$$\begin{bmatrix} F_S(s) \\ F_T(s) \end{bmatrix} = \begin{bmatrix} s\,c_S X_g + k_S X_g \\ 0 \end{bmatrix} . \tag{4.25}$$

Then, the dynamic response of the structural system in terms of displacement as a function of s can be obtained by just an algebraic manipulation of Eq. (4.23):

$$\mathbf{X}(s) = \mathbf{Z}(s)^{-1}\mathbf{F}(s) = \mathbf{H}(s)\mathbf{F}(s)\,, \tag{4.26}$$

where $\mathbf{H}(s) = \mathbf{Z}(s)^{-1}$ is the receptance matrix of the system:

$$\begin{aligned} \mathbf{H}(s) = &\begin{bmatrix} H_{SS}(s) & H_{ST}(s) \\ H_{TS}(s) & H_{TT}(s) \end{bmatrix} = \\ =&\frac{1}{D(s)}\begin{bmatrix} s^2 m_T + s\,c_T + k_T & s\,c_T + k_T \\ s\,c_T + k_T & s^2 m_S + s(c_S + c_T) + (k_S + k_T) \end{bmatrix}, \end{aligned} \tag{4.27}$$

where:

$$\begin{aligned} D(s) =& \det(\mathbf{Z}(s)) = \\ =& s^4(m_S m_T) + s^3(c_S m_T + c_T m_S + c_T m_T) + \\ &+ s^2(c_S c_T + k_T m_S + k_S m_T + k_T m_T) + \\ &+ s(c_S k_T + c_T k_S) + k_S k_T\,. \end{aligned} \tag{4.28}$$

Hence, one obtains:

$$\begin{aligned} X_S(s) &= H_{SS}(s)F_S(s) + H_{ST}(s)F_T(s) = \frac{N_S(s)}{D(s)}\,, \\ X_T(s) &= H_{TS}(s)F_S(s) + H_{TT}(s)F_T(s) = \frac{N_T(s)}{D(s)}\,, \end{aligned} \tag{4.29}$$

where:

$$\begin{aligned} N_S(s) =& s^2\big(m_T F_S(s)\big) + s\big(c_T F_S(s) + c_T F_T(s)\big) + k_T F_S(s) + k_T F_T(s)\,, \\ N_T(s) =& s^2\big(m_S F_T(s)\big) + s\big(c_T F_S(s) + c_S F_T(s) + c_T F_T(s)\big) + k_T F_S(s) + \\ &+ k_S F_T(s) + k_T F_T(s)\,. \end{aligned} \tag{4.30}$$

Finally, the transfer function relevant to the i-th degree of freedom can suitably be expressed in the following simplified form, based on a partial fraction expansion [74]:

$$X_i(s) = G_p \sum_{n=1}^{N} \frac{R_n}{s - p_n}\,, \tag{4.31}$$

where G_p is a constant gain factor, and the time-invariant amplitude of the input signal R_n is a n-th constant called residue and p_n is the n-th root of the denominator $D(s)$, also called pole of the system. Hence, the total number of poles N corresponds to the degree of the denominator $D(s)$, which is of the fourth order here. Such an expression for the transfer function is quite useful in view of the inverse Laplace transform, which returns the response in the time domain:

$$x_i(t) = G_p \sum_{n=1}^{N} R_n \mathrm{e}^{p_n\, t}\,. \tag{4.32}$$

Despite that it could be possible, in principle, to derive the analytical expressions for the residues, even for a relatively simple system as that assumed in this study, such analytical expressions take quite complex and lengthy forms. Hence, the residues will always be evaluated numerically here.

At the same time, the poles could be evaluated analytically if the degree of the denominator is lower than five (Abel-Ruffini theorem), as it is for the case of the transfer function involved in this study (fourth order denominator). However, such analytical expressions are not strictly necessary for the present study, thus the evaluation of the poles will be carried out numerically as well here.

4.2.3 Tuning process

In this section, the optimum tuning process of the free parameters of the Tuned Mass Damper is explored. In order to develop a first investigation and to outline a general method for the optimum design of the TMD for the cases of pulse loading, here the optimisation aims only at reducing the dynamic response in terms of the primary structure displacement. Thus, some additional features, such as the limitation of the stroke of the TMD (which may display significant relevance in practical cases), will not be considered, although it could be possible to implement it in further actions of the optimisation process, through appropriate bounds on the tuning variables.

The present TMD tuning process can be interpreted and managed as a classical optimisation procedure, where the objective (or cost) function is assumed to be a quantity representative of the dynamic response, either in the time or in the frequency domain, which is supposed to be minimised [87, 91–93], as previously explained in Chapter 3:

$$\min_{\mathbf{p}} \mathrm{J}(\mathbf{p}), \qquad \mathbf{l}_b \leq \mathbf{p} \leq \mathbf{u}_b, \tag{4.33}$$

where $\mathbf{p}$, $\mathrm{J}(\mathbf{p})$, $\mathbf{l}_b$ and $\mathbf{u}_b$ represent the optimisation variables, the objective function, the lower and the upper bounds on the optimisation variables, respectively. In the present context, following the usual approach in the literature [7, 11, 26, 41, 50, 53, 83, 84, 126], only the frequency ratio f and the TMD damping ratio ζ_T are taken as optimisation variables, while the mass ratio μ is assumed to be given from scratch (two-variables optimisation).

This approach is motivated by two main reasons. First, the amount of added mass which composes the Tuned Mass Damper is limited by a matter of practical design, i.e. excessive masses of the device could be counterproductive in terms of suitability and safety of the primary structure. The second issue, which is instead related to rather theoretical aspects, is pointed out by the optimisation process itself. Indeed, several previous trials [87, 91–95] have shown that, in case of free mass ratio, this parameter tends to reach the upper limit that has been set, leading to a lower TMD damping ratio, so that to transfer the dynamic response entirely to the TMD. In this sense, a suitable upper limit to design guidelines could be established within the optimisation process, which will likely correspond to the assumed value of μ in the optimisation process.

The characteristics of the considered objective function play a fundamental role for the real effectiveness of the optimisation process. Above all, despite that a relatively simple system has been assumed here, namely a 2DOF structure, the analytical expressions of its dynamic response take quickly a complex form, which lead to consider a numerical optimisation method as a suitable mean to develop the tuning process.

Another important issue, within the tuning process, as inspected in Chapter 3, regards the choice of the optimisation algorithm. In this sense, despite that a large series of modern algorithms could assure high optimisation performances (see e.g. [53]), a classical numerical method looks sufficient for the solution of the present tuning problem. Indeed, the computational time required by the optimisation process has been noted to be quite short, probably because of the simplicity of the structural system and the deterministic nature of the dynamic excitation. In particular, a classical nonlinear gradient-based optimisation algorithm available within MATLAB [109] has been adopted, which is based on Sequential Quadratic Programming (SQP). Further details on this framework are provided in [92].

All the parameters involved in the optimization process have been gathered in Table 4.1 below. The ranges of values of the mass ratio μ and the TMD damping ratio ζ_T have been chosen so as to be similar to those typically appearing in engineering applications, while the values assumed for the tolerances and the maximum value of iterations and function evaluation were able to ensure both fast convergence and high level of accuracy.

Table 4.1: *Characteristics of the optimisation process.*

Tuning variables	$\mathbf{p} = [f;\, \zeta_T]$
Lower bounds	$\mathbf{l}_b = [10^{-3}; 10^{-3}]$
Upper bounds	$\mathbf{u}_b = [5; 1]$
Mass ratio	$\boldsymbol{\mu} = [0.0025 : 0.0025 : 0.1]$
Primary structure damping ratio	$\boldsymbol{\zeta}_S = [0, 0.01, 0.02, 0.03, 0.05]$
Tolerance on variable parameter	10^{-6}
Tolerance on constraint violation	10^{-6}
Tolerance on objective function	10^{-6}
Max. number of iterations	50
Max. number of function evaluation	300

In order to find out the best objective function for the tuning process, a preliminary investigation has been carried out on several objective functions, which are norms of the time response of the primary structure. Such investigation regarded both the type of norms considered and the response quantity. The H_i norm can be defined as follows [133]:

$$\|x\|_i := \left(\sum_{n=1}^{N} |x_n|^i \right)^{\frac{1}{i}} , \qquad 1 < i < \infty \,, \tag{4.34}$$

where N is the total number of time samples of variable x in the assumed time interval. The norms H_1, H_2 and H_∞ have been considered. The former two allow to optimise the dynamic response all along the time window, while the latter is devoted only to the peak response:

$$\|x\|_1 := \sum_{n=1}^{N} |x_n| \ , \qquad \|x\|_2 := \sqrt{\sum_{n=1}^{N} |x_n|^2} \ , \qquad \|x\|_\infty := \max_{1\leq n\leq N} |x_n| \, . \tag{4.35}$$

The response quantities assumed as possible alternative objective functions within this preliminary optimisation session are displacement, velocity and acceleration of the primary structure $x_S(t)$, $\dot{x}_S(t)$, $\ddot{x}_S(t)$ and the kinetic energy of the primary structure $T_S(t)$:

$$T_S(t) = \frac{1}{2} m_S \dot{x}_S^2(t) \, . \tag{4.36}$$

The numerical trials have been developed by assuming the following values of the structural parameters for the primary structure:

$$m_S = 100 \text{ kg}, \quad k_S = 10000 \text{ N/m} \, , \tag{4.37}$$

leading to:

$$\omega_S = 10 \text{ rad/s} \, , \quad \mathrm{f}_S = \frac{\omega_S}{2\pi} = 1.592 \text{ Hz} \, , \quad \mathrm{T}_S = \frac{1}{\mathrm{f}_S} = 0.6283 \text{ s} \, , \tag{4.38}$$

while the damping coefficient of the primary structure c_1 will take different values, so as to explore the influence of such parameter on the results. In case of different structural parameters, the general trends and guidelines presented here would still remain reliable for practical design of TMD devices.

The results obtained from the trials provided the following indications. In general, the assumption of a H_∞ response quantity leads to undesired results, since the frequency ratio f takes values corresponding to the upper bound, while the TMD damping ratio ζ_T displays abnormal decreasing trends, and the dynamic response appears not to take appreciable advantage from the presence of the so conceived TMD. In fact, the peak response is just a little reduced, while the overall response appears to be even increased. The H_1 norm presents some problems as well. Indeed, despite that a good performance of the TMD is obtained, the optimal TMD parameters exhibit irregular trends. Actually, the norm which rendered the best results is the H_2 norm of the considered response quantities, because of the regular trends of the optimal TMD parameters and the remarkable efficiency in reducing the overall dynamic response. Hence, the H_2 norm has been assumed as reference norm within the final optimisation process. However, for all the considered norms, the peak of the primary structure displacement has not been affected significantly, i.e. it has been obtained less than a 3% reduction. The same problem affects as well velocity and acceleration of the primary structure, since they are characterised by a behaviour quite similar to that of the displacement.

The primary structure displacement $x_S(t)$ has been assumed as final objective function in the present study, because in general its reduction represents the most important design goal. However, all the other considered response quantities still cover a role of valid alternative objective functions, with slight preference of kinematic indexes instead of kinetic energy.

4.2.4 Optimum TMD parameters for impulse loading

The optimum TMD parameters obtained by following the tuning methodology have been represented in Fig. 4.3. The main trends of the optimal TMD parameters substantially provide a decreasing f and an increasing ζ_T at increasing μ. In general, the optimum f^{opt} slightly decreases at increasing primary structure damping ratio ζ_S, while ζ_T^{opt} exhibits a general high level of insensitivity with respect to this parameter.

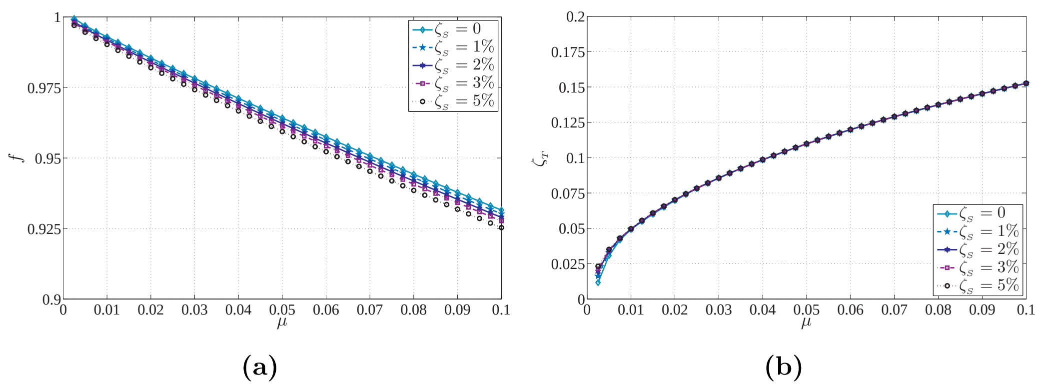
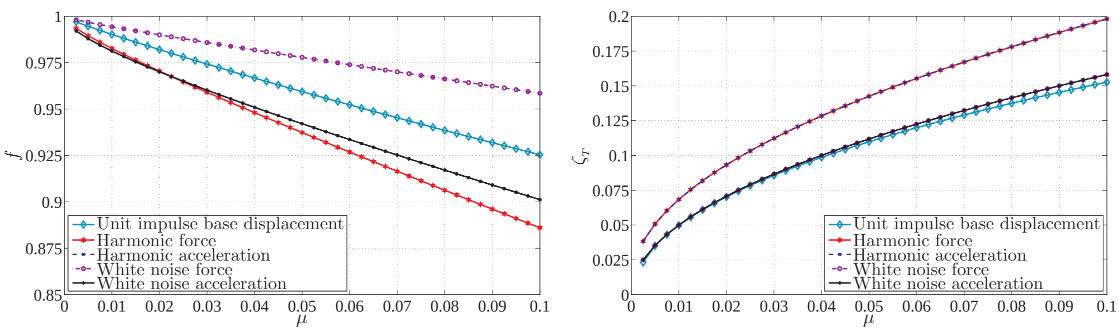


Figure 4.3: Optimum TMD parameters at variable μ for different values of ζ_S: (a) frequency ratio f; (b) TMD damping ratio ζ_T.

A comparison of the presented optimum parameters with respect to those obtainable in case of different excitations is proposed. In particular, the tuning formulas for the cases of harmonic and white noise excitations, applied either as force on the primary structure or base acceleration, taken from the previous Chapter 3, and reported in Table 4.2, have been considered for this analysis.

The relevant results are represented by Fig. 4.4, and will be discussed in the following. First, the main trends are confirmed, i.e. for all the loading cases the frequency ratio f^{opt} decreases and the TMD damping ratio ζ_T^{opt} increases at increasing mass ratio μ. The trends of f^{opt} are a bit different from each other, and in this sense the trend related to impulse loading is placed halfway with respect to the others, quite similarly to the values of f^{opt} for the case of excitations applied as base acceleration (both harmonic and white noise loadings). However, this fact appears not to be related to the present unit impulse applied as base displacement, since, from further tests, almost the same parameters have been obtained also in case of unit impulse force on the primary structure.



(a) **(b)**

Figure 4.4: *Optimum TMD parameters at variable μ, with $\zeta_S = 0.02$, for different dynamic excitations: (a) frequency ratio f; (b) TMD damping ratio ζ_T.*

On the other hand, the different trends of ζ_T look narrower, with two main streams defined by harmonic and white noise excitations respectively [93]. In this context, the parameters obtained for the case of unit impulse base displacement follow very closely the trend outlined for the white noise excitations. In general, the results discussed above seem to support the concept of a sort of unified tuning, i.e. a tuning of TMD parameters that could reasonably approximate the optimum tuning of the three different excitation cases.

Table 4.2: *Tuning formulas for the optimum TMD parameters as outlined in Chapter 3 for different dynamic excitations.*

Dynamic loading	f^{opt}	ζ_T^{opt}
Harmonic Force	$1-\sqrt{3\,\mu}\left(\frac{1}{2}\sqrt{\mu}+\ \ \zeta_S\right)$	$\frac{3}{5}\sqrt{\mu}+\frac{1}{6}\zeta_S$
Harmonic Acceleration	$1-\sqrt{3\,\mu}\left(\frac{2}{3}\sqrt{\mu}+\frac{3}{2}\zeta_S\right)$	$\frac{3}{5}\sqrt{\mu}+\frac{1}{6}\zeta_S$
White Noise Force	$1-\sqrt{3\,\mu}\left(\frac{2}{5}\sqrt{\mu}+\frac{1}{4}\zeta_S\right)$	$\frac{1}{2}\sqrt{\mu}$
White Noise Acceleration	$1-\sqrt{3\,\mu}\left(\frac{2}{3}\sqrt{\mu}+\frac{3}{2}\zeta_S\right)$	$\frac{1}{2}\sqrt{\mu}$

4.2.5 Time response reduction

The effectiveness of the proposed TMD, in terms of reduction of the primary structure time response $x_S(t)$, is the subject of the present section, where the unit impulse base displacement is still considered as the dynamic excitation.

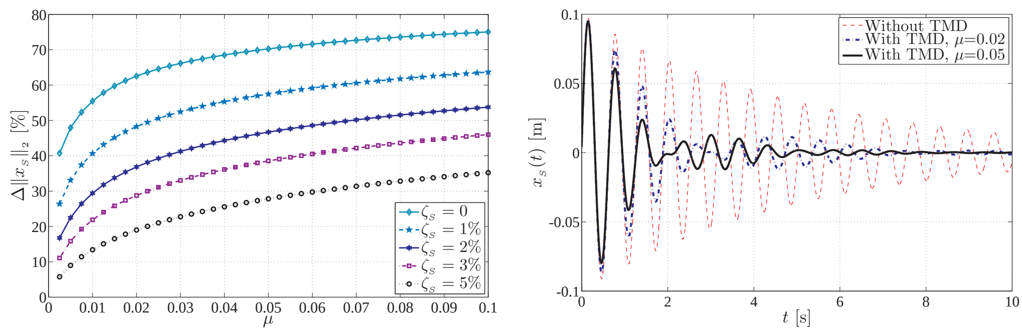
In Table 4.3, the percentage numerical magnitude of response reduction, in terms of $\|x_S(t)\|_\infty$ and $\|x_S(t)\|_2$, for $\boldsymbol{\zeta}_S = [0, 0.02, 0.05]$ and $\boldsymbol{\mu} = [0.02, 0.05, 0.1]$, have been gathered. One may observe that the peak response abatement is very small, also for significant values (from the engineering applications point of view) of μ, and it exhibits high insensitivity with respect to ζ_S. On the other hand, the overall response decrease is characterised by remarkable values, especially for low values of μ and ζ_S, and it results to be quite sensitive on these structural parameters, i.e. $\Delta\|x_S(t)\|_2$ increases at increasing μ and decreasing ζ_S.

These considerations on the overall response reduction are confirmed in Fig. 4.5a. The percentage reduction of the H_2 norm of the primary structure displacement, adopted here as suitable TMD performance index, shows similar trend shapes as a function of μ, for all the different values of ζ_S, but the values are quite different, i.e. one may note that the lower ζ_S, the higher the response reduction. As a further investigation, the optimisation has been carried out also for the case of base displacement written in terms of relative coordinates (i.e. the case of base acceleration,

Table 4.3: *Percent reduction of H_∞ and H_2 norm of the primary structure displacement $x_S(t)$ as a function of different values of the mass ratio μ and the primary structure damping ratio ζ_S.*

ζ_S	μ	$\Delta\|x_S(t)\|_\infty$ [%]	$\Delta\|x_S(t)\|_2$ [%]
	0.02	0.94	62.54
0	0.05	2.37	70.25
	0.10	4.67	75.05
	0.02	0.91	36.85
0.02	0.05	2.30	46.68
	0.10	4.53	53.76
	0.02	0.87	19.00
0.05	0.05	2.20	27.84
	0.10	4.34	35.23

represented by Eqs. (4.19)–(4.21)), which, as expected, gave the same results of the first formulation.



(a) **(b)**

Figure 4.5: *(a) Percentage reduction of the H_2 norm of the displacement of the primary structure at variable μ for different values of ζ_S and (b) displacement time history with ζ_S = 0.02.*

From Fig. 4.5b, it can be observed that the increase in mass ratio turns out in a remarkable increase of TMD efficiency, especially after the very beginning of the dynamic response (i.e. the first two/three peaks of oscillation). Further investigations pointed out that such a fact is more evident for lower values of ζ_S. However, the amplitude of the primary structure displacement $x_S(t)$ tends to zero after more or less the same duration time (e.g. 6–8 seconds for the considered primary structure), independently of the assumed magnitude of μ. Despite the general considerable efficiency of the TMD, the maximum response, which in case of impulsive loading for a system initially at rest occurs at the first peak of the dynamic response, seems not to be affected by the insertion of a passive TMD.

4.3 Hybrid TMD

4.3.1 Feedback closed loop control statement

From the results presented in the previous section, the passive Tuned Mass Damper appears not to be effective in reducing significantly also the peak displacement, which occurs during the early dynamic response for the case of impulsive loading. Such a fact is likely due to the intrinsic inertial behaviour of the TMD, which usually needs a few seconds to affect visibly the dynamic response of the primary structure.

It is therefore meaningful to attempt upgrading such a control device from the previously analysed merely passive version, to a hybrid form, by the introduction of a feedback closed loop controller, apt to supply an external relative control force $f_c(t)$ (Fig. 4.6). In this sense, the improvement expected from the introduction of the active controller is represented by its capacity to contribute to the response mitigation just after the beginning of the excitation. Such behaviour should allow to achieve further significant benefits with respect to the case of a passive TMD. Indeed, it is worth noting, in view of the following contents, that the active controller has been added after the addition and the tuning of the passive Tuned Mass Damper, in order to guarantee a minimum threshold value of response reduction. Thus, the further introduction of the controller is motivated only by the possibility of additional response reduction in terms of peak response.

4.3.2 Adopted control laws

The control force is taken as a linear combination of terms of the dynamic response of the structural system, where the constant coefficient values of the combination

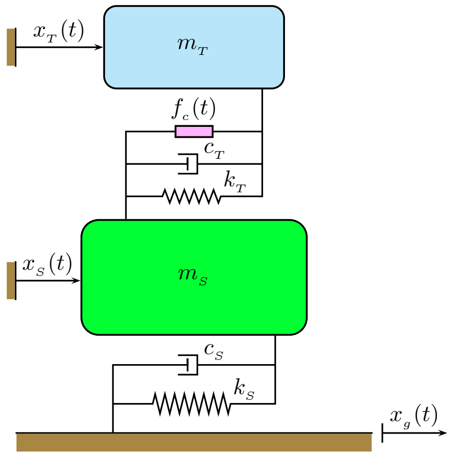


Figure 4.6: *Structural parameters and absolute dynamic degrees of freedom of a 2DOF mechanical system, subjected to base displacement, comprising a SDOF primary structure (S), equipped with an added hybrid TMD (T), apt to supply a control force $f_c(t)$.*

are called gains. Such a structural system is represented in Fig. 4.6 and its dynamic behaviour is described by the following equations of motion, in terms of the absolute degrees of freedom $x_S(t)$ and $x_T(t)$:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t) + \mathbf{D}f_c(t)\,, \tag{4.39}$$

where

$$\mathbf{D} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \tag{4.40}$$

defines the location vector for the control force $f_c(t)$. Different control strategies have been analysed, in order to discover which one would correspond to the best choice for the considered structural context. Two main final ones based on two gain parameters are extensively analysed in the following.

A first control strategy, which for the sake of simplicity will be reported as "Control Law 1" (CL1), is merely based on the kinematic response of the primary structure, and starts from often adopted references in the literature [40, 103]:

$$f_c(t) = g_a\ddot{x}_S(t) + g_v\dot{x}_S(t) + g_d x_S(t)\,, \tag{4.41}$$

where g_a, g_v and g_d are three gain constants related to acceleration, velocity and displacement of the primary structure, respectively. Preliminary optimisation tests developed in the present setting have pointed out that the acceleration gain g_a appears to play a negligible role within the global amount of supplied control force ($g_a = 0$). Hence, the final representation of CL1 is considered in the following two-gains simplified version:

$$f_c(t) = f_c^{CL1}(t) = g_v \dot{x}_S(t) + g_d x_S(t)\,. \tag{4.42}$$

The second control strategy alternatively proposed in this study, which will be labelled "Control Law 2" (CL2), is based on the acceleration of the primary structure $\ddot{x}_S(t)$ and the relative velocity between the primary structure and the Tuned Mass Damper $\dot{x}_S(t) - \dot{x}_T(t)$:

$$f_c(t) = f_c^{CL2}(t) = g_a \ddot{x}_S(t) + g_v \left(\dot{x}_S(t) - \dot{x}_T(t)\right), \tag{4.43}$$

where g_a, g_v are two gain constants related to the acceleration of the primary structure and the relative velocity, respectively. The introduction of this latter control law is motivated by the aim of controlling both the motion of the primary structure and the relative motion between structure and TMD, which under particular conditions could inadvertently amplify the motion of the primary structure instead of reducing it.

4.3.3 Dynamic response

By firstly considering CL1, the Laplace transform of Eq. (4.42) for a system initially at rest takes the form:

$$F_c^{CL1}(s) = s\, g_v X_S(s) + g_d X_S(s)\,. \tag{4.44}$$

The substitution of Eq. (4.44) into the transformed of Eq. (4.39), by considering Eqs. (4.23)–(4.26), gives the following impedance and receptance matrices:

$$\mathbf{Z}_{CL1}(s) = \begin{bmatrix} s^2 m_S + s(c_S + c_T + g_v) + k_S + k_T + g_d & -s\,c_T - k_T \\ -s(c_T + g_v) - k_T - g_d & s^2 m_T + s\,c_T + k_T \end{bmatrix}, \tag{4.45}$$

$$\mathbf{H}_{CL1}(s) = \\ = \frac{1}{\det(\mathbf{Z}_{CL1}(s))} \begin{bmatrix} s^2 m_T + s\,c_T + k_T & s(c_T + g_v) + k_T + g_d \\ s\,c_T + k_T & s^2 m_S + s(c_S + c_T + g_v) + k_S + k_T + g_d \end{bmatrix}. \tag{4.46}$$

Similarly, the Laplace transform of CL2, described in Eq. (4.43), for a system initially at rest, is the following:

$$F_{c}^{CL2}(s) = s^2 g_a X_S(s) + s\, g_v \big(X_S(s) - X_T(s)\big)\,. \tag{4.47}$$

As before, the substitution of Eq. (4.47) into the transformed of Eq. (4.39), by considering Eqs. (4.23)–(4.26), gives the following impedance and receptance matrices:

$$\mathbf{Z}_{CL2}(s) = \begin{bmatrix} s^2(m_S + g_a) + s(c_S + c_T + g_v) + k_S + k_T & -s(c_T + g_v) - k_T \\ -s^2 g_a - s(c_T + g_v) - k_T & s^2 m_T + s(c_T + g_v) + k_T \end{bmatrix}, \tag{4.48}$$

$$\mathbf{H}_{CL2}(s) = \\ = \frac{1}{\det(\mathbf{Z}_{CL2}(s))} \begin{bmatrix} s^2 m_T + s(c_T + g_v) + k_T & s^2 g_a + s(c_T + g_v) + k_T \\ s(c_T + g_v) + k_T & s^2(m_S + g_a) + s(c_S + c_T + g_v) + k_S + k_T \end{bmatrix}. \tag{4.49}$$

4.3.4 BIBO stability analysis

In this section, a preliminary stability analysis has been developed, so as to firmly establish the bounds (which will be further considered in the optimisation process) on the values that the control gains of the different feedback control laws may assume in order to ensure *a priori* a limited magnitude of the dynamic response in time for a given bounded input signal.

This principle is represented by the so called Bounded-Input-Bounded-Output (BIBO) stability, which requires, as necessary and sufficient condition for the stability of the motion of the system, negative real parts of the closed loop poles [34], which for the considered structural system are the roots of the following characteristic equation:

$$D(s) = 0\,, \tag{4.50}$$

where $D(s)$ is the denominator of the transfer function, obtained as determinant of the impedance matrix $\mathbf{Z}(s)$. Such condition, in practical terms, ensures the decay of the amplitude of the vibration modes of the structural systems.

In this sense, for Control Law 1, Eqs. (4.42), (4.44), the related characteristic equation becomes:

$$\begin{aligned} D_{CL1}(s) = &\det(\mathbf{Z}_{CL1}(s)) = \\ &= s^4 m_S m_T + s^3(c_S m_T + c_T m_S + c_T m_T + g_v m_T) + \\ &+ s^2(c_S c_T + k_T m_S + k_S m_T + k_T m_T + g_d m_T) + \\ &+ s(c_S k_T + c_T k_S) + k_S k_T = 0\,. \end{aligned} \tag{4.51}$$

Similarly, for Control Law 2, Eqs. (4.43), (4.47), the following characteristic equation is obtained:

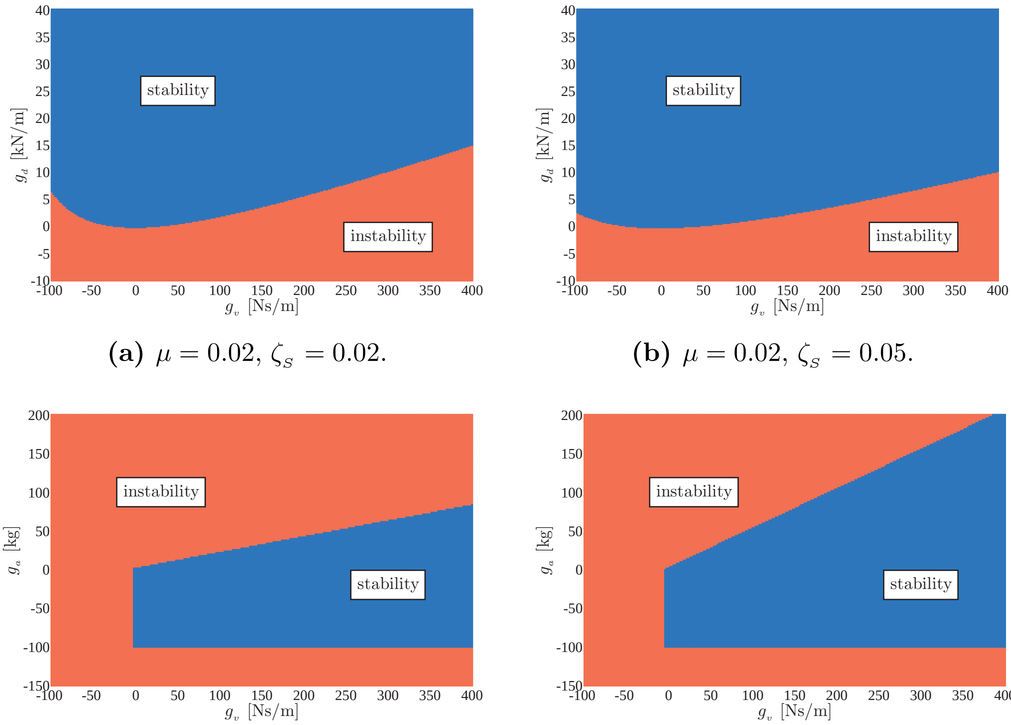
$$\begin{aligned} D_{CL2}(s) = &\det(\mathbf{Z}_{CL2}(s)) = \\ &= s^4(m_S m_T + g_a m_T) + s^3\big(c_S m_T + c_T m_S + c_T m_T + \\ &+ g_v(m_S + m_T)\big) + s^2(c_S c_T + k_T m_S + k_S m_T + k_T m_T + g_v c_S) + \\ &+ s(c_S k_T + c_T k_S + g_v k_S) + k_S k_T = 0\,. \end{aligned} \tag{4.52}$$

In order to satisfy the stability criterion described above, for each control strategy and set of given values of the gains, the sign of the less negative (or more positive) real part of the closed loop poles of the system has been investigated, in order to establish a sort of stability threshold for the system. The primary structure described in the previous section has been assumed again for this study. In this sense, it is important to confirm that possible changes in the structural parameters should not affect the theoretical principles outlined within the present investigation.

The results of the analysis described above are represented in Fig. 4.7, where the stability regions are shown, for both considered control laws, as a function of the two control gains, for $\mu = 0.02$ and $\zeta_S = [0.02,0.05]$. Such range of values can be considered as a suitable reference, and sufficient to outline important guidelines about the stability of the system.

The outcomes relevant to CL1 will be discussed first. From Figs. 4.7a–4.7b it can be noted that, as a general consideration, the stability region is outlined by positive values of g_d, while an increase in the value of g_v tends to narrow this area. It appears that the minimum values of the control gains assuring the stability of the system are of the same order of magnitude of that of the primary structure parameters c_S, k_S (for g_v, g_d, respectively). Moreover, a brief comparison between Fig. 4.7a and

Fig. 4.7b points out that the higher the primary structure damping ratio, the wider the stability region.



(a) $\mu = 0.02$, $\zeta_S = 0.02$.

(b) $\mu = 0.02$, $\zeta_S = 0.05$.

(c) $\mu = 0.02$, $\zeta_S = 0.02$.

(d) $\mu = 0.02$, $\zeta_S = 0.05$.

Figure 4.7: *BIBO stability region for (a),(b) Control Law 1 and (c),(d) Control Law 2, for mass ratio μ = 0.02 and different values of primary structure damping ratio ζ_S.*

The results related to CL2, reported in Figs. 4.7c–4.7d, exhibit a sharper contour of the stability region. This fact basically means that for a stable motion it is necessary to assume a positive value of g_v, while the acceptable range of values of acceleration gain g_a is limited to negative values for $g_v = 0$ and becomes larger, extended to positive values of g_a at increasing g_v. Also, for CL2 a sort of stability threshold corresponding to $g_a = -m_S$ can be established, which apparently represents a constant outcome, independent of the assumed control law. A possible physical interpretation of such a result can be the following: for $g_a < -m_S$ one obtains a sort of second virtual primary system, which moves in the opposite direction with respect to the main one. Such mass magnitudes, the real and the virtual one, and most of all their interaction, may easily create dynamic instability within the global system. However, these conditions are likely far from real applications, since an inertial force of the same magnitude of that of the primary structure is not feasible, especially in the case of buildings and, in general, for large systems. As found previously for CL1, by observing Figs. 4.7c–4.7d also for CL2 the amplitude of the stability region increases at increasing ζ_S.

4.3.5 Optimum feedback control for the hybrid TMD

The stability analysis developed and discussed in Section 4.3.4 has provided important guidelines towards the optimisation process of the control gains, especially concerning the bounds on the range of values that such gains may assume. As a further step before the optimisation of the control gains, a preliminary analysis of the objective function for both control laws will be presented first, in order to better define the context and, most important, to check if the present optimisation problem is well posed.

The objective function considered for this study is the peak displacement of the primary structure $x_S(t)$:

$$\mathrm{J}(\mathbf{v}) = \|x_S(t)\|_\infty \,, \tag{4.53}$$

where $\mathbf{v}$ is the vector of the control gains, which play the role of optimisation variables. The minimisation of the peak displacement is motivated by the fact that the passive Tuned Mass Damper has already been tuned by considering the H_2 norm, namely the overall response of the primary structure, and it turned out actually unable to reduce appreciably the peak of response as well. This is, in the end, one main motivation in the addition of the active controller to the existing system with optimum passive TMD.

A significant extract of the results of this investigation have been reported in Fig. 4.8, leading to the following considerations. First, Control Law 1 is characterised by the velocity gain g_v and the displacement gain g_d, which play here the role of optimisation variables:

$$\mathbf{v}_{CL1} = [g_v; g_d] \,. \tag{4.54}$$

In this sense, Figs. 4.8a–4.8b represent the shape of the objective function for CL1, which basically takes smaller values for higher velocity gain g_v, whilst less sensitivity is obtained for the acceleration gain g_a, since an actual minimum point is recovered for values almost twice the stiffness of the primary structure k_S, but such a minimum area tends to be enlarged at increasing g_v.

From these features and previous stability analysis, the values of the bounds on the two control gains have been assumed as follows:

$$0 < g_v < 50 \text{ Ns/m}\,, \qquad 10000 \text{ N/m} < g_d < 25000 \text{ N/m}\,, \tag{4.55}$$

In particular, the upper bound on g_v has been assumed in order to limit the motion of the TMD, which may lead to large magnitude for higher values of the control gain. This fact could be physically explained by a motion of the TMD progressively joined to that of the primary structure, which for high values of g_v may lead to system instability.

CL2 is based instead on the acceleration gain g_a and the velocity gain g_v, which play here the role of optimisation variables:

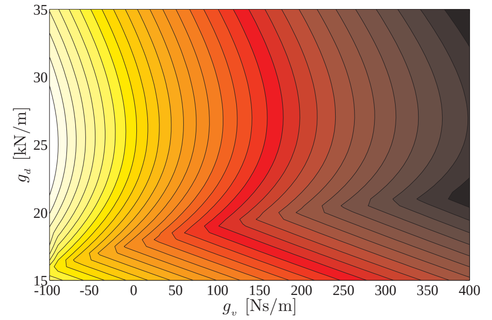
$$\mathbf{v}_{CL2} = [g_a; g_v]\,. \tag{4.56}$$

The results in Figs. 4.8c–4.8d display an objective function characterised by a clear minimum area corresponding to a value of g_v about one third of the primary structure damping coefficient c_S and a value of g_a slightly lower than the threshold value, obtained from the stability analysis, $g_a = -m_S$. In general, the presence of a well defined minimum area may lead already to consider this control law as better than the previous one, at least from the point of view of the optimisation process, which should therefore result better posed.

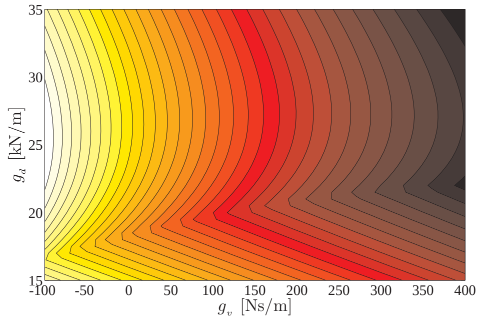
Finally, the plots and previous stability analysis suggest the following lower and upper bounds for the two control gains:

$$-95 \text{ kg} < g_a < -75 \text{ kg}\,, \qquad 20 \text{ Ns/m} < g_v < 45 \text{ Ns/m}\,. \tag{4.57}$$

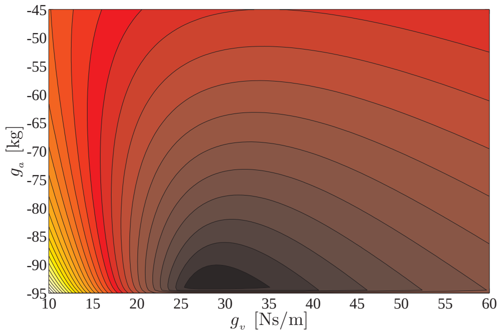
The parameters and bounds adopted within the optimisation process have been shown in Table 4.1. The lower and the upper bounds on the values that the gains may assume will be successively specified for each control law, for their influence within the amount of the supplied control force.



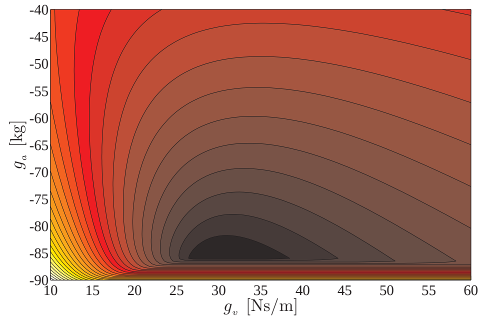
(a) $\mu = 0.02$, $\zeta_S = 0.02$



(b) $\mu = 0.02$, $\zeta_S = 0.05$



(c) $\mu = 0.02$, $\zeta_S = 0.02$



(d) $\mu = 0.02$, $\zeta_S = 0.05$

Figure 4.8: *Objective function peak displacement of the primary structure for (a),(b) Control Law 1 and (c),(d) Control Law 2, as a function of the control gains, for* $\mu = 0.02$ *and different values of primary structure damping ratio* ζ_S.

The validity of the optimisation outcomes presented in the following sections, and the subsequent achieved dynamic response, will be evaluated also with respect to the results previously obtained with the passive TMD only, so as to point out the further benefit coming from the addition of the active controller.

4.3.6 Optimum control gains and hybrid TMD performance

The numerical results of the optimisation process on the control gains, for the considered case, have been summarised in Table 4.4, where the structural parameters, the optimum values of the control gains and the relevant percentage response reduction are reported.

The optimum control gains for CL1 exhibit a value corresponding to the upper value for g_v and a value of g_a increasing at increasing ζ_S. In particular, the results on g_v make necessary the assumption of an upper limit for this gain, based on further

Table 4.4: *Optimum control gains and response reduction (μ = 0.02).*

Control Law		CL1		CL2	
Primary structure damping ratio ζ_S		0.02	0.05	0.02	0.05
Control gains	g_d [N/m] (CL1), g_a [kg] (CL2)	18387.7	19128.7	−94.2662	−85.8264
	g_v [Ns/m]	50	50	28.2240	30.0005
Response reduction [%]	$\Delta\|x_S(t)\|_\infty$	39.66	38.98	28.09	24.25
	$\Delta\|x_S(t)\|_2$	52.40	35.29	70.52	54.12
	$\Delta\|x_S(t)\|_\infty - \Delta\|x_S(t)\|_\infty^p$	38.76	38.11	27.17	23.37
	$\Delta\|x_S(t)\|_2 - \Delta\|x_S(t)\|_2^p$	15.55	16.29	33.67	35.12
	$\Delta\|x_T(t)\|_\infty$	50.90	41.82	77.39	71.91
	$\Delta\|x_T(t)\|_2$	63.35	58.65	89.05	85.81
Control force [kN]	$\|f_c(t)\|_\infty$	1.07723	1.08793	34.3592	7.84583
	$\|f_c(t)\|_2$	30.7000	27.8247	53.1595	17.5010

features, besides the minimisation of the primary structure response. On the other hand, for CL2 a decreasing g_a and an increasing g_v at increasing ζ_S have been obtained, consistently with the preliminary analysis outlined before. Such values of the optimum gain could lead to the physical meaning that the active controller attempts to counteract the inertial force of the primary mass in the largest possible way, by trying to create a sort of "virtual" mass and, at the same time, it supplies a further quantity of damping between the passive TMD and the primary structure, so as to reduce the movement of the control device, which under particular conditions could amplify the response of the primary structure, instead of reducing it.

The peak response of the primary structure, i.e. the H_∞ norm of $x_S(t)$, has been reduced significantly, i.e. of about 39% for CL1 and 24–28% for CL2, with a performance in general a bit lower for larger inherent damping, especially in the case of CL2. This is perhaps the most important outcome of the present analysis, since it strongly supports the introduction of the hybrid TMD for the purposes of the present control problem, i.e. the abatement of the peak response also.

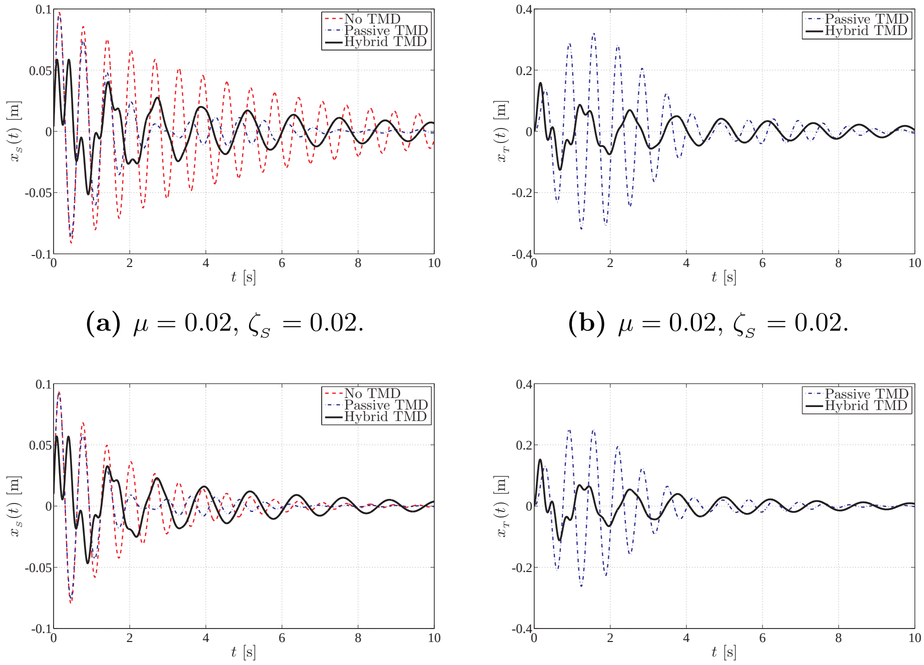
Remarkable results have been obtained also in terms of reduction of the H_2 displacement, which in general decreases at increasing ζ_S, especially for CL2. The difference in these indexes between the passive and the hybrid Tuned Mass Damper exhibits a general improvement in device performance. Most of all, the reduction of the peak displacement appears to get greater improvement from the introduction of the active controller. On the other hand, the overall response, represented by the H_2 norm, is reduced by a smaller amount. This is likely due to the previous beneficial effective optimisation already achieved with the passive TMD, based on this index.

An important consequence and benefit due to the hybrid TMD is the large reduction of the TMD stroke, considered as either H_∞ or H_2 norms. This fact is a very interesting and additional consequence of the optimisation process, even if the objective function has been established by neglecting the minimisation of the TMD response, and it could play an important role in view of practical applications.

The peak supplied force appears to be almost constant for each value of ζ_S for CL1, while it takes values quite different for CL2. On the other hand, the overall control force magnitude takes similar values for CL1, while it remarkably decreases for increasing ζ_S. These results reflect the time history of the control forces, whose amount is more distributed in time for CL1 and almost all concentrated at the beginning of the response for CL2. The contribution of a larger inherent damping is noticeable in reducing the required amount of control force in the case of CL2.

The dynamic responses of the different cases in the time domain, in terms of displacement of the primary structure and of the TMD, have been presented in Figs. 4.9–4.10. For CL1 (Fig. 4.9), the most noticeable fact is the almost equal shape of the dynamic response in the case of hybrid TMD. Indeed, it exhibits a sort of “double peak” at the very beginning of response, characterised by a constant amplitude, and the rest of response shows an oscillating behaviour, of higher period with respect to that of the passive system. Such results explain the numerical outcomes discussed above, since the amplitude of the peak response remains almost the same for the different cases, while the overall response decreases for increasing values of ζ_S. As a consequence, the reduction in the peak response is a sort of constant result, while any further reduction of the overall response is smaller at higher ζ_S.

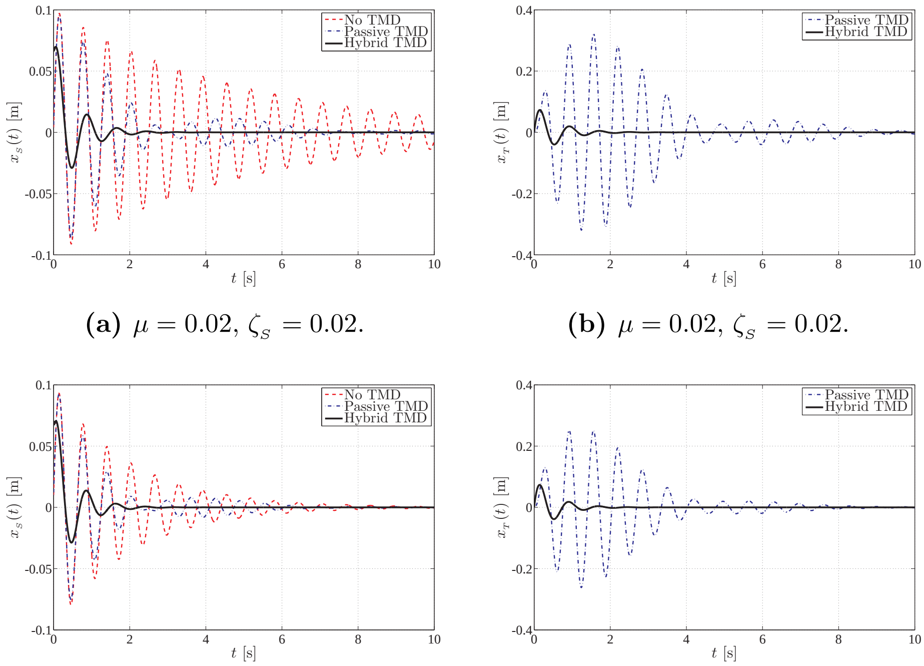
The time histories for CL2 are presented in Fig. 4.10. The dynamic response of the hybrid TMD is characterised by a first peak in the response, followed by a rapid decrease in the oscillations, which end after about 2 seconds. In terms of the settling time this is a much more efficient behaviour with respect to the one obtained with CL1, which leads to an oscillating primary structure for several seconds after the excitation. A further consideration regards the constant shape, from the point of view of the amplitude, of the minimised dynamic response. This was also found for the previous tests with CL1.



(a) $\mu = 0.02$, $\zeta_S = 0.02$. **(b)** $\mu = 0.02$, $\zeta_S = 0.02$.

(c) $\mu = 0.02$, $\zeta_S = 0.05$. **(d)** $\mu = 0.02$, $\zeta_S = 0.05$.

Figure 4.9: *Time history of (a),(c) the primary structure displacement $x_S(t)$ and (b),(d) the TMD displacement $x_T(t)$, for Control Law 1, with and without TMD, for $\mu = 0.02$ and different values of ζ_S ($\zeta_S = 0.02$ in (a),(b), $\zeta_S = 0.05$ in (c),(d)).*



(a) $\mu = 0.02$, $\zeta_S = 0.02$. **(b)** $\mu = 0.02$, $\zeta_S = 0.02$.

(c) $\mu = 0.02$, $\zeta_S = 0.05$. **(d)** $\mu = 0.02$, $\zeta_S = 0.05$.

Figure 4.10: *Time history of (a),(c) the primary structure displacement $x_S(t)$ and (b),(d) the TMD displacement $x_T(t)$, for Control Law 2, with and without TMD, for $\mu = 0.02$ and different values of ζ_S ($\zeta_S = 0.02$ in (a),(b), $\zeta_S = 0.05$ in (c),(d)).*

4.3.7 General closing considerations on the assumed control laws

The results obtained from the previous sections allow the following general remarks and considerations to be outlined, with the aim of establishing the best control strategy among the control laws analysed for the present structural context.

The dynamic behaviour obtained by assuming the first control law (CL1), typically quoted in the literature, is characterised by two peaks of significant amplitude at the very beginning of the response time history, which could seriously jeopardise the resistance of the structure. Moreover, during the remaining dynamic response, the oscillating response exhibits non-negligible magnitudes.

On the other hand, the second control law (CL2) leads, in terms of dynamic response, to a unique initial peak, and then the remaining time history shows a response characterised by a rapid decay. Such difference detected from the dynamic responses achieved by the two control laws points out the higher performance obtainable in the case of the newly proposed CL2, since the primary structure would be less stressed at all points in time. A further element in supporting this preference also comes from the numerical results, given in Tables 4.4. In general, the percentage reduction in the dynamic response, for both primary structure and TMD, is only slightly lower in case of CL2 than in case of CL1. Also, this performance is reached through an amount of supplied control force required by CL2 that is higher (more or less one quarter) with respect to CL1. Additionally, the issues on stability analysis in Section 4.3.4 and plots of the objective function in Fig. 4.8 display strong favour for the newly proposed CL2. Hence, by considering all these qualitative and quantitative features, CL2 overall shows as the best choice for the hybrid TMD under investigation here.

Chapter 5

Optimum TMDs for earthquake excitation

5.1 Introduction

As introduced in Chapter 2, the actual role of TMDs in earthquake engineering, including the tuning method and the potential performance in reducing the structural seismic response, represents a wide open research topic. In this context, the effectiveness of TMDs is usually measured by assuming the seismic signal in post-tuning trials, while the optimisation of the TMD parameters is carried out by means of tuning formulas provided for ideal excitations, such as those of harmonic or white noise loading [2].

In recent years, just a few studies considered the seismic signal as input within the tuning process, focusing therefore on the TMD optimisation for a specific earthquake event [28, 91, 92, 100]. However, an exhaustive study on this framework still appears to be lacking in the present literature. Moreover, from a general point of view, the actual level of benefit due to the addition of a TMD for the purposes of seismic response abatement seems to be an issue still under wide debate [64].

The contents presented in the present chapter insert themselves into this line of research in TMD tuning and explore the efficiency of the proposed optimisation method through numerical tests on several frame buildings, characterised by different structural parameters, so that to carry out as well a first investigation on the variability of the optimum TMD parameters and on the device effectiveness in reducing the earthquake response. The main novelty aspect of the optimum tuning procedure proposed here is the systematic application of a nonlinear optimisation algorithm on specific cases of frame structures and of seismic events, so that the obtained TMD parameters turn out optimum for each considered case.

The proposed selection of structures and earthquakes is composed of 16 shear-type frame buildings, mainly partitioned in 8 typologies based on the number of floors and 2 floor masses, which play the role of primary structures, and 18 seismic signals (horizontal ground acceleration), which exhibit diversified features. This set of $(2 \times 8) \times 18 = 288$ cases should allow for the analysis of a wide range of different situations, especially in terms of the interaction between the structural and the seismic loading parameters. Also, a quite high structural damping (relevant to the first mode of vibration) has been assumed for all the structures, namely $\zeta_{S,I} = 0.05$, in order to test the TMD within an unfavourable but real structural context, and therefore point out a minimum level of response abatement, i.e. a sort of lower performance threshold. Indeed, in case of lower structural damping, quite greater benefit should be expected.

The outcomes of the present study are exposed in the form of plots, tables and bar charts, first concerning the optimum TMD parameters, evaluated for all the 288 considered cases and for 6 different values of mass ratio (1728 cases in total). Then, for a selected value of mass ratio, namely $\mu = 0.02$, the seismic response reduction has been evaluated through several kinematic and energy response indexes, in order to outline a general overview of the TMD performance. Finally, a comprehensive analysis of the structural systems, concerning modal and response parameters, before and after the TMD insertion, has been presented, conceived as a complementary instrument aimed at the investigation on possible connections between the TMD performance and the structural properties, in order to provide significant explanations on the behaviour of the control device in earthquake engineering.

5.2 Analysis of the primary structures

5.2.1 Statement of the main features

In the present study, in order to explore the influence of the structural context in the tuning process and in the efficiency of the added Tuned Mass Damper in the seismic engineering context, different primary structures have been considered, represented by $2 \times 8 = 16$ shear-type frame buildings, i.e. structures with horizontal dynamic degrees of freedom only, characterised by different structural parameters, specifically 8 typologies based on the number of floors and 2 floor masses (assumed equal for each floor of the structure), and supposed to be subjected to different seismic ground accelerations $\ddot{x}_g(t)$. A schematic representation of this framework is displayed in Fig. 5.1.

The selected primary structures, which *inter alia* include both SDOF and MDOF buildings, are supposed to be described by perfectly/linear elastic behaviour, and constant stiffness and damping along the structure. More features could be varied in this study, but the task of this investigation suggests to clarify first the role of these main characteristics on the behaviour of the structure and the inserted TMD, before introducing further variables. The free structural parameters presented above have been chosen as follows:

- Number of storeys, 8 typologies: $n_s = [1, 2, 3, 5, 10, 15, 25, 40]$;
- Floor mass, 2 values: $m_{S,i} = 100000$ kg $= 100$ t; $m_{S,i} = 150000$ kg $= 150$ t;

Besides that, the following constant structural parameters, useful for outlining the dynamic behaviour, have been adopted:

- Squared section columns $l_c = 0.6$ m;
- Inertia moment $J_c = l_c^4/12 = 6.75 \times 10^{-4}$ m^4;
- Column height $h_c = 3$ m;
- Number of columns $n_c = 2$;
- Young's modulus E $=3 \times 10^4$ MPa;

As conceived in this study, the Tuned Mass Damper is a single passive device placed on top of the primary structure, where the maximum dynamic response is expected to occur, since the overall vibration shape is found to be almost totally governed by the first mode of vibration, as it will be clearer in the following of the study.

However, the placement of the TMD on the top of the frame turns out to be anyway a suitable solution, also in view of practical applications for seismic retrofitting, and it allows to emulate efficiently the optimal structural configuration.

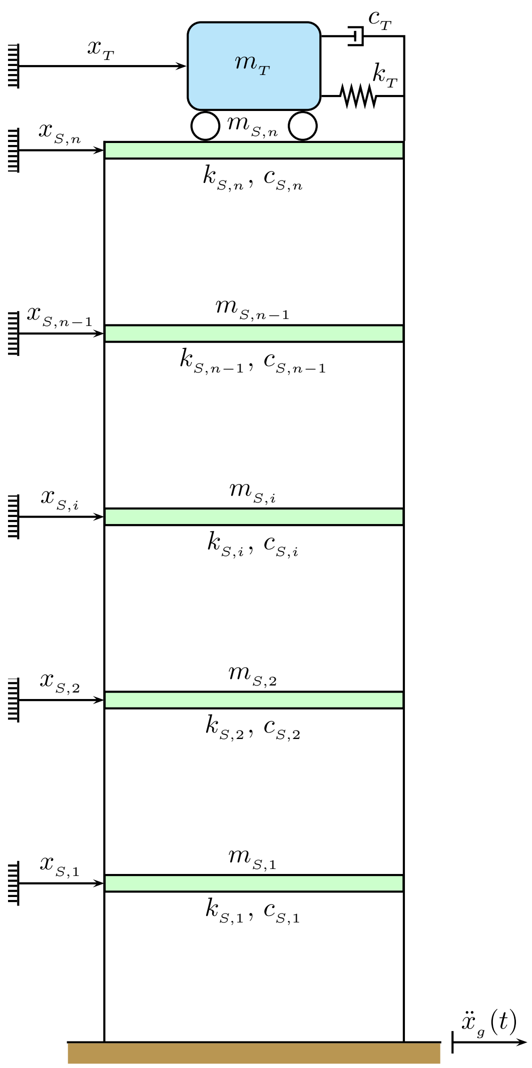


Figure 5.1: *Structural parameters and absolute (relative to the ground) dynamic degrees of freedom of the general structural system, subjected to seismic base acceleration, comprising of a MDOF primary structure (S), equipped with a passive TMD added on top (T).*

5.2.2 Dynamic analysis

The equations of motion of a linear MDOF shear-type primary structure with n storeys (Fig. 5.1), which rule its dynamic behaviour, subjected to a generic seismic base excitation $\ddot{x}_g(t)$, can be written as follows:

$$\mathbf{M}_S\ddot{\mathbf{x}}_S(t) + \mathbf{C}_S\dot{\mathbf{x}}_S(t) + \mathbf{K}_S\mathbf{x}_S(t) = -\mathbf{M}_S\mathbf{r}\,\ddot{x}_g(t) \tag{5.1}$$

where the $(x \times n)$ matrices

$$\mathbf{M}_S = \begin{bmatrix} m_{S,1} & 0 & \dots & \dots & 0 \\ 0 & m_{S,2} & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & m_{S,n-1} & 0 \\ 0 & \dots & \dots & 0 & m_{S,n} \end{bmatrix},$$

$$\mathbf{C}_S = \begin{bmatrix} c_{S,1}+c_{S,2} & -c_{S,2} & 0 & \dots & 0 \\ -c_{S,2} & c_{S,2}+c_{S,3} & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & c_{S,n-1}+c_{S,n} & -c_{S,n} \\ 0 & \dots & 0 & -c_{S,n} & c_{S,n} \end{bmatrix}, \tag{5.2}$$

$$\mathbf{K}_S = \begin{bmatrix} k_{S,1}+k_{S,2} & -k_{S,2} & 0 & \dots & 0 \\ -k_{S,2} & k_{S,2}+k_{S,3} & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & k_{S,n-1}+k_{S,n} & -k_{S,n} \\ 0 & \dots & 0 & -k_{S,n} & k_{S,n} \end{bmatrix}$$

are the diagonal mass matrix with floor masses $m_{S,i}$, the tridiagonal damping matrix composed of the damping coefficients $c_{S,i}$ and the tridiagonal stiffness matrix based on the floor stiffness coefficients $k_{S,i}$, respectively, where the index i denotes the i-th floor. The kinematic response of the primary structure is represented in terms of the $(n \times 1)$ vectors:

$$\mathbf{x}_S(t) = \begin{bmatrix} x_{S,1} \\ x_{S,2} \\ \vdots \\ x_{S,n} \end{bmatrix}, \qquad \dot{\mathbf{x}}_S(t) = \begin{bmatrix} \dot{x}_{S,1} \\ \dot{x}_{S,2} \\ \vdots \\ \dot{x}_{S,n} \end{bmatrix}, \qquad \ddot{\mathbf{x}}_S(t) = \begin{bmatrix} \ddot{x}_{S,1} \\ \ddot{x}_{S,2} \\ \vdots \\ \ddot{x}_{S,n} \end{bmatrix}, \tag{5.3}$$

namely displacement $\mathbf{x}_S(t)$, velocity $\dot{\mathbf{x}}_S(t)$ and acceleration $\ddot{\mathbf{x}}_S(t)$ vectors. The base seismic acceleration $\ddot{x}_g(t)$ is allocated to the frame structure by inertia effects, as proportional to the discrete masses, through the $(n \times 1)$ rigid body translational motion vector $\mathbf{r}$ with unit components:

$$\mathbf{r} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} . \tag{5.4}$$

As often reported in the literature, see e.g. Villaverde and Koyama [121] and Sadek et al. [90], classical Rayleigh damping is assumed here, by taking the $(n \times n)$ damping matrix $\mathbf{C}_S$ as simply proportional to the stiffness matrix $\mathbf{K}_S$, with:

$$\mathbf{C}_S = \beta \, \mathbf{K}_S \,, \qquad \beta = \frac{2\,\zeta_{S,I}}{\omega_{S,I}} \,, \tag{5.5}$$

where $\zeta_{S,I}$ and $\omega_{S,I}$ are respectively the given structural damping ratio and computed proper angular frequency of the primary structure referred to its first mode of vibration.

The seismic response of the MDOF structures may be evaluated also in terms of energy indicators, represented here by elastic energy $\mathbf{E}_S(t)$, kinetic energy $\mathbf{T}_S(t)$ and dissipation power $\mathbf{D}_S(t)$ of the primary structure, which are defined as follows:

$$\begin{aligned} \mathbf{E}_S(t) &= \frac{1}{2}\,\mathbf{x}_S^T(t)\,\mathbf{K}_S\,\mathbf{x}_S(t)\,, \\ \mathbf{T}_S(t) &= \frac{1}{2}\,\dot{\mathbf{x}}_S^T(t)\,\mathbf{M}_S\,\dot{\mathbf{x}}_S(t)\,, \\ \mathbf{D}_S(t) &= \frac{1}{2}\,\dot{\mathbf{x}}_S^T(t)\,\mathbf{C}_S\,\dot{\mathbf{x}}_S(t)\,. \end{aligned} \tag{5.6}$$

In the case of a MDOF primary structure equipped with a TMD added on top of the last storey, equations of motion (5.1) are slightly modified by an added equation and transform into the following $(n+1)$ equations:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\,\mathbf{x}(t) = -\mathbf{M}\,\mathbf{r}\,\ddot{x}_g(t). \tag{5.7}$$

In Eq. (5.7), system matrixes $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are based respectively on $\mathbf{M}_S$, $\mathbf{C}_S$, $\mathbf{K}_S$ in Eqs. (5.1)–(5.2), with appropriate insertion of the TMD parameters m_T, c_T and

k_T, correspondingly to the location of the TMD on the top floor of the primary structure:

$$\mathbf{M} = \left[\begin{array}{ccccc|c} m_{S,1} & 0 & \dots & \dots & \dots & 0 \\ 0 & m_{S,2} & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & m_{S,n-1} & \ddots & \vdots \\ \vdots & & & \ddots & m_{S,n} & 0 \\ \hline 0 & \dots & \dots & \dots & 0 & m_T \end{array}\right],$$

$$\mathbf{C} = \left[\begin{array}{ccccc|c} c_{S,1}+c_{S,2} & -c_{S,2} & \dots & \dots & \dots & 0 \\ -c_{S,2} & c_{S,2}+c_{S,3} & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & c_{S,n-1}+c_{S,n} & -c_{S,n} & 0 \\ \vdots & & & -c_{S,n} & c_{S,n}+c_T & -c_T \\ \hline 0 & \dots & \dots & 0 & -c_T & c_T \end{array}\right], \tag{5.8}$$

$$\mathbf{K} = \left[\begin{array}{ccccc|c} k_{S,1}+k_{S,2} & -k_{S,2} & \dots & \dots & \dots & 0 \\ -k_{S,2} & k_{S,2}+k_{S,3} & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & k_{S,n-1}+k_{S,n} & -k_{S,n} & 0 \\ \vdots & & & -k_{S,n} & k_{S,n}+k_T & -k_T \\ \hline 0 & \dots & \dots & 0 & -k_T & k_T \end{array}\right].$$

Similarly, for vectors $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$, $\ddot{\mathbf{x}}(t)$ in Eq. (5.7), indicating the kinematic response of the structure+TMD system, the kinematic response of the TMD (respectively $x_T(t)$, $\dot{x}_T(t)$ and $\ddot{x}_T(t)$) is included at the bottom of the structural vector components:

$$\mathbf{x}(t) = \begin{bmatrix} x_{S,1} \\ x_{S,2} \\ \vdots \\ x_{S,n} \\ x_T \end{bmatrix}, \qquad \dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_{S,1} \\ \dot{x}_{S,2} \\ \vdots \\ \dot{x}_{S,n} \\ \dot{x}_T \end{bmatrix}, \qquad \ddot{\mathbf{x}}(t) = \begin{bmatrix} \ddot{x}_{S,1} \\ \ddot{x}_{S,2} \\ \vdots \\ \ddot{x}_{S,n} \\ \ddot{x}_T \end{bmatrix}. \tag{5.9}$$

5.2.3 Modal analysis

In Tables 5.1–5.14, the modal parameters of the considered primary structures without TMD have been listed, in terms of modal frequencies $\omega_{S,i}$, $f_{S,i}$, modal periods $T_{S,i}$ and effective modal masses $M_{me,S,i}$. In particular, these latter turn out quite useful in order to establish the hierarchy of the vibration modes and, in general, the relevance within the overall dynamic response. The effective modal mass related to the i-th mode of vibration is defined as follows [20]:

$$M_{me,S,i} = \Gamma_{S,i}\Lambda_{S,i} = \frac{\Lambda^2_{S,i}}{\mathbf{M}_{m,S,i}}\,, \tag{5.10}$$

where:

$$\Gamma_{S,i} = \frac{\Lambda_{S,i}}{\mathbf{M}_{m,S,i}} \tag{5.11}$$

represents the i-th modal participation factor, being:

$$\mathbf{M}_{m,S,i} = \mathbf{\Phi}^T_{S,i}\,\mathbf{M}_S\,\mathbf{\Phi}_{S,i} \tag{5.12}$$

the modal masses and $\mathbf{\Phi}_{S,i}$ is the i-th mode shape of the primary structure, normalised so that to have a unit component at the level of the top storey.

Due to the intrinsic regularity of the structures (regular distribution of mass and stiffness), the outcomes point out general trends, discussed in the following. As a first note, for the two different floor masses, besides an expected shift of frequencies and periods, the same trends of the modal parameters are displayed.

In this sense, at increasing number of floors, the frequency of the first vibration mode tends to decrease and, viceversa, the period to increase, with values characteristic of those in real structures and, most important, corresponding to the interval of acceleration response spectrum where the seismic actions sort the greatest effect, as it will be displayed in a later moment.

The effective modal mass related to the first mode of vibration decreases at increasing number of storeys; however, even for the highest number of floors ($n_S = 40$), the first mode exhibits more than 80% of the total mass, with small contribution of the second mode and an almost negligible role played by the third mode. Moreover, as general indication, the modes higher than the fourth involve less than 1%

of the effective modal mass. Finally, for all the considered primary structures the first mode appears to be the dominant and the most representative of the global dynamic response.

Table 5.1: *Modal parameters for the structure with $n_S = 1$, $m_{S,i} = 100$ t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	53.6656	8.54115	0.11708	100	100	100.00

Table 5.2: *Modal parameters for the structure with $n_S = 2$, $m_{S,i} = 100$ t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	33.1672	5.27872	0.18944	138.197	189.443	94.72
II	86.8328	13.8199	0.0723596	361.803	10.5573	5.28

Table 5.3: *Modal parameters for the structure with $n_S = 3$, $m_{S,i} = 100$ t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	23.8835	3.80117	0.263077	184.117	274.224	91.41
II	66.9199	10.6506	0.0938911	286.294	22.4631	7.49
III	96.7021	15.3906	0.0649746	929.59	3.31306	1.10

Table 5.4: *Modal parameters for the structure with $n_S = 5$, $m_{S,i} = 100$ t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	15.2748	2.43106	0.411342	280.685	439.765	87.95
II	44.587	7.09624	0.14092	332.354	43.5887	8.72
III	70.287	11.1865	0.0893932	481.478	12.1078	2.42
IV	90.2928	14.3705	0.0695868	940.838	3.75466	0.75
V	102.984	16.3903	0.0610115	3464.64	0.783787	0.16

Table 5.5: *Modal parameters for the structure with $n_S = 10$, $m_{S,i} = 100$ t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	8.02088	1.27656	0.783354	527.948	847.925	84.79
II	23.8835	3.80117	0.263077	552.35	91.4079	9.14
III	39.2125	6.24087	0.160234	605.868	30.9147	3.09
IV	53.6656	8.54115	0.11708	700	14.2857	1.43
V	66.9199	10.6506	0.0938911	858.881	7.4877	0.75
VI	78.6794	12.5222	0.0798581	1134.8	4.09968	0.41
VII	88.6813	14.1141	0.0708513	1654.43	2.2135	0.22
VIII	96.7021	15.3906	0.0649746	2788.77	1.10435	0.11
IX	102.563	16.3234	0.0612618	6042.78	0.453081	0.05
X	106.132	16.8915	0.0592014	23634.2	0.108182	0.01

Table 5.6: *Modal parameters for the structure with* $n_S = 15$, $m_{S,i} = 100$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	5.43624	0.865204	1.1558	776.993	1254.23	83.62
II	16.2529	2.58673	0.386588	793.188	137.453	9.16
III	26.9029	4.28172	0.233551	826.955	48.1187	3.21
IV	37.2767	5.93277	0.168555	881.304	23.5175	1.57
V	47.2681	7.52295	0.132927	961.475	13.4066	0.89
VI	56.7744	9.03592	0.110669	1076.09	8.30305	0.55
VII	65.6981	10.4562	0.0956372	1239.35	5.38383	0.36
VIII	73.9477	11.7691	0.084968	1475.28	3.57001	0.24
IX	81.4385	12.9613	0.0771525	1826.6	2.37734	0.16
X	88.0936	14.0205	0.071324	2374.77	1.56272	0.10
XI	93.8447	14.9359	0.066953	3290.62	0.993791	0.07
XII	98.6329	15.6979	0.0637027	4983.39	0.594051	0.04
XIII	102.409	16.2989	0.0613539	8647.79	0.317549	0.02
XIV	105.134	16.7326	0.0597635	19125.8	0.136234	0.01
XV	106.781	16.9947	0.058842	75720.3	0.0333576	0.00

Table 5.7: *Modal parameters for the structure with* $n_S = 25$, $m_{S,i} = 100$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	3.30527	0.52605	1.90096	1276.21	2065.65	82.63
II	9.90328	1.57616	0.634455	1285.95	228.355	9.13
III	16.4637	2.62028	0.381638	1305.72	81.374	3.25
IV	22.9617	3.65447	0.273638	1336.15	40.8816	1.64
V	29.3726	4.67479	0.213913	1378.22	24.2209	0.97
VI	35.6721	5.67738	0.176137	1433.32	15.7903	0.63
VII	41.8362	6.65844	0.150185	1503.42	10.9448	0.44
VIII	47.8417	7.61424	0.131333	1591.13	7.90814	0.32
IX	53.6656	8.54115	0.11708	1700	5.88235	0.24
X	59.286	9.43566	0.105981	1834.82	4.46576	0.18
XI	64.6815	10.2944	0.0971403	2002.1	3.43832	0.14
XII	69.8317	11.1141	0.0899761	2210.87	2.67131	0.11
XIII	74.7169	11.8916	0.0840932	2473.82	2.08539	0.08
XIV	79.3188	12.624	0.0792144	2809.2	1.62951	0.07
XV	83.6197	13.3085	0.07514	3244	1.26968	0.05
XVI	87.6034	13.9425	0.071723	3819.39	0.982552	0.04
XVII	91.2549	14.5237	0.0688531	4600.72	0.751718	0.03
XVIII	94.5602	15.0497	0.0664464	5696.61	0.565406	0.02
XIX	97.5067	15.5187	0.0644385	7298.61	0.415034	0.02
XX	100.083	15.9288	0.0627795	9770.45	0.294275	0.01
XXI	102.28	16.2784	0.0614309	13873.5	0.198436	0.01
XXII	104.09	16.5664	0.0603633	21430.8	0.124034	0.00
XXIII	105.504	16.7914	0.0595541	37762.2	0.0685171	0.00
XXIV	106.518	16.9528	0.0589872	84428.5	0.0300649	0.00
XXV	107.128	17.0499	0.0586514	336434	0.00745914	0.00

Table 5.8: *Modal parameters for the structure with $n_S = 40$, $m_{S,i} = 100$ t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	2.0813	0.331249	3.01888	2025.76	3281.98	82.05
II	6.24076	0.993247	1.0068	2031.87	363.933	9.10
III	10.3908	1.65375	0.604685	2044.16	130.49	3.26
IV	14.5253	2.31177	0.432569	2062.78	66.1745	1.65
V	18.6379	2.96631	0.337119	2087.96	39.7079	0.99
VI	22.7224	3.61639	0.276519	2120.02	26.3114	0.66
VII	26.7728	4.26103	0.234685	2159.36	18.6071	0.47
VIII	30.783	4.89926	0.204112	2206.5	13.7743	0.34
IX	34.7468	5.53012	0.180828	2262.07	10.5452	0.26
X	38.6583	6.15266	0.162531	2326.86	8.28202	0.21
XI	42.5117	6.76595	0.147799	2401.79	6.63498	0.17
XII	46.3012	7.36907	0.135702	2488	5.39954	0.13
XIII	50.021	7.9611	0.125611	2586.86	4.44954	0.11
XIV	53.6656	8.54115	0.11708	2700	3.7037	0.09
XV	57.2295	9.10836	0.109789	2829.43	3.10781	0.08
XVI	60.7073	9.66187	0.1035	2977.55	2.62453	0.07
XVII	64.0938	10.2008	0.0980311	3147.33	2.2275	0.06
XVIII	67.3839	10.7245	0.0932447	3342.4	1.89768	0.05
XIX	70.5726	11.232	0.0890315	3567.24	1.62102	0.04
XX	73.6552	11.7226	0.0853054	3827.45	1.387	0.03
XXI	76.627	12.1956	0.081997	4130.09	1.1876	0.03
XXII	79.4835	12.6502	0.0790502	4484.11	1.01663	0.03
XXIII	82.2205	13.0858	0.0764187	4901.06	0.869245	0.02
XXIV	84.8338	13.5017	0.0740646	5395.98	0.741624	0.02
XXV	87.3196	13.8973	0.0719562	5988.77	0.630713	0.02
XXVI	89.674	14.2721	0.070067	6706.18	0.534053	0.01
XXVII	91.8935	14.6253	0.0683747	7584.89	0.449649	0.01
XXVIII	93.9748	14.9565	0.0668603	8676.17	0.375873	0.01
XXIX	95.9147	15.2653	0.065508	10053.6	0.311387	0.01
XXX	97.7104	15.5511	0.0643042	11825.6	0.255088	0.01
XXXI	99.3591	15.8135	0.0632371	14157.4	0.20606	0.01
XXXII	100.858	16.0521	0.0622971	17311	0.163549	0.00
XXXIII	102.206	16.2666	0.0614757	21721.9	0.126924	0.00
XXXIV	103.4	16.4566	0.0607659	28157.6	0.0956658	0.00
XXXV	104.438	16.6218	0.0601618	38075.6	0.069347	0.00
XXXVI	105.319	16.7621	0.0596584	54526.3	0.0476178	0.00
XXXVII	106.042	16.8771	0.0592517	84813	0.0301976	0.00
XXXVIII	106.606	16.9668	0.0589386	150250	0.0168663	0.00
XXXIX	107.009	17.0309	0.0587167	337215	0.00745846	0.00
XL	107.251	17.0695	0.0585842	1346830	0.001859	0.00

Table 5.9: *Modal parameters for the structure with* $n_S = 1$, $m_{S,i} = 150$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	43.8178	6.97382	0.143393	150	150	100.00

Table 5.10: *Modal parameters for the structure with* $n_S = 2$, $m_{S,i} = 150$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	27.0809	4.31006	0.232015	207.295	284.164	94.72
II	70.8987	11.2839	0.088622	542.705	15.8359	5.28

Table 5.11: *Modal parameters for the structure with* $n_S = 3$, $m_{S,i} = 150$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	19.5008	3.10364	0.322202	276.175	411.336	91.41
II	54.6399	8.69621	0.114993	429.44	33.6946	7.49
III	78.957	12.5664	0.0795774	1394.38	4.96959	1.10

Table 5.12: *Modal parameters for the structure with* $n_S = 5$, $m_{S,i} = 150$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	12.4718	1.98496	0.503789	421.027	659.648	87.95
II	36.4051	5.79406	0.172591	498.531	65.3831	8.72
III	57.3891	9.13376	0.109484	722.218	18.1617	2.42
IV	73.7238	11.7335	0.085226	1411.26	5.632	0.75
V	84.0858	13.3827	0.0747235	5196.97	1.17568	0.16

Table 5.13: *Modal parameters for the structure with* $n_S = 10$, $m_{S,i} = 150$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	6.54902	1.04231	0.959409	791.923	1271.89	84.79
II	19.5008	3.10364	0.322202	828.525	137.112	9.14
III	32.0169	5.09565	0.196246	908.801	46.3721	3.09
IV	43.8178	6.97382	0.143393	1050	21.4286	1.43
V	54.6399	8.69621	0.114993	1288.32	11.2315	0.75
VI	64.2414	10.2243	0.0978058	1702.21	6.14952	0.41
VII	72.4079	11.5241	0.0867748	2481.65	3.32025	0.22
VIII	78.957	12.5664	0.0795774	4183.15	1.65653	0.11
IX	83.7422	13.328	0.0750301	9064.16	0.679622	0.05
X	86.6568	13.7919	0.0725066	35451.3	0.162273	0.01

Table 5.14: *Modal parameters for the structure with* $n_S = 15$, $m_{S,i} = 150$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	4.43867	0.706436	1.41556	1165.49	1881.35	83.62
II	13.2705	2.11206	0.473471	1189.78	206.179	9.16
III	21.9661	3.49601	0.28604	1240.43	72.1781	3.21
IV	30.4363	4.84409	0.206437	1321.96	35.2763	1.57
V	38.5942	6.14246	0.162801	1442.21	20.1099	0.89
VI	46.3561	7.3778	0.135542	1614.14	12.4546	0.55
VII	53.6423	8.53743	0.117131	1859.03	8.07575	0.36
VIII	60.378	9.60946	0.104064	2212.92	5.35501	0.24
IX	66.4942	10.5829	0.0944922	2739.9	3.566	0.16
X	71.9281	11.4477	0.0873537	3562.16	2.34409	0.10
XI	76.6239	12.1951	0.0820003	4935.93	1.49069	0.07
XII	80.5334	12.8173	0.0780196	7475.09	0.891077	0.04
XIII	83.6166	13.308	0.0751428	12971.7	0.476324	0.02
XIV	85.8417	13.6621	0.073195	28688.8	0.20435	0.01
XV	87.186	13.8761	0.0720665	113581	0.0500365	0.00

Table 5.15: *Modal parameters for the structure with* $n_S = 25$, $m_{S,i} = 150$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	2.69874	0.429518	2.32819	1914.32	3098.47	82.63
II	8.08599	1.28693	0.777046	1928.92	342.533	9.13
III	13.4426	2.13945	0.467409	1958.58	122.061	3.25
IV	18.7482	2.98386	0.335136	2004.23	61.3224	1.64
V	23.9826	3.81695	0.261989	2067.32	36.3314	0.97
VI	29.1261	4.63557	0.215723	2149.99	23.6855	0.63
VII	34.1591	5.43659	0.183939	2255.13	16.4172	0.44
VIII	39.0626	6.217	0.160849	2386.69	11.8622	0.32
IX	43.8178	6.97382	0.143393	2550	8.82353	0.24
X	48.4068	7.70419	0.1298	2752.22	6.69864	0.18
XI	52.8123	8.40533	0.118972	3003.15	5.15748	0.14
XII	57.0173	9.07459	0.110198	3316.31	4.00696	0.11
XIII	61.0061	9.70943	0.102993	3710.73	3.12808	0.08
XIV	64.7635	10.3074	0.0970174	4213.8	2.44427	0.07
XV	68.2752	10.8663	0.0920273	4866	1.90452	0.05
XVI	71.5279	11.384	0.0878424	5729.09	1.47383	0.04
XVII	74.5093	11.8585	0.0843275	6901.07	1.12758	0.03
XVIII	77.208	12.288	0.0813799	8544.91	0.848108	0.02
XIX	79.6139	12.6709	0.0789207	10947.9	0.62255	0.02
XX	81.7178	13.0058	0.0768888	14655.7	0.441413	0.01
XXI	83.5117	13.2913	0.0752372	20810.3	0.297653	0.01
XXII	84.9887	13.5264	0.0739296	32146.2	0.186051	0.00
XXIII	86.1434	13.7102	0.0729386	56643.4	0.102776	0.00
XXIV	86.9714	13.8419	0.0722443	126643	0.0450973	0.00
XXV	87.4694	13.9212	0.071833	504651	0.0111887	0.00

Table 5.16: *Modal parameters for the structure with* $n_S = 40$, $m_{S,i} = 150$ *t.*

Mode	$\omega_{S,i}$ [rad/s]	$f_{S,i}$ [Hz]	$T_{S,i}$ [s]	$M_{m,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [$\times 10^3$ kg]	$M_{me,S,i}$ [%]
I	1.69937	0.270463	3.69736	3038.64	4922.98	82.05
II	5.09556	0.810983	1.23307	3047.8	545.9	9.10
III	8.48408	1.35028	0.740585	3066.24	195.735	3.26
IV	11.8598	1.88755	0.529787	3094.17	99.2618	1.65
V	15.2178	2.42198	0.412885	3131.94	59.5619	0.99
VI	18.5528	2.95277	0.338665	3180.02	39.467	0.66
VII	21.8599	3.47912	0.287429	3239.04	27.9107	0.47
VIII	25.1342	4.00023	0.249986	3309.75	20.6614	0.34
IX	28.3706	4.51532	0.221468	3393.11	15.8179	0.26
X	31.5644	5.02363	0.199059	3490.29	12.423	0.21
XI	34.7107	5.52438	0.181016	3602.69	9.95247	0.17
XII	37.8048	6.01682	0.166201	3732	8.09932	0.13
XIII	40.842	6.50021	0.153841	3880.28	6.6743	0.11
XIV	43.8178	6.97382	0.143393	4050	5.55556	0.09
XV	46.7277	7.43694	0.134464	4244.14	4.66171	0.08
XVI	49.5673	7.88888	0.126761	4466.33	3.93679	0.07
XVII	52.3324	8.32895	0.120063	4721	3.34125	0.06
XVIII	55.0187	8.7565	0.114201	5013.6	2.84651	0.05
XIX	57.6223	9.17087	0.109041	5350.86	2.43153	0.04
XX	60.1392	9.57145	0.104477	5741.18	2.0805	0.03
XXI	62.5657	9.95764	0.100425	6195.13	1.7814	0.03
XXII	64.898	10.3288	0.0968163	6726.17	1.52494	0.03
XXIII	67.1328	10.6845	0.0935934	7351.59	1.30387	0.02
XXIV	69.2665	11.0241	0.0907102	8093.97	1.11244	0.02
XXV	71.2961	11.3471	0.088128	8983.15	0.94607	0.02
XXVI	73.2185	11.6531	0.0858142	10059.3	0.801079	0.01
XXVII	75.0307	11.9415	0.0837415	11377.3	0.674474	0.01
XXVIII	76.7301	12.212	0.0818869	13014.3	0.56381	0.01
XXIX	78.314	12.4641	0.0802306	15080.4	0.467081	0.01
XXX	79.7802	12.6974	0.0787562	17738.3	0.382631	0.01
XXXI	81.1264	12.9117	0.0774493	21236	0.309091	0.01
XXXII	82.3505	13.1065	0.0762981	25966.5	0.245323	0.00
XXXIII	83.4508	13.2816	0.0752921	32582.8	0.190385	0.00
XXXIV	84.4256	13.4368	0.0744228	42236.5	0.143499	0.00
XXXV	85.2734	13.5717	0.0736828	57113.4	0.10402	0.00
XXXVI	85.9929	13.6862	0.0730663	81789.5	0.0714268	0.00
XXXVII	86.5831	13.7801	0.0725683	127219	0.0452964	0.00
XXXVIII	87.043	13.8533	0.0721848	225375	0.0252994	0.00
XXXIX	87.3721	13.9057	0.071913	505822	0.0111877	0.00
XL	87.5697	13.9372	0.0717507	2020250	0.00278851	0.00

5.3 Seismic input signals

5.3.1 General issues

In this section, the main characteristics of the seismic input signals assumed in these trials, listed in following Table 5.17, are presented in detail. The 18 earthquake signals considered for the present analysis cover a wide time window within the last two centuries, from the earthquake of Long Beach, California, USA on 1933 to the recent very strong motion of Tohoku, Japan, 2011. Moreover, several signals that are adopted here have been already assumed in different literature works on the tuning of TMDs [45, 79, 119, 121].

Some of the selected signals concern the same earthquake event recorded at different stations. This choice is motivated by the differences that hold between these signals, from the points of view of the intensity and the distribution of the peaks of ground acceleration, even if such accelerograms state the same earthquake event.

For each earthquake event, the accelerograms have been selected by assuming as main criterion the adoption of the time history with the highest PGA, with the expectation that this index could be related to the highest signal intensity, and therefore could represent the strongest record of the earthquake event.

The proposed selection concerns seismic input signals characterised by quite different values of several parameters apt to thoroughly state the signal in time domain, at least for the purposes of the present investigation. In particular, such parameters are:

- Magnitude M, 5.8<M<9.0;
- Peak Ground Acceleration PGA, 0.142 g<PGA<2.612 g (g = 9.8066 m/s^2);
- Frequency content f (or period content T);
- Maximum Pseudo-acceleration response spectrum S_{pa}^{max} and relevant period $T(S_{pa}^{max})$;
- Duration and shape of the time history;
- Epicentral distance and fault depth.

The motivation of such variability is the investigation on the influence of the different quantities on the optimum tuning and related performance of the TMDs.

The seismic input signals have been represented in Figs. 5.3–5.38 in terms of:

(a) Time history of ground acceleration $\ddot{x}_g(t)$;

(b) Fourier spectrum (frequency amplitude) FAS(f);

(c) Displacement response spectrum S_d(T);

(d) Pseudo-velocity response spectrum S_{pv}(T);

(d) Pseudo-acceleration response spectrum S_{pa}(T).

The list above points out two response spectra, related to the response in terms of displacement (S_d(T)), velocity (S_{pv}(T)) and acceleration (S_{pa}(T)). In this sense, it could be possible to gather, by means of a comprehensive plot, namely the Displacement-Velocity-Acceleration (D-V-A) spectrum [20, 25] the whole information about the response spectra. However, the separated representation displayed here has been assumed for the sake of comprehension, since it allows for an immediate reading of the seismic characteristics, as confronted to the structural parameters.

5.3.2 Signal processing

In general, the accelerograms could originally exhibit some intervals where the amplitude of the acceleration displays negligible values. Most of these parts are represented by pre-events, i.e. the very beginning of the signal record previous to the earthquake event, where the signal is usually almost equal to zero. Within a preliminary stage, these intervals turn out quite useful, especially during the signal processing, as it will be clear in the following. Other intervals of the accelerogram of small amplitude are usually those at the end of the record, when the seismic event already took place. In view of the seismic tuning, these intervals, where present, have been removed so that to reduce the computational time required by the optimisation procedure. This measure is in principle allowed with consideration of the purposes of the present study, since preliminary tests strongly proved that it does not affect significantly the tuning results.

Among the signal features, in Table 5.17 the status of the original signal is also indicated, i.e. if either it has been preliminary processed after the record or if it could be affected by disturbances. In this sense, it is possible to note that all the displayed signals have been provided as already processed, except for those related to the Tohoku 2011 earthquake, which therefore have been processed here.

In the present context, the main motivation of the signal processing concerns the correction of the potential misplacement of the optimum region due to possible disturbances of the earthquake signal, which could therefore affect the goodness of the tuning results and the related TMD performance.

The signal processing procedure adopted has been based on the following steps [109]:

1. Removal of the accelerogram baseline, assumed in principle as the average value of the whole signal. However, when in the presence of a pre-event, the evaluation of the mean acceleration has been narrowed down just on this interval of the accelerogram.

2. Application of a Butterworth filter, which is the most adopted for seismic signal processing, thoroughly stated by the following parameters:

    - Filter order n_{Bf};
    - Passband cutoff frequencies vector $\omega_P = [\omega_{P,l}\,;\,\omega_{P,u}]$;
    - Stopband cutoff frequencies vector $\omega_S = [\omega_{S,l}\,;\,\omega_{S,u}]$;
    - Ripple in the passband R_P;
    - Attenuation in the stopband R_S.

In particular, for the case of the Tohoku 2011 earthquake signals, due to very similar characteristics of the T2011T and the T2011S records, especially from the point of view of the frequency content, it has been possible to provide a filter valid for the processing of both signals, based on the following parameters:

- $n_{Bf} = 7$;
- $\omega_P = [0.4\,;\,44]$ Hz;
- $\omega_S = [0.1\,;\,50]$ Hz;

- $R_P = 3$ dB;
- $R_S = 10$ dB.

The adopted Butterworth filter, represented in Fig. 5.2, allowed to take off efficiently the original noise in the input signal at both low and high frequencies, for the considered accelerograms. Among these parameters, ω_P and ω_S have been chosen from a preliminary inspection of the recorded signal, whose sample frequency is 100 Hz, and which exhibits low frequency noise at about 0.005 Hz. R_P and R_S have been assumed so as to ensure minimum conditions of stability of the filter, which has been found to be jeopardised especially at low frequencies, i.e. nearby the threshold between the low-stop and the low-pass values. Finally, the filter order n_{Bf} has been evaluated through proper built-in functions *buttord* and *butter* in MATLAB [109], which is based on the four parameters and vectors introduced previously.

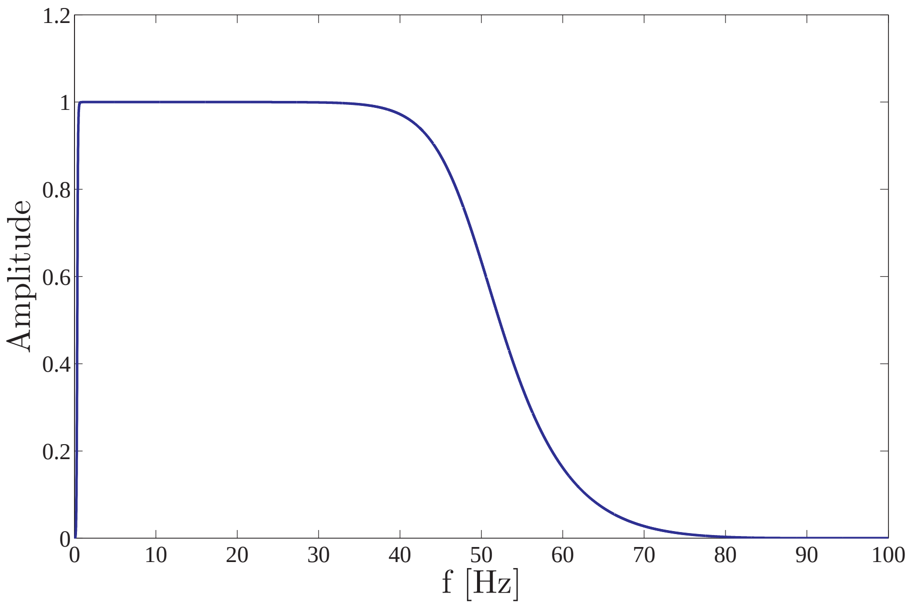


Figure 5.2: *Butterworth filter adopted for the processing of Tohoku 2011 seismic input signals (Sendai and Tsukidate stations).*

The filter in Fig. 5.2 displays the following main features. First, as mainly required, the filter exhibits unit amplitude along the passband and zero amplitude along the stopband. In general, the shape reflects that usually obtainable for this typology of filter, i.e. quite steep at the transition of bands for the low frequencies interval and instead quite smooth at the transition of bands for the high frequencies, due to the polynomial statement of the Butterworth filter. In particular, for the low frequencies high precision is required for the determination of the pass and the stop frequencies, since even slight changes could lead to a misuse of the filter. On the other hand, the tolerance appears to be larger for the assumption of the high stop and pass frequencies, since the transition of the bands occurs along a moderate interval of frequencies.

Besides a visual inspection, useful to assess the presence of clear inconsistencies, the validation of the goodness of the processed signal is based on the following assessment:

1. Double integration of the processed time history of the ground acceleration, so that to obtain the time history of ground velocity and displacement;

2. Double time derivation of the displacement time history, so that to get back the accelerogram;

3. Qualitative and quantitative comparisons between the former and the latter acceleration time history.

In this sense, the processed signal has been considered reliably representing the actual seismic record if a tolerance on the whole time window has been fulfilled, quantitatively fixed in an overall difference between the two signal amplitudes equal to 5%:

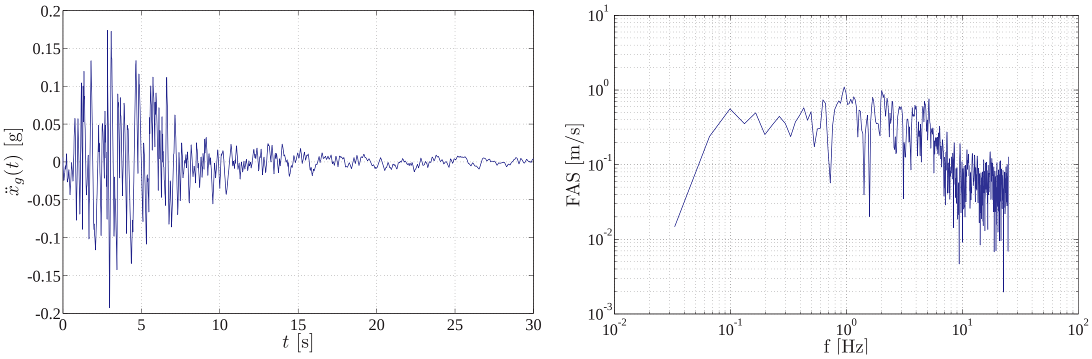
$$\frac{|\ddot{x}_g^P - \ddot{x}_g^{P,d}|}{|\ddot{x}_g^P|} < 5\% \tag{5.13}$$

where $\ddot{x}_g^P$ and $\ddot{x}_g^{P,d}$ denote the merely processed and the derived processed seismic ground acceleration, respectively.

Table 5.17: *Main characteristics of the strong motions considered in the present study.*

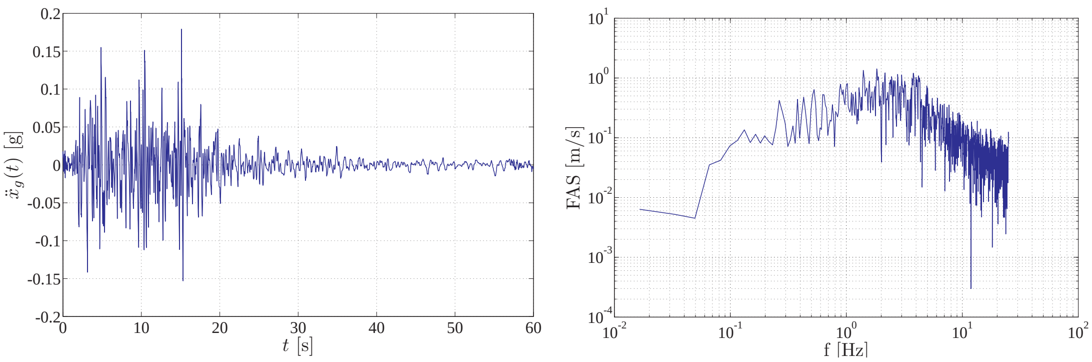
Earthquake (Code)	St., Comp. (P/R)	M	Duration [s]	Epicentre [km]	Fault [km]	PGA [g]	S_{pa}^{max} [g]	$T(S_{pa}^{max})$ [s]
Long Beach, 10/03/1933 (L1933)	Long Beach, N-S (P)	6.4	30	19.2	-	0.216	0.669	0.188
El Centro, 30/12/1934 (E1934)	El Centro, S90W (P)	6.5	60	65.6	-	0.052	0.639	0.256
Imperial Valley, 18/05/1940 (I1940)	El Centro, S00E (P)	6.9	50	16.9	-	0.359	0.907	0.253
Kern County, 21/07/1952 (K1952)	Taft, S69E (P)	7.5	50	46.4	-	0.196	0.582	0.440
Borrego Mountain, 08/04/1968 (B1968)	El Centro, S00W (P)	6.5	80	70.0	-	0.142	0.286	0.266
San Fernando, 09/02/1971 (S1971)	Pacoima Dam, S16E (P)	6.6	20	6.7	-	1.251	2.993	0.387
Imperial Valley, 15/10/1979 (I1979)	El Centro, N-S (P)	6.6	40	27.2	-	0.337	1.753	0.226
Chile, 03/03/1985 (C1985)	San Isidro, 0 (P)	7.8	100	-	-	0.721	3.456	0.337
Loma Prieta, 17/10/1989 (L1989)	Watsonville, 0 (P)	7.0	30	18.1	-	0.801	2.693	0.329
Northridge, 17/01/1994 (N1994)	Tarzana, 90 (P)	6.4	30	5.5	0.4	1.927	4.994	0.333
Kobe, 17/01/1995 (K1995TZ)	Takarazuka, 90 (P)	7.0	20	-	-	0.694	2.506	0.471
Kobe, 17/01/1995 (K1995TK)	Takatori, 90 (P)	7.0	40	-	-	0.616	2.348	0.187
L'Aquila, 06/04/2009 (A2009)	V. Aterno (C. Valle), W-E (P)	5.8	20	4.9	3.8	0.676	1.803	0.111
Chile, 27/02/2010 (C2010A)	Angol, N-S (P)	8.8	200	209.3	65.6	0.928	3.720	0.175
Chile, 27/02/2010 (C2010C)	Concepcion San Pedro, 7 (P)	8.8	160	109.1	36.2	0.651	2.260	0.196
New Zealand, 03/09/2010 (N2010)	Greendale, N55W (P)	7.0	40	6.9	3.5	0.772	1.490	0.504
Tohoku, 11/03/2011 (T2011T)	Tsukidate-MYG004, N-S (R)	9.0	180	125.9	75.1	2.612	13.02	0.237
Tohoku, 11/03/2011 (T2011S)	Sendai-MYG013, N-S (R)	9.0	180	126.1	71.8	1.402	2.562	0.660

5.3.3 Seismic signals



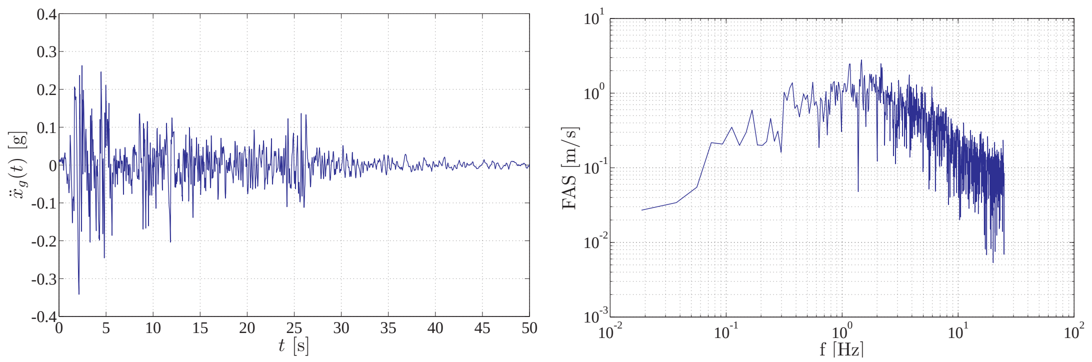
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.3: *Long Beach 1933 earthquake, ground acceleration.*



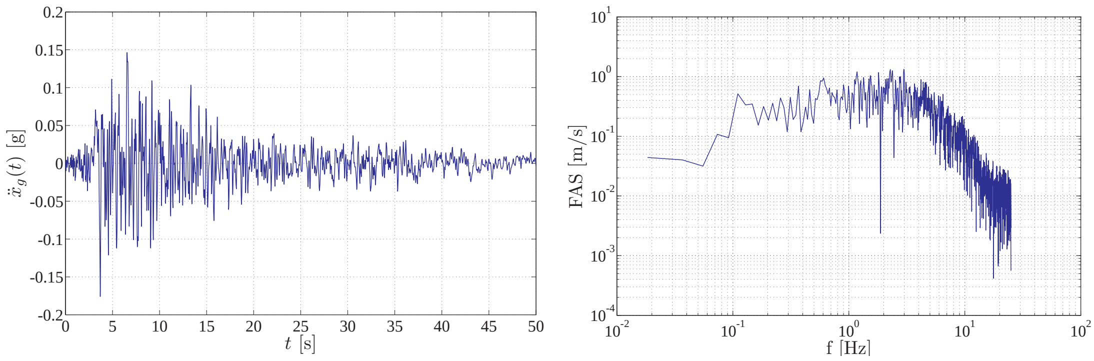
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.4: *El Centro 1934 earthquake, ground acceleration.*



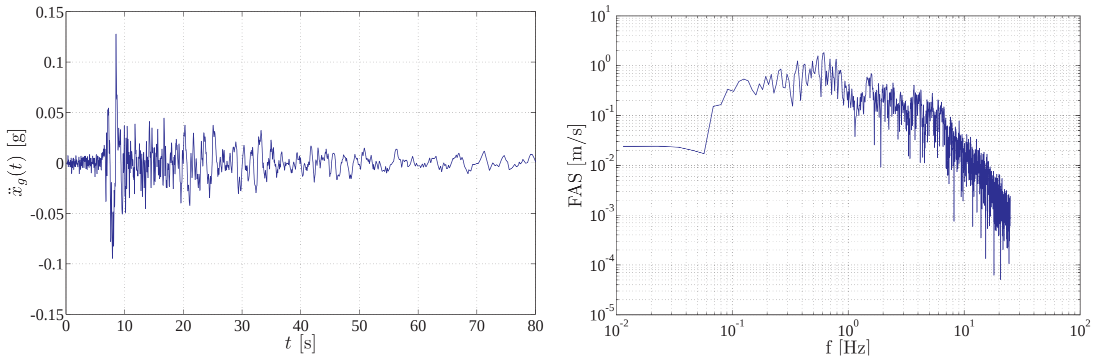
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.5: *Imperial Valley 1940 earthquake, ground acceleration.*



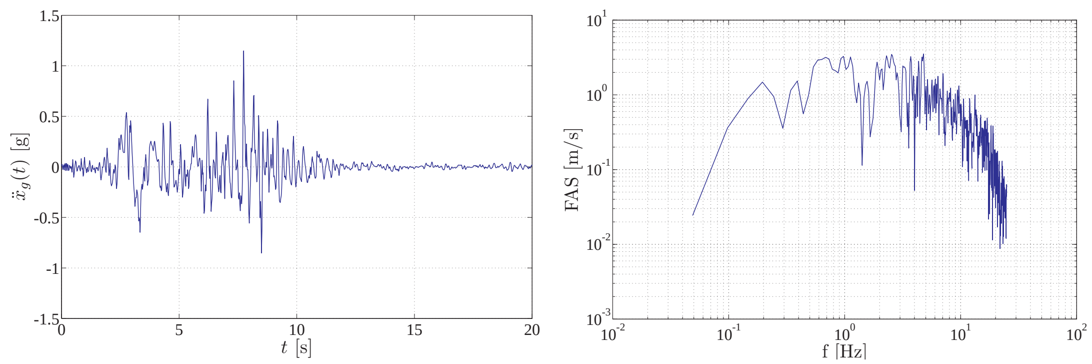
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.6: *Kern County 1952 earthquake, ground acceleration.*



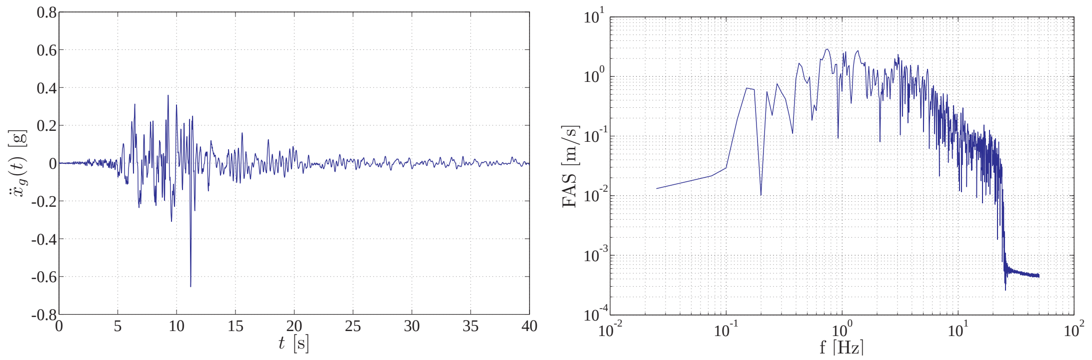
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.7: *Borrego Mountain 1968 earthquake, ground acceleration.*



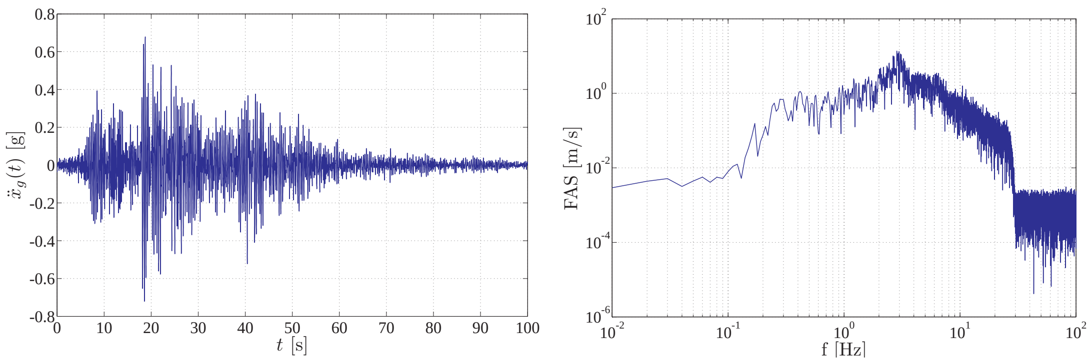
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.8: *San Fernando 1971 earthquake, ground acceleration.*



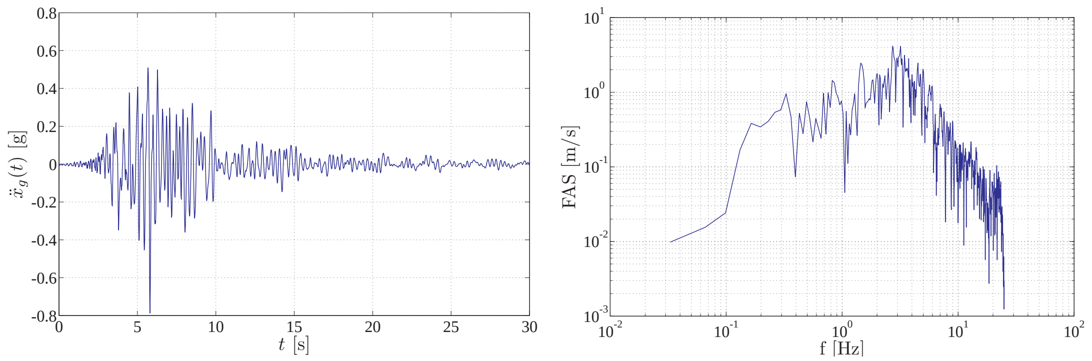
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.9: *Imperial Valley 1979 earthquake, ground acceleration.*



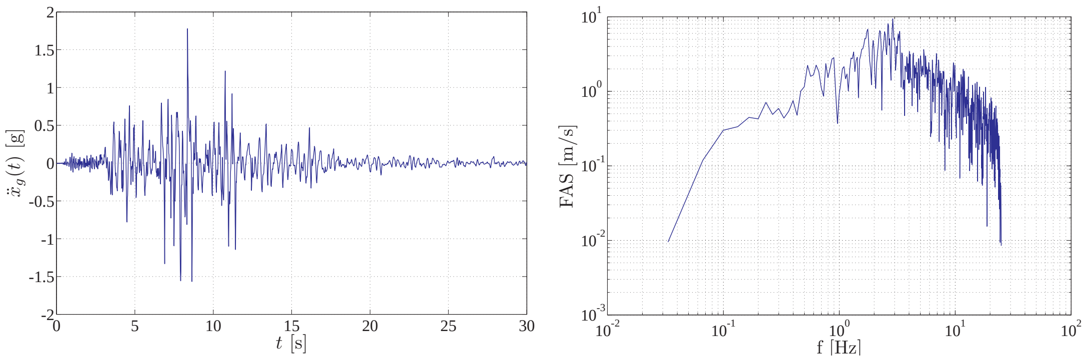
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.10: *Chile 1985 earthquake, ground acceleration.*



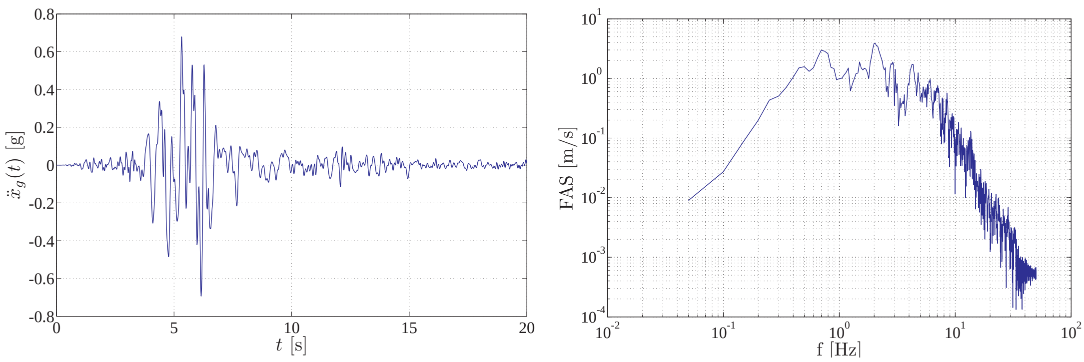
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.11: *Loma Prieta 1989 earthquake, ground acceleration.*



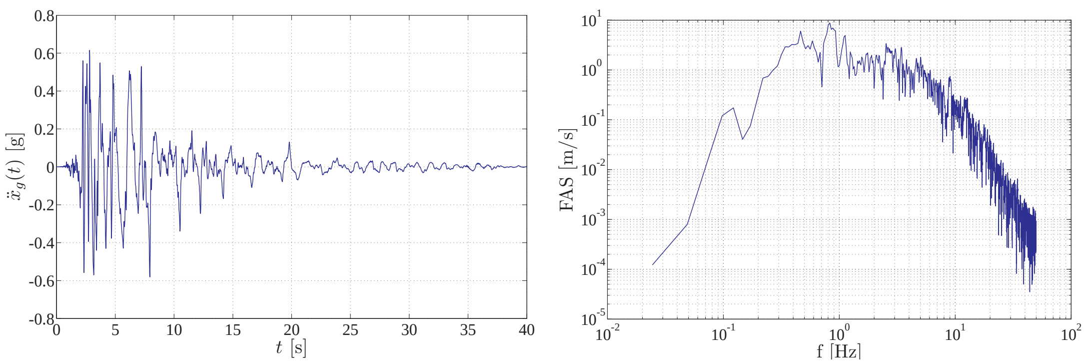
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.12: *Northridge 1994 earthquake, ground acceleration.*



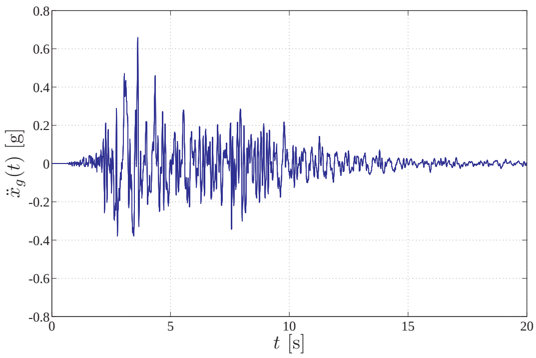
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.13: *Kobe 1995 earthquake (Takarazuka station), ground acceleration.*

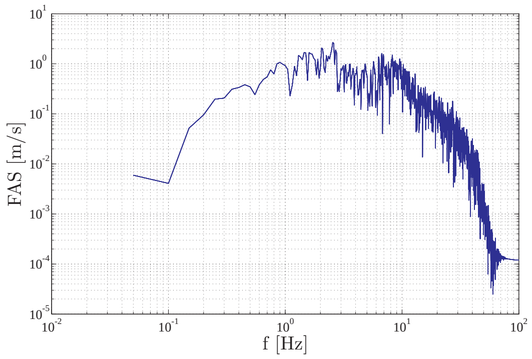


(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.14: *Kobe 1995 earthquake (Takatori station), ground acceleration.*

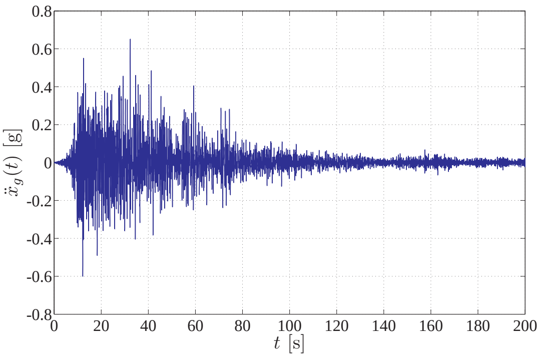


(a) Accelerogram.

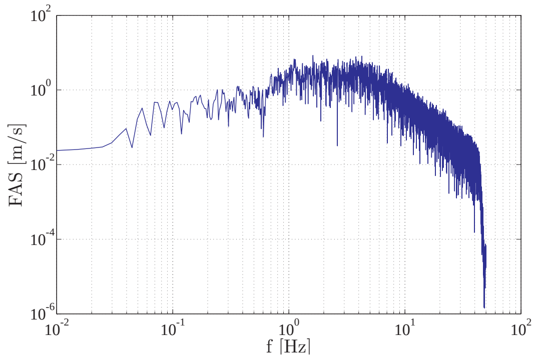


(b) Fourier amplitude spectrum.

Figure 5.15: *L'Aquila 2009 earthquake, ground acceleration.*

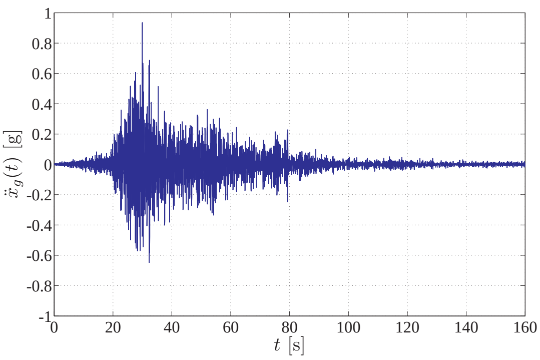


(a) Accelerogram.

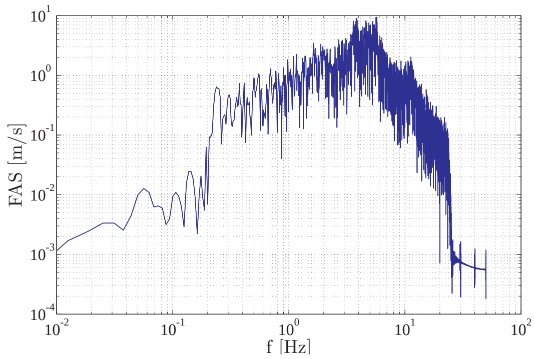


(b) Fourier amplitude spectrum.

Figure 5.16: *Chile 2010 earthquake (Concepcion San Pedro station), ground acceleration.*

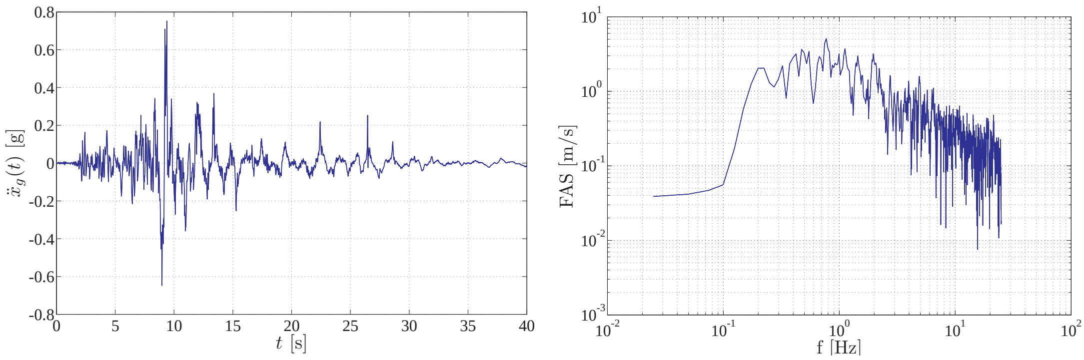


(a) Accelerogram.



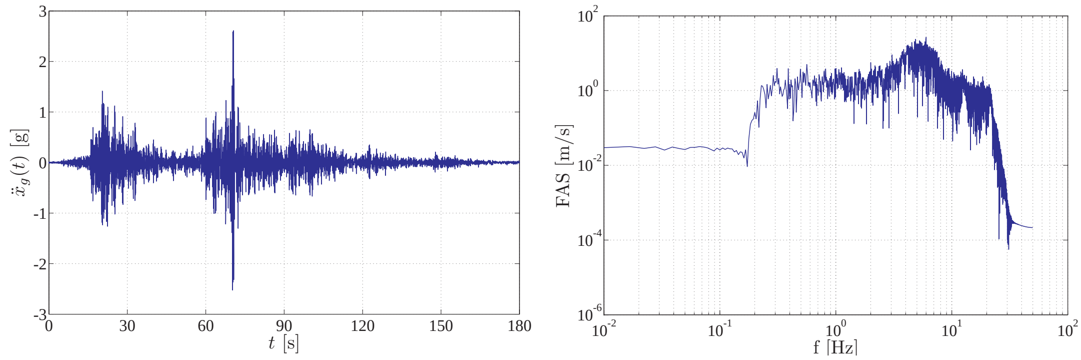
(b) Fourier amplitude spectrum.

Figure 5.17: *Chile 2010 earthquake (Angol Station), ground acceleration.*



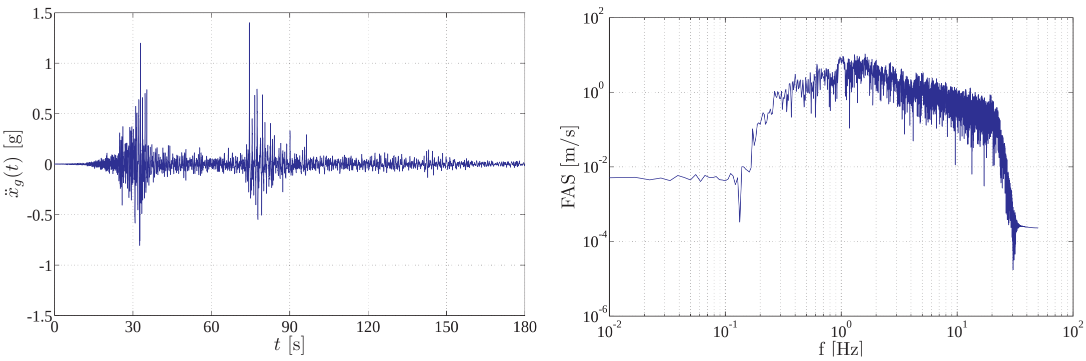
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.18: *New Zealand 2010 earthquake, ground acceleration.*



(a) Accelerogram. **(b)** Fourier amplitude spectrum.

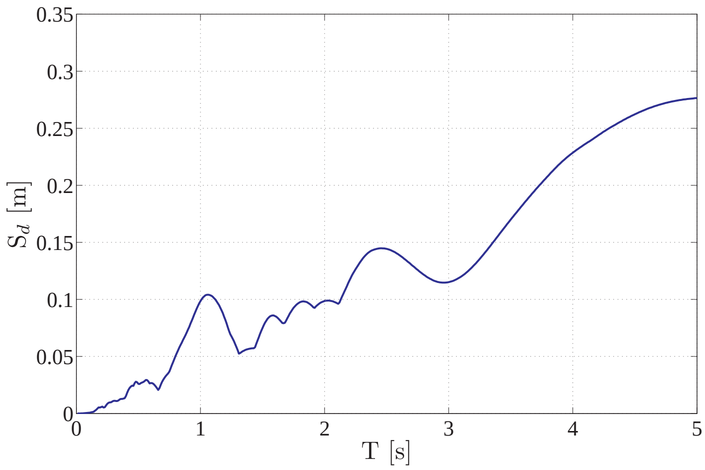
Figure 5.19: *Tohoku 2011 earthquake (Tsukidate station), ground acceleration.*



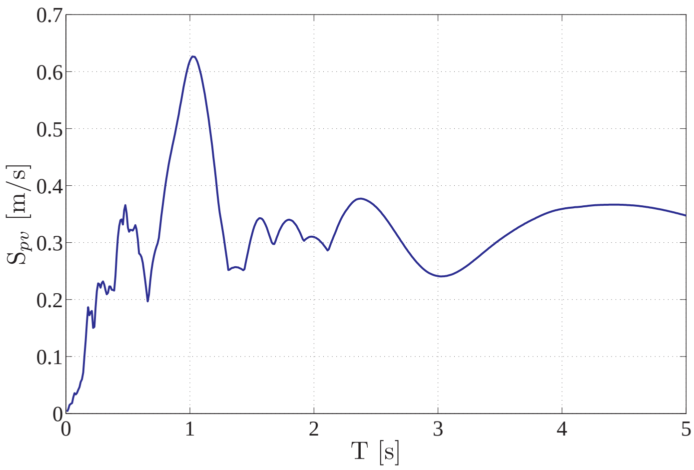
(a) Accelerogram. **(b)** Fourier amplitude spectrum.

Figure 5.20: *Tohoku 2011 earthquake (Sendai station), ground acceleration.*

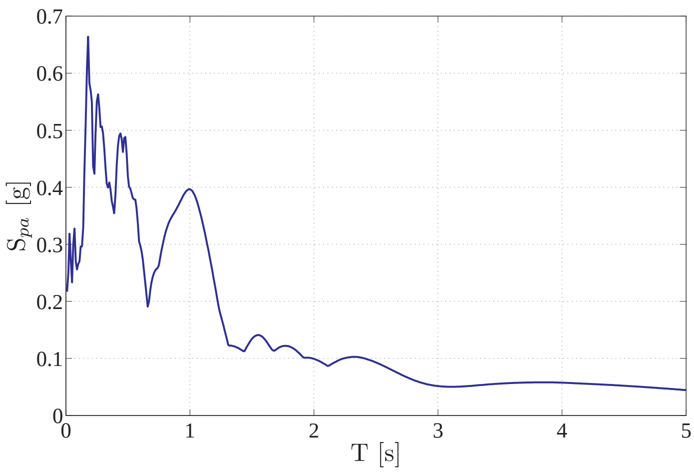
5.3.4 Response spectra



(a) Displacement response spectrum.

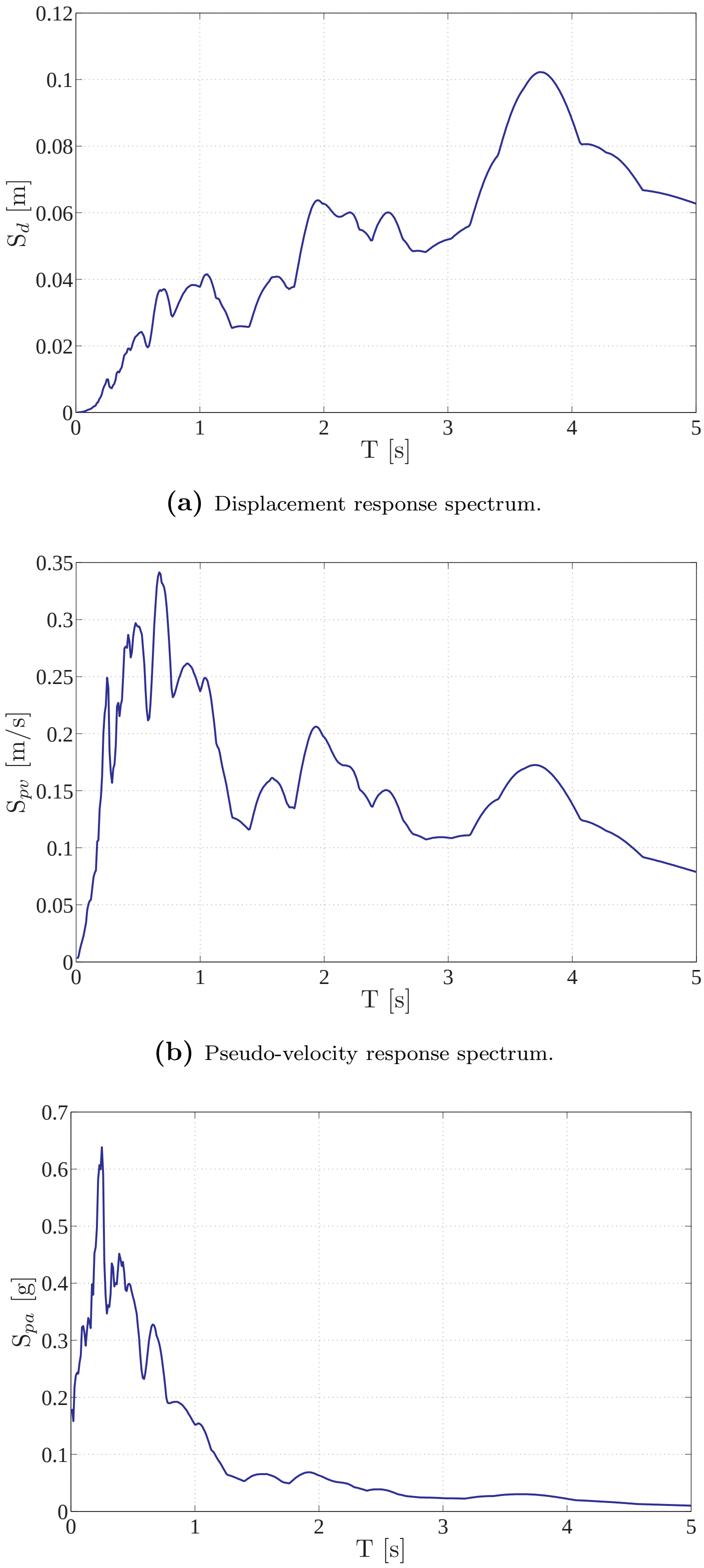


(b) Pseudo-velocity response spectrum.



(c) Pseudo-acceleration response spectrum.

Figure 5.21: *Long Beach 1933 earthquake, response spectra.*

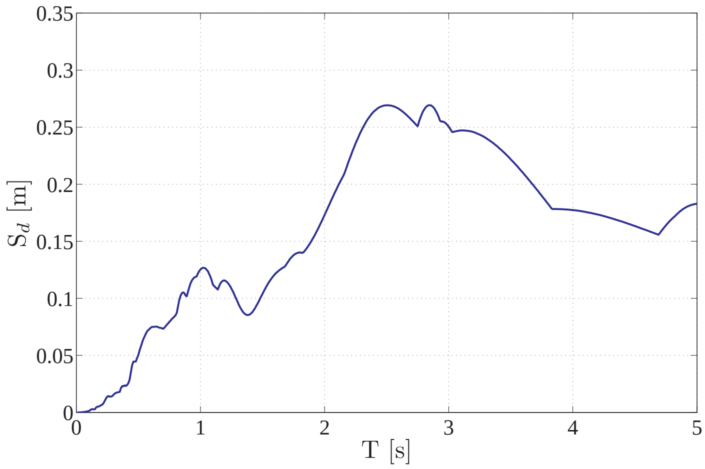


(a) Displacement response spectrum.

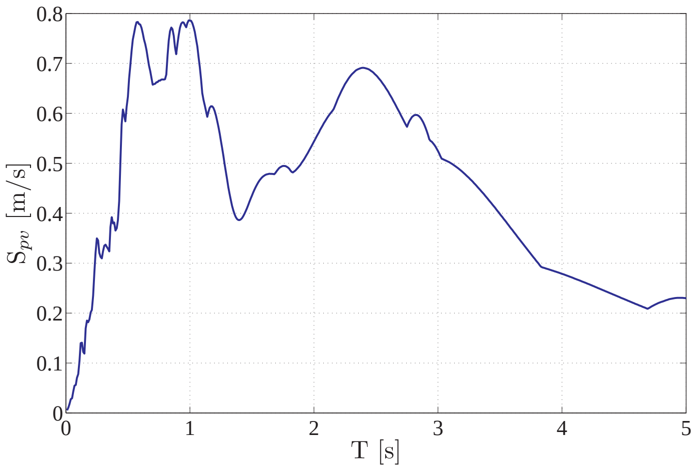
(b) Pseudo-velocity response spectrum.

(c) Pseudo-acceleration response spectrum.

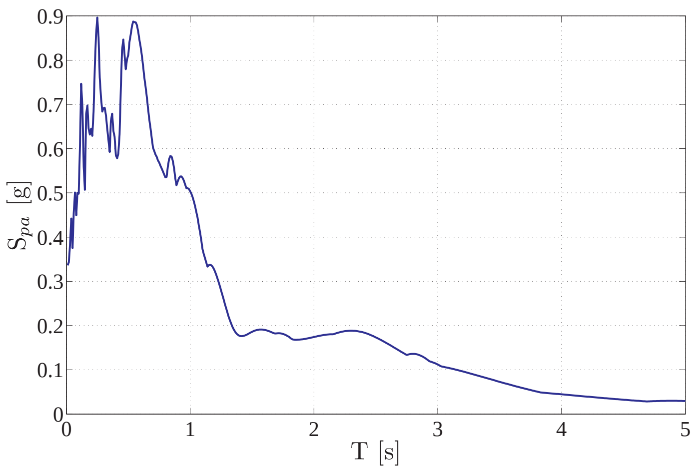
Figure 5.22: *El Centro 1934 earthquake, response spectra.*



(a) Displacement response spectrum.

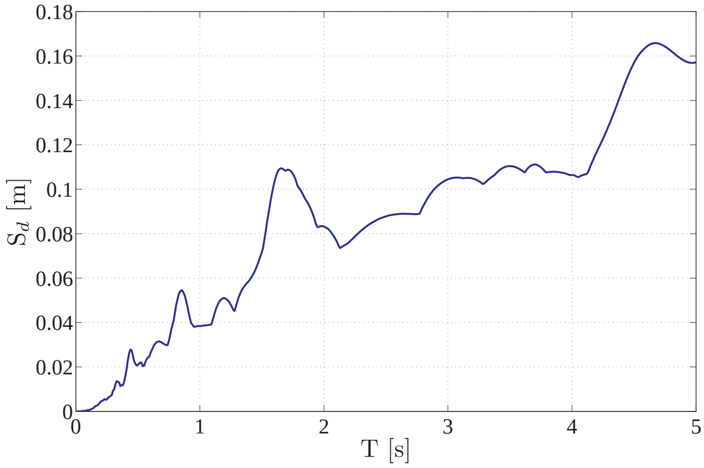


(b) Pseudo-velocity response spectrum.

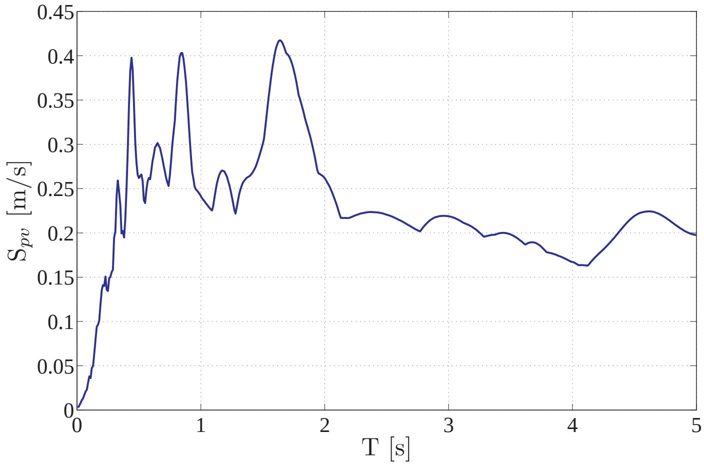


(c) Pseudo-acceleration response spectrum.

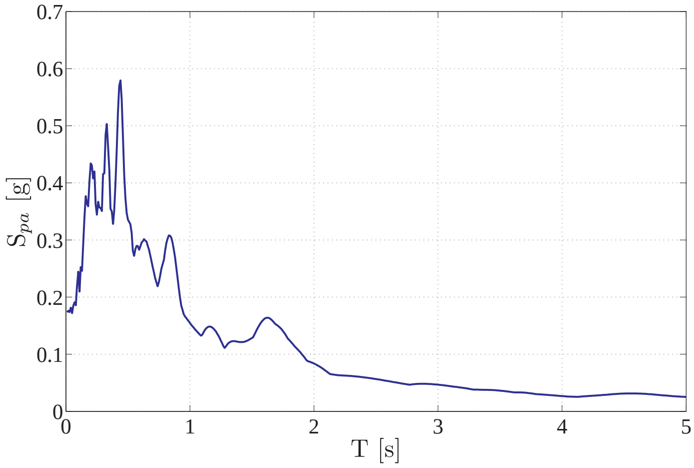
Figure 5.23: *Imperial Valley 1940 earthquake, response spectra.*



(a) Displacement response spectrum.

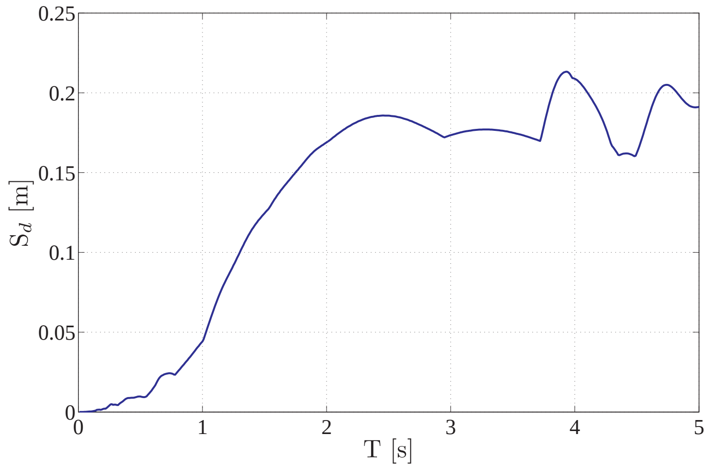


(b) Pseudo-velocity response spectrum.

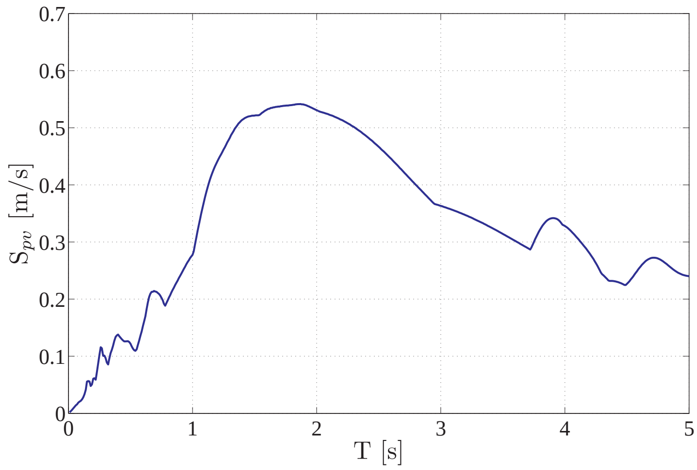


(c) Pseudo-acceleration response spectrum.

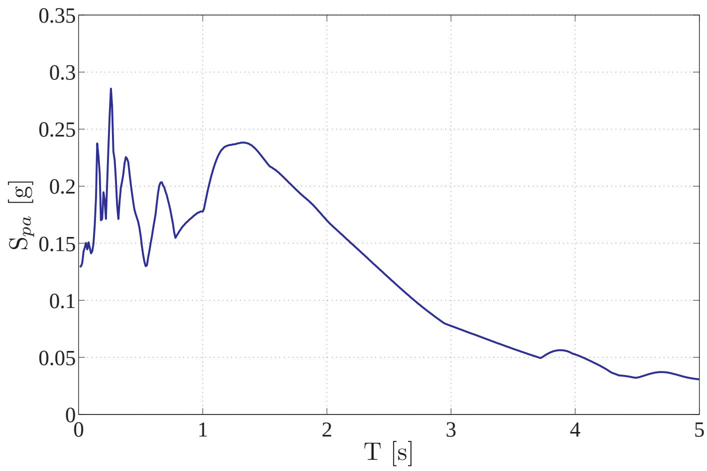
Figure 5.24: *Kern County 1952 earthquake, response spectra.*



(a) Displacement response spectrum.

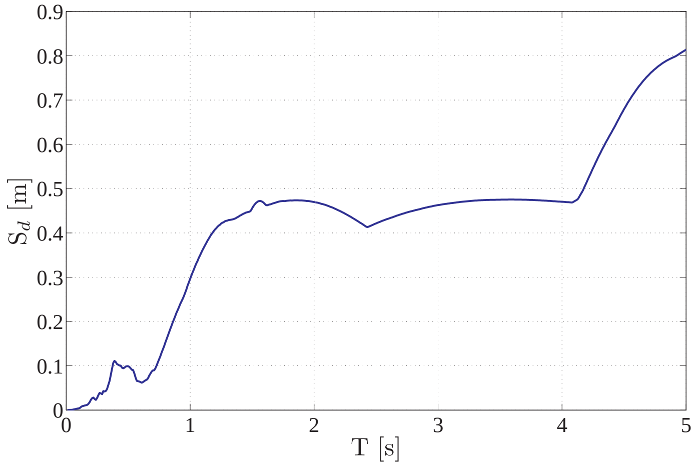


(b) Pseudo-velocity response spectrum.

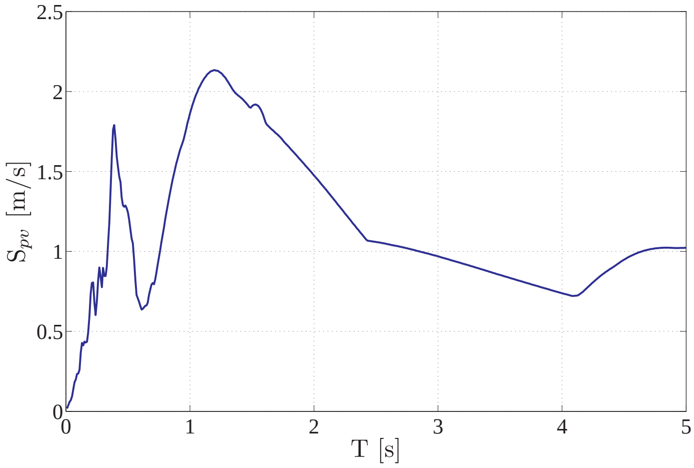


(c) Pseudo-acceleration response spectrum.

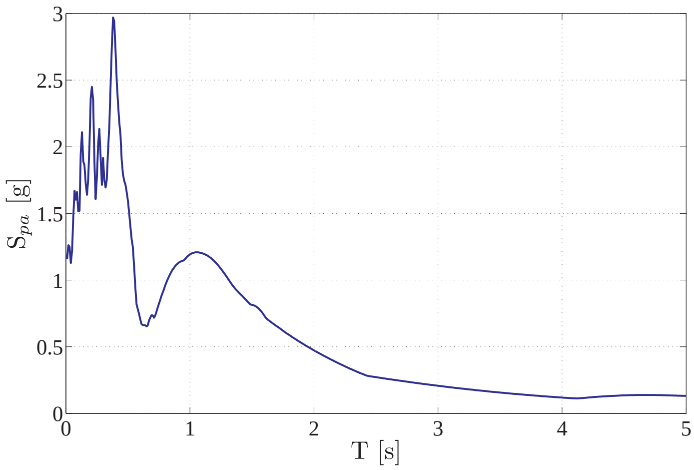
Figure 5.25: *Borrego Mountain 1968 earthquake, response spectra.*



(a) Displacement response spectrum.

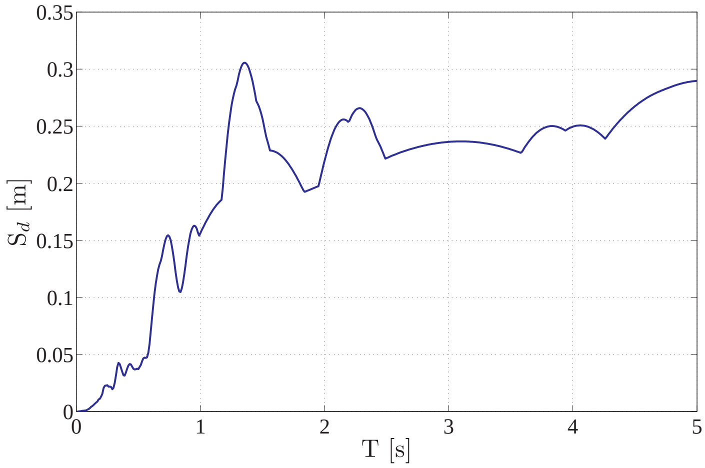


(b) Pseudo-velocity response spectrum.

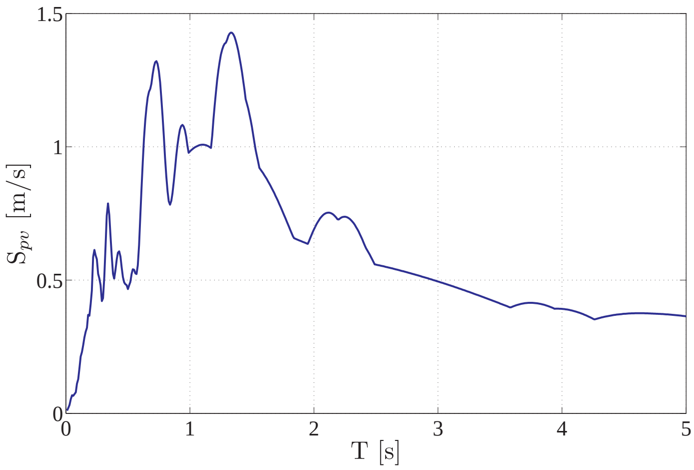


(c) Pseudo-acceleration response spectrum.

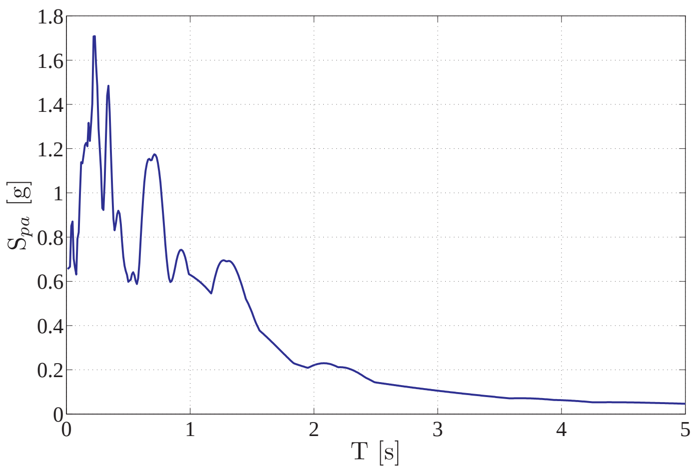
Figure 5.26: *San Fernando 1971 earthquake, response spectra.*



(a) Displacement response spectrum.

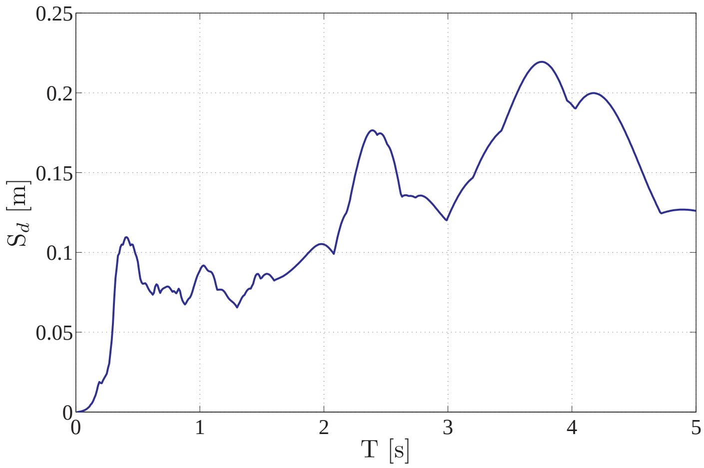


(b) Pseudo-velocity response spectrum.

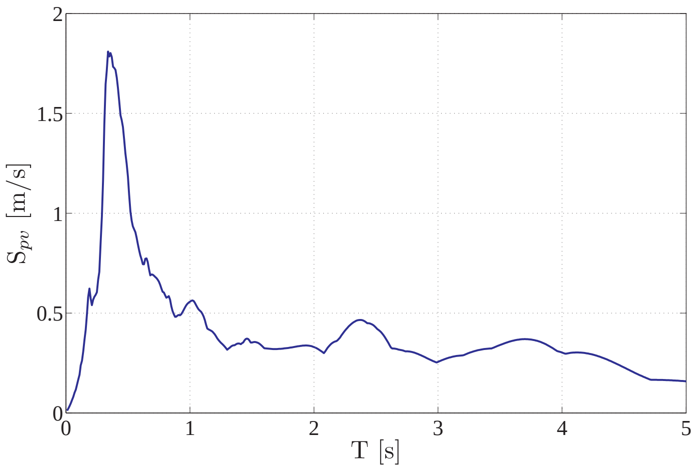


(c) Pseudo-acceleration response spectrum.

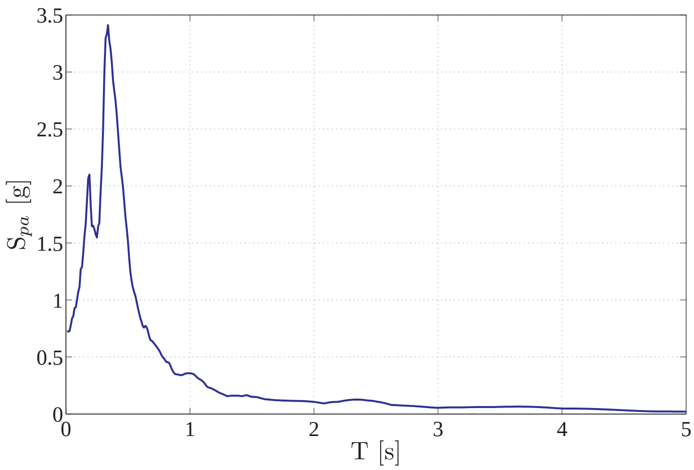
Figure 5.27: *Imperial Valley 1979 earthquake, response spectra.*



(a) Displacement response spectrum.

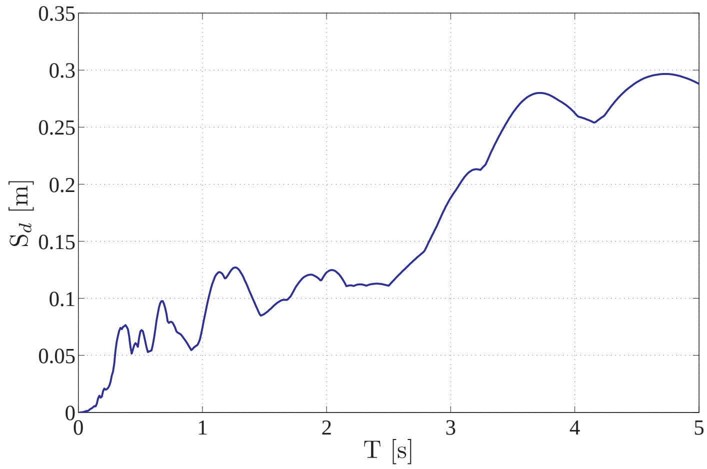


(b) Pseudo-velocity response spectrum.

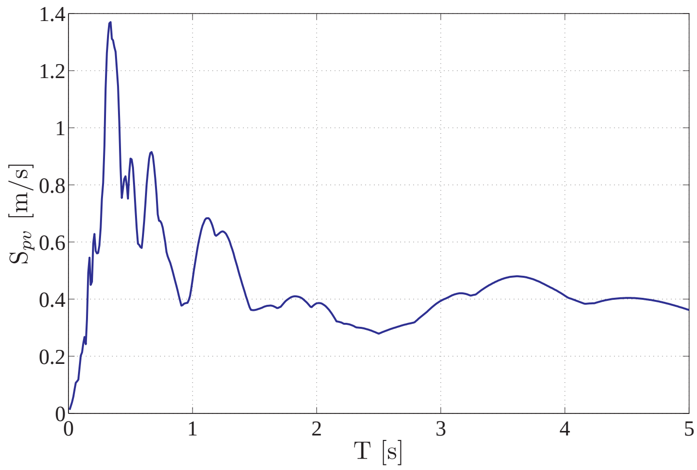


(c) Pseudo-acceleration response spectrum.

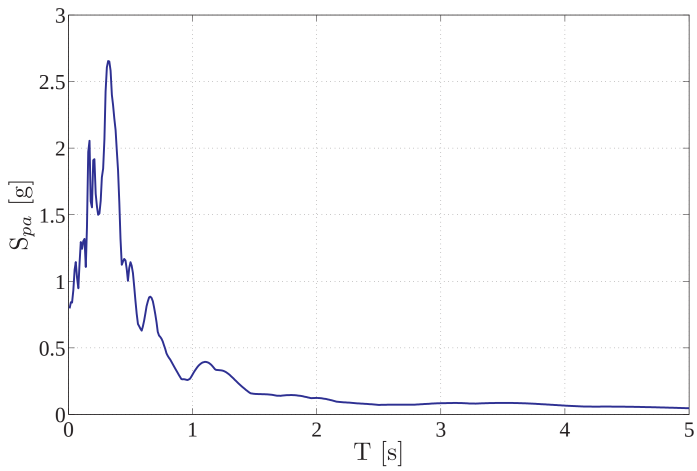
Figure 5.28: *Chile 1985 earthquake, response spectra.*



(a) Displacement response spectrum.

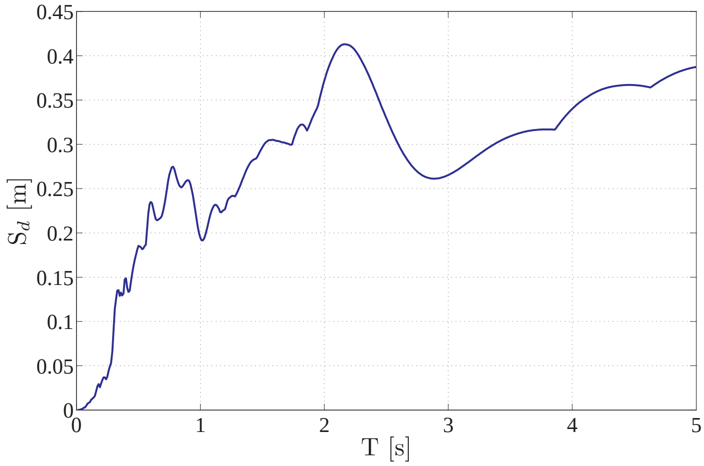


(b) Pseudo-velocity response spectrum.

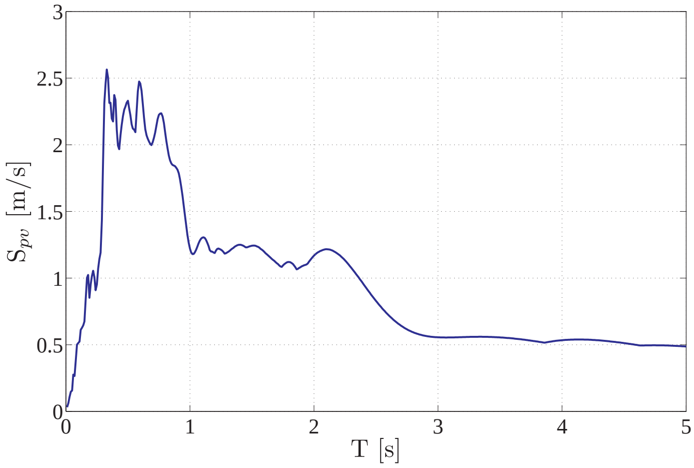


(c) Pseudo-acceleration response spectrum.

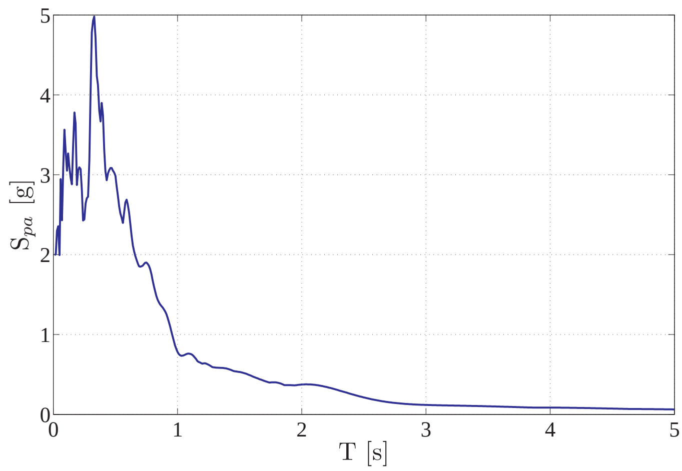
Figure 5.29: *Loma Prieta 1989 earthquake, response spectra.*



(a) Displacement response spectrum.

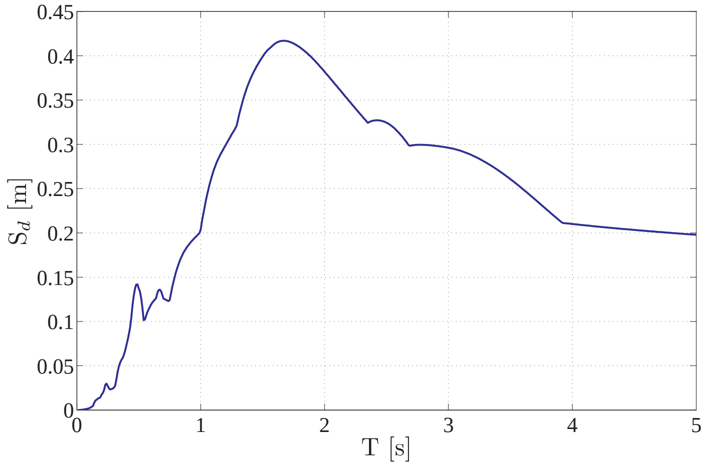


(b) Pseudo-velocity response spectrum.

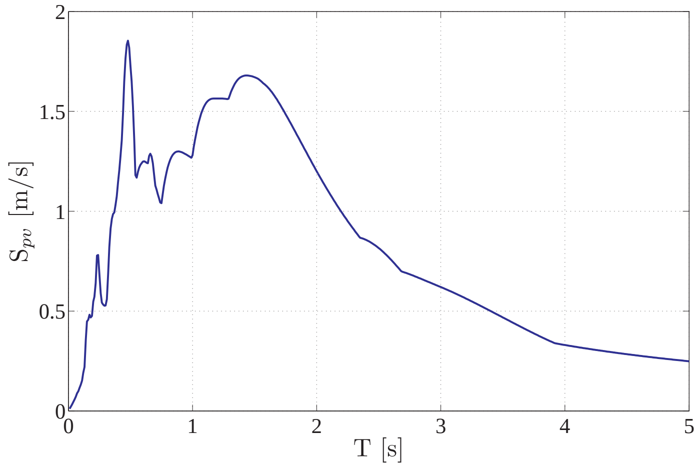


(c) Pseudo-acceleration response spectrum.

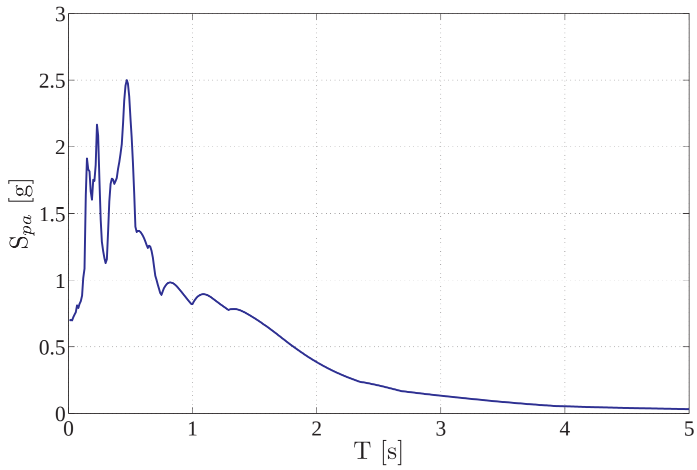
Figure 5.30: *Northridge 1994 earthquake, response spectra.*



(a) Displacement response spectrum.

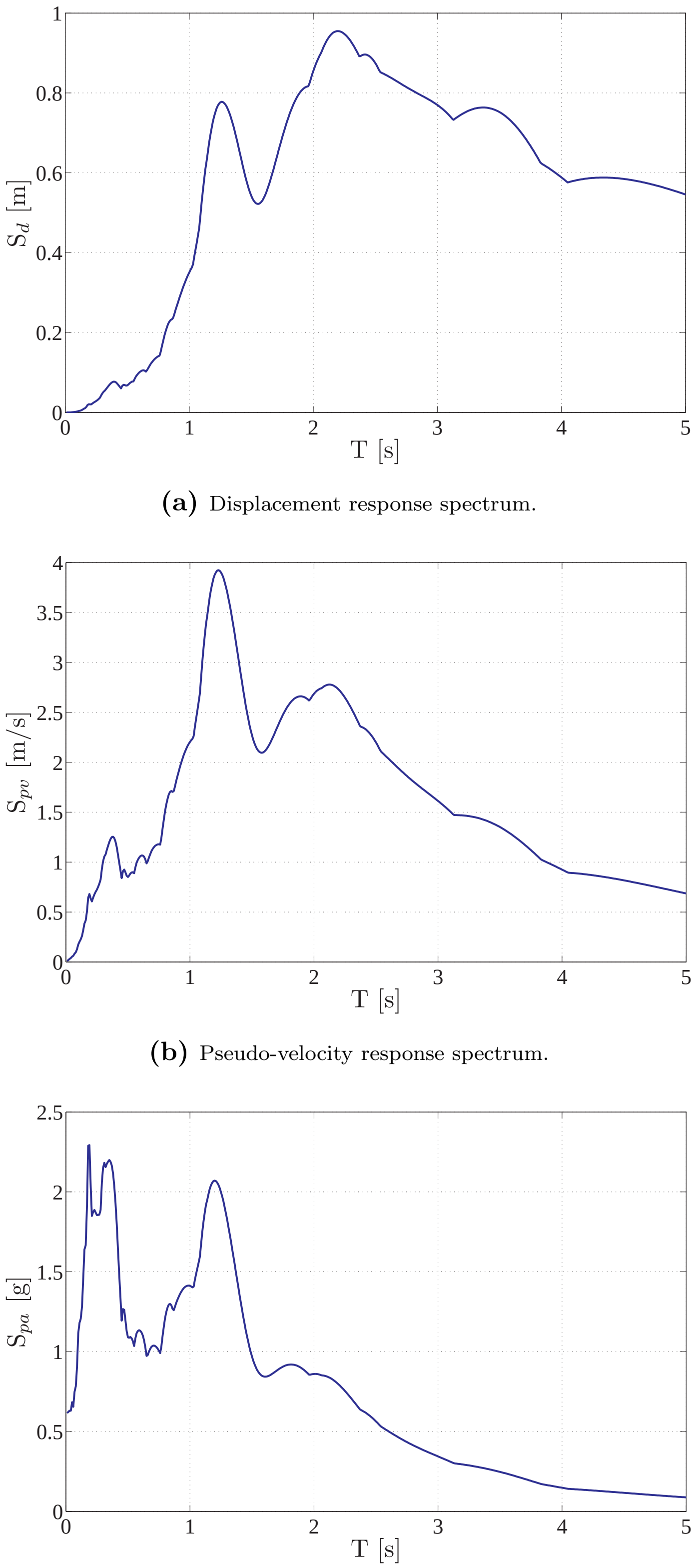


(b) Pseudo-velocity response spectrum.



(c) Pseudo-acceleration response spectrum.

Figure 5.31: *Kobe 1995 earthquake (Takarazuka station), response spectra.*

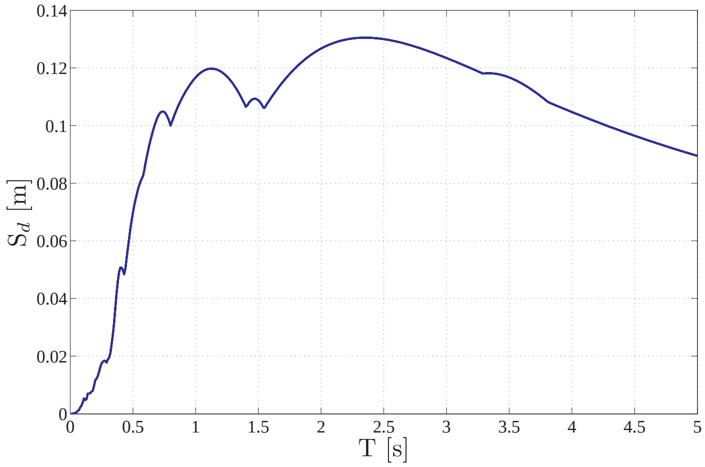


(a) Displacement response spectrum.

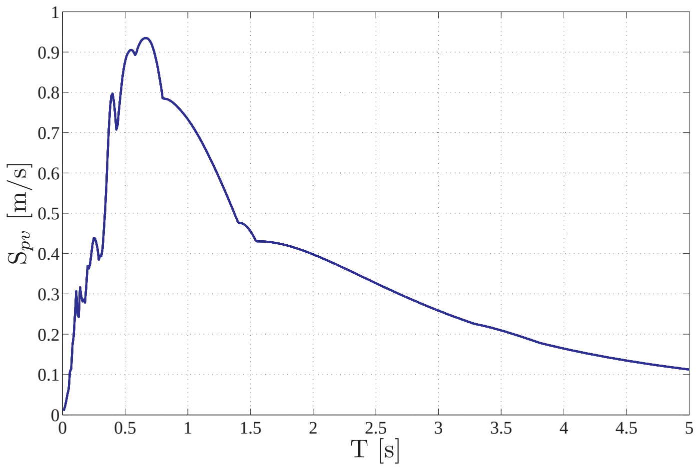
(b) Pseudo-velocity response spectrum.

(c) Pseudo-acceleration response spectrum.

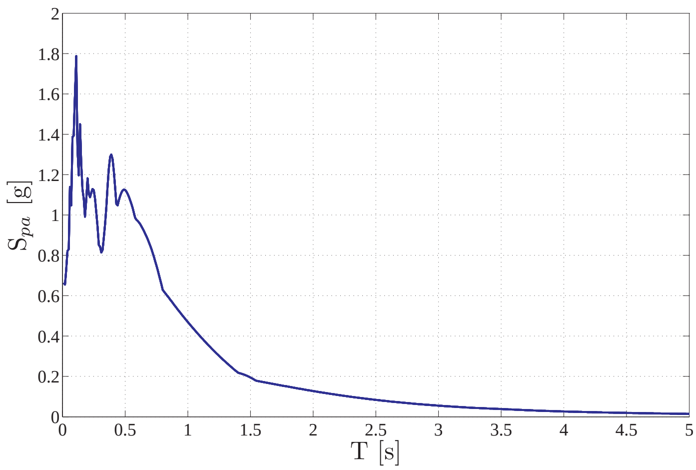
Figure 5.32: *Kobe 1995 earthquake (Takatori station), response spectra.*



(a) Displacement response spectrum.

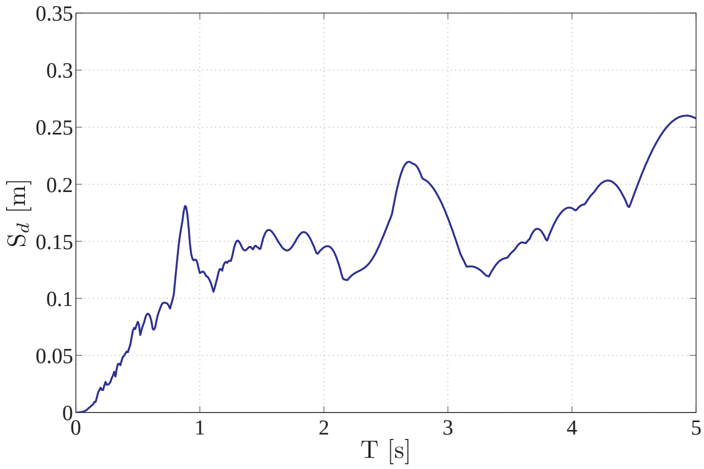


(b) Pseudo-velocity response spectrum.

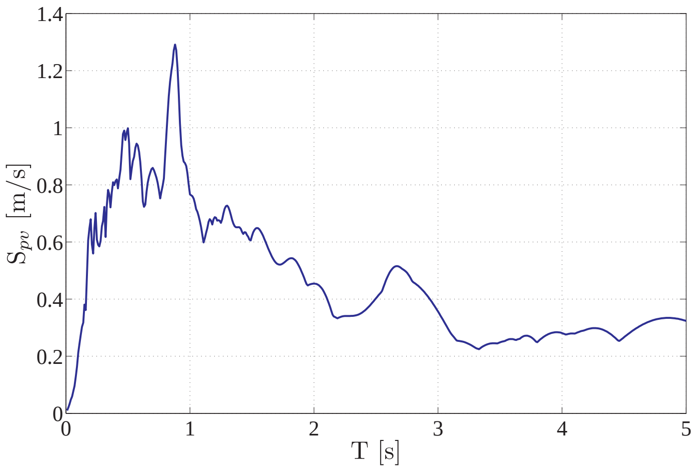


(c) Pseudo-acceleration response spectrum.

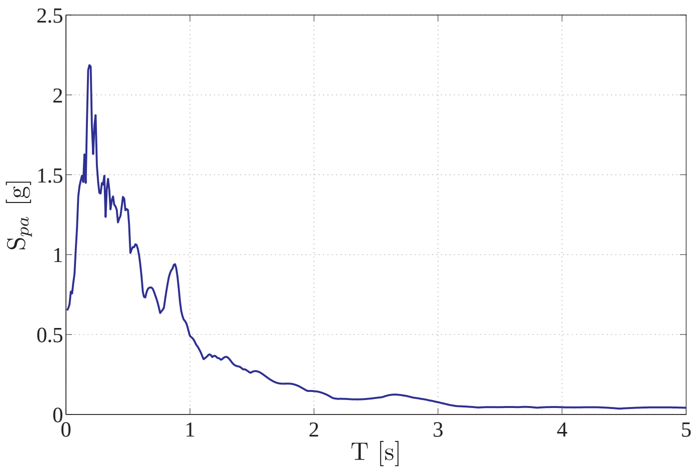
Figure 5.33: *L'Aquila 2009 earthquake, response spectra.*



(a) Displacement response spectrum.

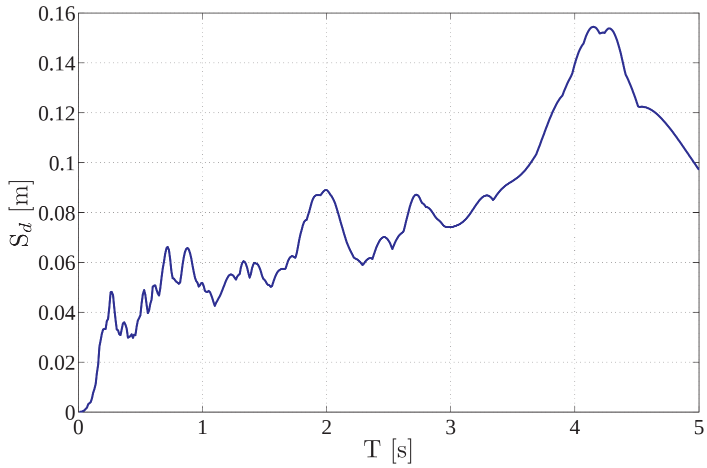


(b) Pseudo-velocity response spectrum.

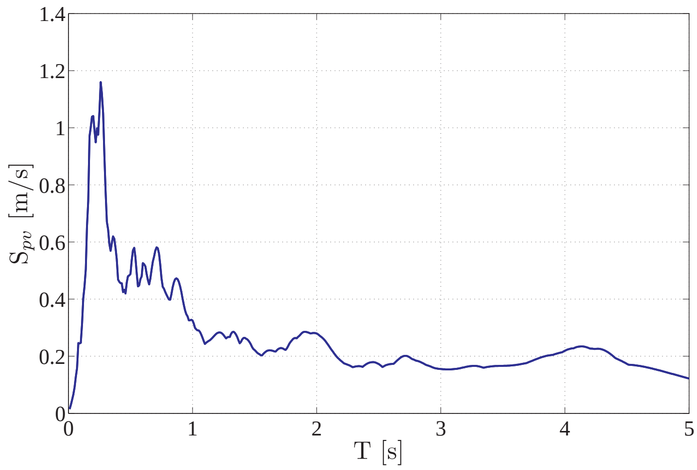


(c) Pseudo-acceleration response spectrum.

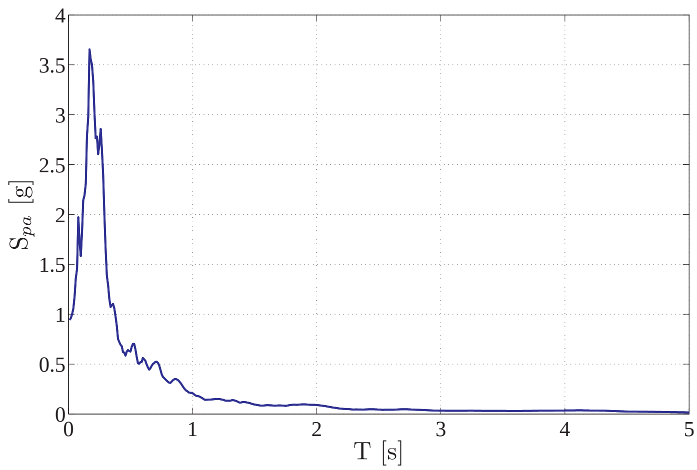
Figure 5.34: *Chile 2010 earthquake (Concepcion San Pedro station), response spectra.*



(a) Displacement response spectrum.

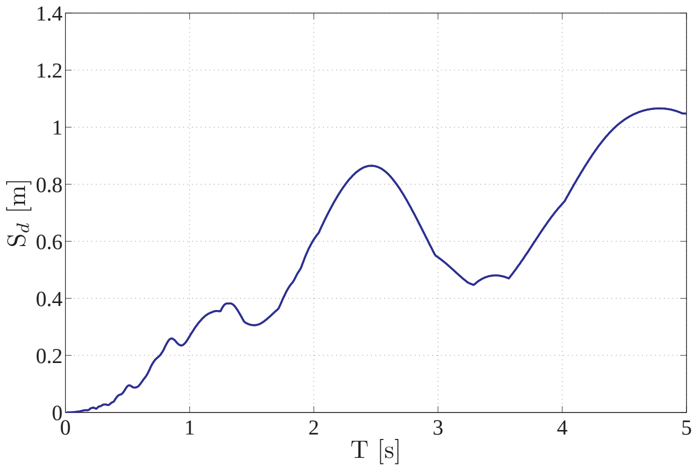


(b) Pseudo-velocity response spectrum.

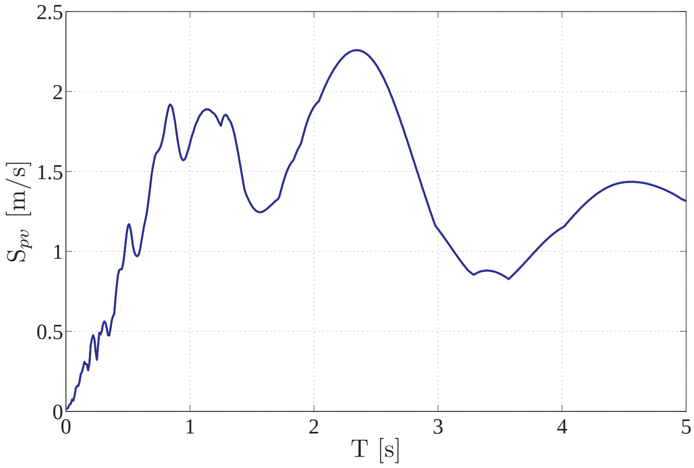


(c) Pseudo-acceleration response spectrum.

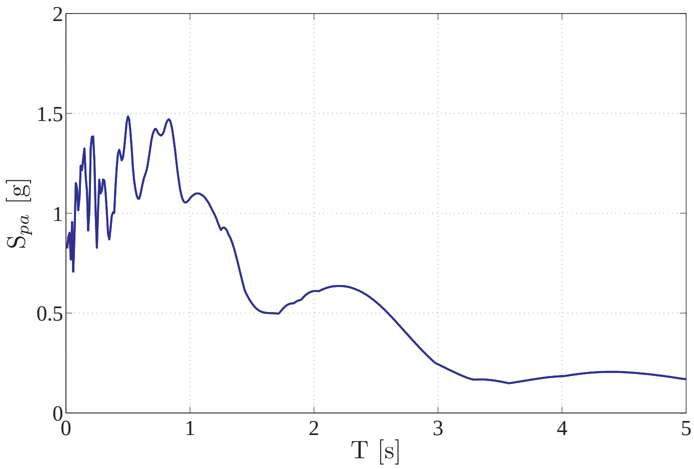
Figure 5.35: *Chile 2010 earthquake (Angol Station), response spectra.*



(a) Displacement response spectrum.

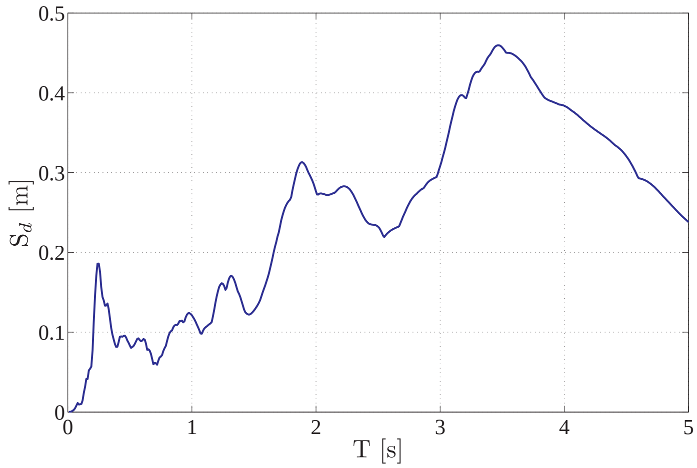


(b) Pseudo-velocity response spectrum.

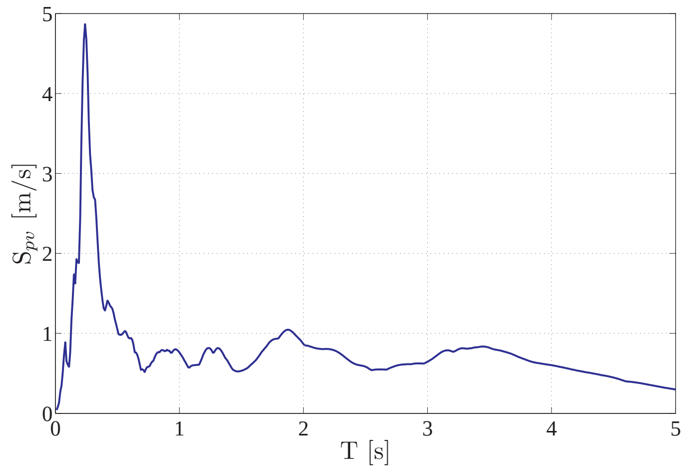


(c) Pseudo-acceleration response spectrum.

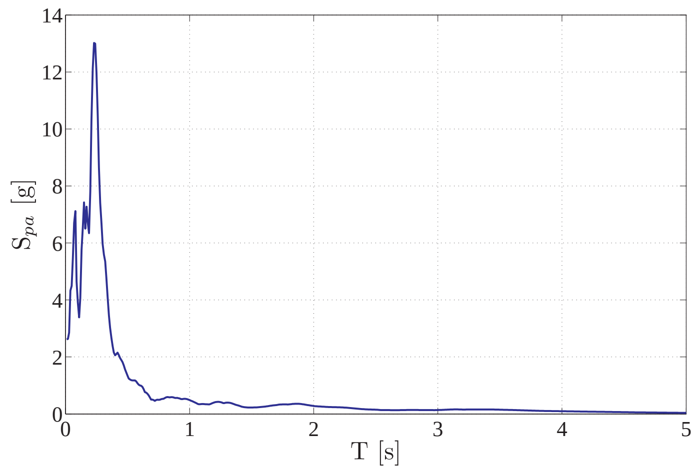
Figure 5.36: *New Zealand 2010 earthquake, response spectra.*



(a) Displacement response spectrum.

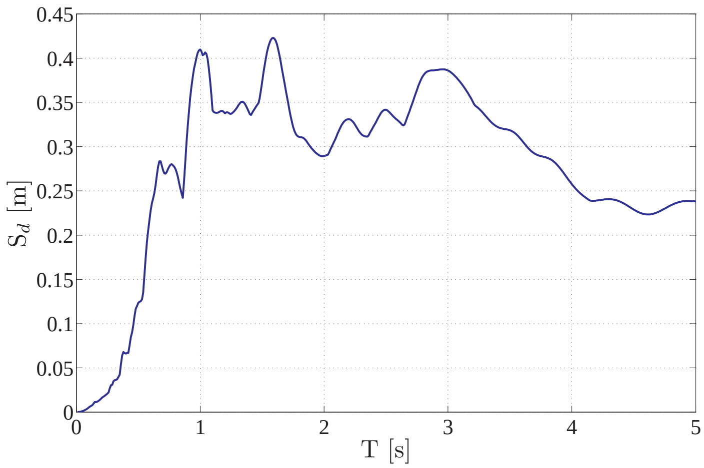


(b) Pseudo-velocity response spectrum.

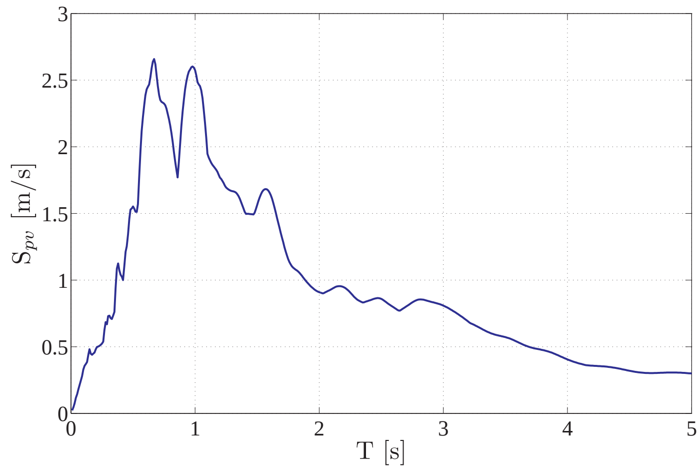


(c) Pseudo-acceleration response spectrum.

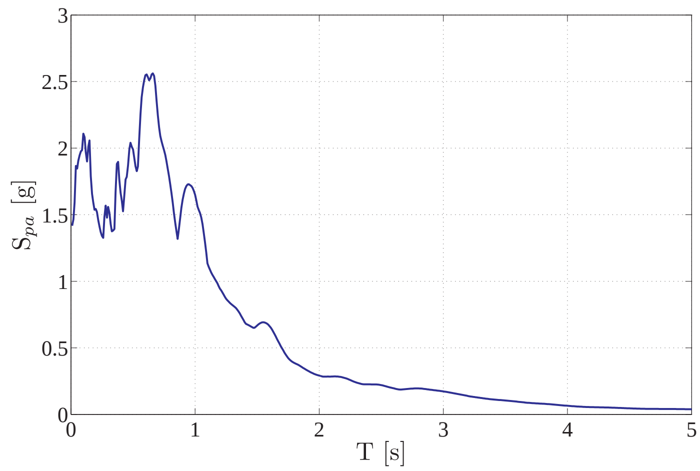
Figure 5.37: *Tohoku 2011 earthquake (Tsukidate station), response spectra.*



(a) Displacement response spectrum.



(b) Pseudo-velocity response spectrum.



(c) Pseudo-acceleration response spectrum.

Figure 5.38: *Tohoku 2011 earthquake (Sendai station), response spectra.*

5.4 Seismic tuning method

5.4.1 Statement of the tuning procedure

The proposed seismic tuning method is presented and explained in detail in this section. In principle, it shares some important features with the tuning methods previously described in Chapters 3–4. However, for the present purpose of the TMD seismic optimisation, new fundamental features will be introduced within the tuning process. In this sense, the innovative character of this tuning methodology concerns the embedding of the specific seismic input signal within the optimisation process, with the aim of achieving the optimum TMD parameters for the given case of structure and of earthquake event. This concept is certainly mainly endowed of a theoretical character but shall have also important implications in view of practical applications in the earthquake engineering context.

In this study, the target priority is the assessment of the TMD effectiveness in reducing the primary structure response, which therefore will play a main role within the tuning process. Hence, different suitable objectives, such as for instance the limitation of the stroke of the TMD (which may display significant relevance in practical cases), will be left to further investigations. However, the proposed method consists of a general procedure, which could easily allow for achieving optimum results also along different and multiple tuning indexes.

As pointed out in Section 5.2.3, the dynamic response of all the considered primary structures is substantially represented by the first mode of vibration, which therefore will be assumed as the reference objective within the tuning process. Specifically, the single passive TMD is tuned so as to reduce the amplitude of the response of the first mode of vibration, occurring at the top storey for the considered buildings. Further analyses with a larger number of control devices could reliably consider at least the second mode of vibration within the optimisation process.

Hence, considering the first mode of vibration only, as compared to Eq. (3.3), in the case of a MDOF primary structure, the mass ratio and the tuning frequency ratio are defined as follows:

$$\mu = \frac{m_{_T}}{M_{m,S,I}}\,, \qquad f = \frac{\omega_{_T}}{\omega_{S,I}}\,, \tag{5.14}$$

where $\omega_{S,I}$ is the first mode angular frequency of the primary structure, as displayed in Eq. (5.5). The TMD damping ratio is still defined as in Eq. (3.2).

As described for previous tuning problems presented in Chapters 3–4, the tuning process mainly concerns an optimisation process that aims at minimising a selected response quantity of the primary structure:

$$\min_{\mathbf{p}} \mathrm{J}(\mathbf{p}) \,, \qquad \mathbf{l}_b \leq \mathbf{p} \leq \mathbf{u}_b \,, \tag{5.15}$$

where $\mathbf{p}$, $\mathrm{J}(\mathbf{p})$, $\mathbf{l}_b$ and $\mathbf{u}_b$ represent the optimisation variables, the objective function, and the lower and the upper bounds on the optimisation variables, respectively. Again, the optimisation process will consider as free variables the frequency ratio f and the TMD damping ratio ζ_T, the mass ratio μ being instead fixed *a priori*.

This approach is motivated by two main reasons. First, the amount of added mass which composes the Tuned Mass Damper may be limited by a matter of practical design, i.e. excessive masses of the device could be counterproductive in terms of suitability and safety of the primary structure. The second issue, which is instead related to rather theoretical aspects, is pointed out by the optimisation process itself. Indeed, several previous trials [87, 91–93] have shown that, in case of a free mass ratio, this parameter tends to reach the upper limit that has been set, leading to a lower TMD damping ratio, so that to transfer the dynamic response entirely to the TMD. In this sense, a suitable upper limit to design guidelines could be established within the optimisation process.

The choice of the optimisation algorithm is, in principle, an important feature within the tuning process, especially when the excitation is non-deterministic, i.e. not modeled by analytical relationships, such as for the seismic input signals considered here. In this sense, despite that a large series of modern algorithms could ensure high optimisation performances (see e.g. [53, 106]), a classical numerical method shall look effective and sufficient for the solution of the present tuning problem. In particular, a nonlinear gradient-based optimisation algorithm available within MATLAB [109] has been adopted, based on Sequential Quadratic Programming (SQP). Further details on this framework are provided in Chapters 3–4.

In view of the analysis developed in the following, it is necessary to recall the relationship of the H_i norm, which can be defined as follows [133]:

$$\|x\|_i := \left(\sum_{n=1}^{N} |x_n|^i \right)^{\frac{1}{i}} , \qquad 1 < i < \infty , \tag{5.16}$$

where N is the total number of time samples of variable x in the assumed time window. In particular, for the present study the H_2 and H_∞ norms have been considered. The former is actually the Euclidian norm and concerns the whole time history, while the latter represents the peak response:

$$\|x\|_2 := \sqrt{\sum_{n=1}^{N} |x_n|^2} , \qquad \|x\|_\infty := \max_{1 \leq n \leq N} |x_n| . \tag{5.17}$$

5.4.2 Analysis of the objective function

The single objective function that is finally adopted in all the numerical tests that will be presented in the next section is the RMS displacement of the top storey of the frame structure:

$$\mathrm{J}(\mathbf{p}) = x_{S,n}^{RMS} = \frac{\|x\|_2}{N} , \tag{5.18}$$

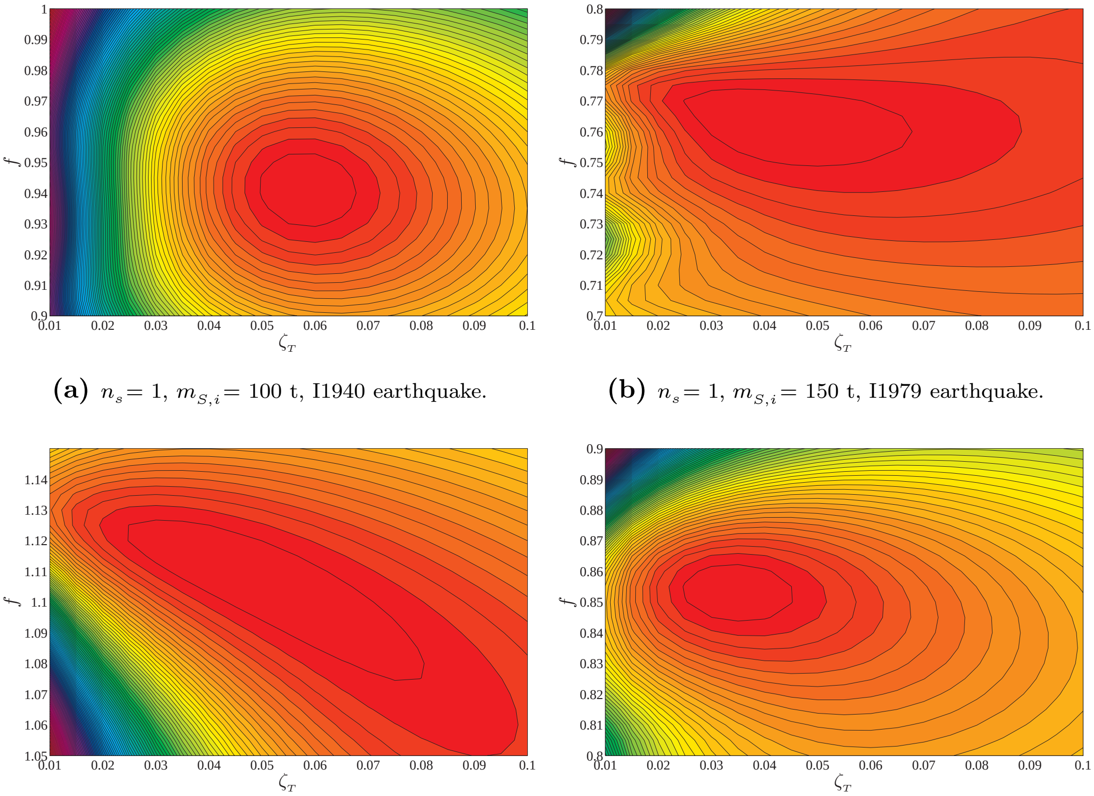
which reflects the choice of a H_2 approach, just normalised to the number of samples. The motivations of such a choice are the following. First of all, the considered structures display the common property of showing a dominant first bending mode of vibration; therefore, the maximum displacement is expected to occur at the level of the top storey. Second, the choice of a RMS indicator assures better efficiency within the optimisation process. Indeed, in preliminary trials, where different response indicators, in type (maximum value vs. RMS average, global vs. local, kinematic index vs. energy value) and location (different floors) have been investigated, it has been experienced that, in case of a seismic excitation, an average response quantity, rather than a maximum value, turns out a better objective function towards TMD tuning. This feature has been also spotted within the study presented in Chapter 4 and in several past researches [91, 92, 94]. Particularly, the present optimisation converges easily and consistently (conversely, the assumption of a maximum value as objective function may bring to convergence complications in some cases).

Moreover, following this choice, one can obtain a higher reduction of the global seismic response of the structure. In other words, a RMS estimate seems to set a reliable and efficient objective function for the global response of the structure in the entire time window of analysis. Besides the RMS displacement of the primary structure (at the top floor), it has been observed that RMS velocity and RMS acceleration (at the top floor) are also very suitable objective functions and even at almost the same computational cost.

Further experience gained in handling RMS objective functions has been reported in [91], where also the RMS average of the kinetic energy of the primary structure, as defined in Eq. (5.6), has been considered. It has been demonstrated that this choice could lead to a quite efficient TMD, but implies a higher computational cost, especially in case of primary structures with many degrees of freedom.

As seen in Chapters 3–4, a preliminary inspection of the objective function, nearby the expected minimum region, turn out useful in order to understand not only if the optimisation problem is well posed but also the sensitivity of the optimum point with respect to the optimisation variables. Hence, such an investigation has been proposed also for this study. Indeed, Fig. 5.39, which represents a meaningful extract of the outcomes of a wider analysis, displays the assumed objective function, reported in Eq. (5.18) (i.e. the RMS displacement of the top storey), for different cases of primary structure and seismic input signals, nearby the optimum region in the space (plane) of the free TMD parameters.

Such study turns out helpful in order to understand if the optimisation process is numerically well posed, and in particular if it is possible to locate the values of the free TMD parameters that correspond to a global minimum. For classical deterministic excitations this task results quite easy to be achieved, while in the case of seismic input the uncertainties related to the intrinsic nature of the earthquake signal may produce unexpected complications within the optimisation process. As proof of this expectation, the plots in Fig. 5.39 display quite different situations. The minimum region could be represented by an almost circular area, but it could also take a lengthened or irregular shape. Moreover, the parameters nearby the minimum point could assume values far from those usually achieved in case of ideal excitations (see e.g. Chapters 3–4). This issue mainly concerns the frequency ratio f^{opt} while the TMD damping ratio ζ_T^{opt} appears to be less sensitive to different input



(a) $n_s = 1$, $m_{S,i} = 100$ t, I1940 earthquake.

(b) $n_s = 1$, $m_{S,i} = 150$ t, I1979 earthquake.

(c) $n_s = 2$, $m_{S,i} = 150$ t, S1971 earthquake.

(d) $n_s = 10$, $m_{S,i} = 100$ t, N2010 earthquake.

Figure 5.39: *Global minimum regions related to different primary structures and earthquakes.*

conditions.

Hence, the preliminary investigation on the objective function outlined above suggests, within the tuning stage, to adopt quite lack bounds for the optimisation variables, so that to adapt the tuning process to this wide range of cases.

5.4.3 Optimisation method

In order to start the optimisation process, it is necessary to initialise the values of the two variable parameters f and ζ_T. In this sense, the initial evaluation of the free parameters has been based on the tuning formulas by Den Hartog [26], which provide general good initial estimates of the TMD parameters [91, 95]:

$$f = \frac{1}{1+\mu}, \quad \zeta_T = \sqrt{\frac{3\mu}{8(1+\mu)}}. \tag{5.19}$$

The lower and upper bound vectors on the two parameters f and ζ_T are also taken as follows:

$$\mathbf{l}_b = [0.45; 0.005], \qquad \mathbf{u}_b = [1.5; 0.5], \tag{5.20}$$

which represent quite wide intervals for the optimisation variables.

Besides the aforementioned bounds, as operated in Chapters 3–4 and in previous works [91–95] in the implemented algorithm various tolerances have been set for the different quantities involved in the optimisation process. In the present context, the tolerances adopted in the numerical algorithm have been taken as follows:

- Tolerance on the variable parameters: 10^{-6};
- Tolerance on the constraint violations: 10^{-6};
- Tolerance on the objective function: 10^{-6}.

These values assure a good compromise between obtained convergence and achieved accuracy. Other given external constraints are the maximum number of iterations and the maximum number of function evaluations, fixed at 200 and 500 instances, respectively.

The dynamic response of the structural system is computed in the time domain, by direct integration of the equations of motion through the (unconditionally stable) implicit Newmark average acceleration method, with the following usual integration parameters, see e.g. [20] (and typical Newmark's equations in Step 3 below):

$$\alpha = \frac{1}{4}, \quad \beta = \frac{1}{2}. \tag{5.21}$$

The Newmark time integration method is implemented through a standard numerical procedure composed of several steps, which are briefly resumed as follows (see e.g. [20]):

Step 1: appropriate choice of the time step. The integration method assumed here is unconditionally stable, therefore the time step selection is not limited by stability requirements. However, to reach good accuracy, the time interval should be assumed quite small. Hence, the recording sample of the seismic signal has been considered in principle.

Step 2: setting of the initial conditions on the vectors of initial displacement and velocity of the structural system. It has been assumed that the system is initially at rest (homogeneous initial conditions):

$$\mathbf{x}_0 = \mathbf{0}\,, \qquad \dot{\mathbf{x}}_0 = \mathbf{0}\,. \tag{5.22}$$

Then, the initial acceleration vector is obtained from equations of motion (5.7) as follows:

$$\ddot{\mathbf{x}}_0 = \mathbf{M}^{-1}(\mathbf{F}_0 - \mathbf{C}\dot{\mathbf{x}}_0 - \mathbf{K}\mathbf{x}_0)\,, \tag{5.23}$$

where $\mathbf{F}_0$ is the value of the external force vector $\mathbf{F}$ at $t = t_0 = 0$. Thus, in the seismic case:

$$\mathbf{F}(t) = -\mathbf{M}\,\mathbf{r}\,\ddot{x}_g(t)\,, \quad \mathbf{F}_0 = \mathbf{F}(t_0) = -\mathbf{M}\,\mathbf{r}\,\ddot{x}_g(t_0)\,. \tag{5.24}$$

Step 3: evaluation of displacements, velocities and accelerations at each generic time instant t_{i+1}, starting with $i = 0$:

$$\begin{aligned}\mathbf{x}_{i+1} = {} & \left(\frac{1}{\alpha\,\Delta t^2}\mathbf{M} + \frac{1}{\beta\,\Delta t}\mathbf{C} + \mathbf{K}\right)^{-1} \cdot \\ & \cdot \left\{\mathbf{F}_{i+1} + \mathbf{M}\left[\frac{1}{\alpha\,\Delta t^2}\mathbf{x}_i + \frac{1}{\alpha\,\Delta t}\dot{\mathbf{x}}_i + \left(\frac{1}{2\alpha} - 1\right)\ddot{\mathbf{x}}_i\right] + \right. \\ & \left. + \mathbf{C}\left[\frac{\beta}{\alpha\,\Delta t}\mathbf{x}_i + \left(\frac{\beta}{\alpha} - 1\right)\dot{\mathbf{x}}_i + \left(\frac{\beta}{2\alpha} - 1\right)\Delta t\,\ddot{\mathbf{x}}_i\right]\right\},\end{aligned} \tag{5.25}$$

$$\ddot{\mathbf{x}}_{i+1} = \frac{1}{\alpha\,\Delta t^2}(\mathbf{x}_{i+1} - \mathbf{x}_i) - \frac{1}{\alpha\,\Delta t}\dot{\mathbf{x}}_i - \left(\frac{1}{2\alpha} - 1\right)\ddot{\mathbf{x}}_i\,, \tag{5.26}$$

$$\dot{\mathbf{x}}_{i+1} = \dot{\mathbf{x}}_i + \big[(1-\beta)\ddot{\mathbf{x}}_i + \beta\,\ddot{\mathbf{x}}_{i+1}\big]\Delta t\,. \tag{5.27}$$

Step 4: repetition of Steps 1-3 until the end of the time window of analysis.

The Newmark time solver has been linked to the optimisation process in a closed-loop algorithm that allows for best tuning at given seismic input. A synoptical flowchart of the tuning algorithm is sketched in Fig. 5.40 and can be briefly resumed as follows.

First, a preliminary analysis is carried out (initialisation): the primary structure parameters are defined, by operating a modal analysis of the main structure and the starting values of the TMD tuning parameters are set, as defined above. A first time integration of the equations of motion gives the initial seismic response. The optimisation process is then started. The optimisation algorithm takes control of the iterative procedure by varying the TMD parameters within the chosen admissible bounds and calls iteratively the Newmark time solver to update the system response and to relate it to the previous one. Once all set tolerances on both TMD parameters and response indexes assumed as objective functions are fulfilled, the final optimum TMD parameters are obtained and the corresponding seismic response is recorded. The iterative loops may continue only until when the maximum number of iterations is not exceeded (Fig. 5.40).

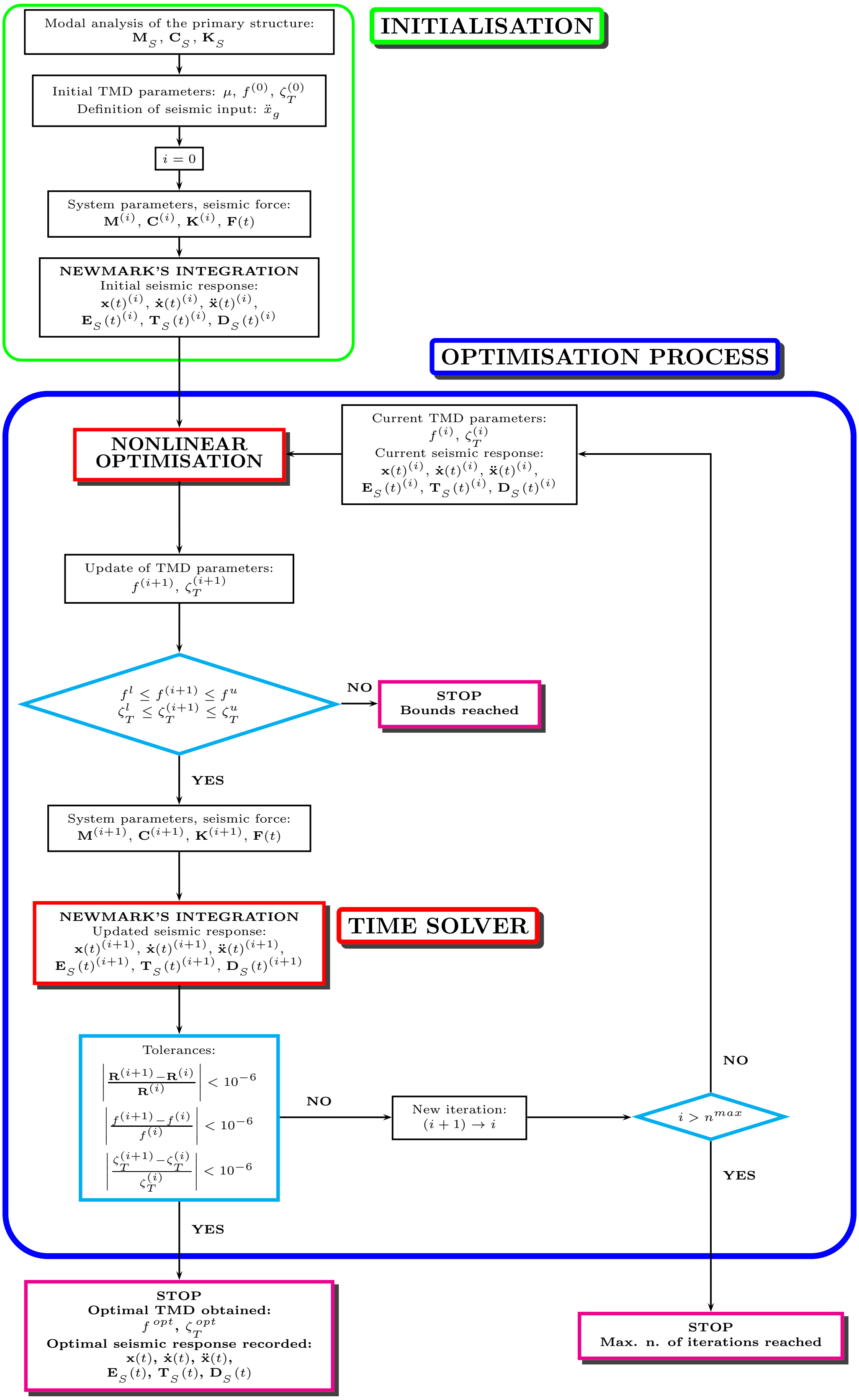


Figure 5.40: *Flowchart of the proposed numerical algorithm.*

5.5 Optimum TMD parameters

The optimum TMD parameters, obtained by means of the tuning methodology presented previously, for all the considered primary structures and seismic input signals, have been gathered in Tables 5.18–5.29 and will be discussed in this section.

In particular, for the present study, the optimum TMD parameters have been evaluated for a fixed primary structure damping ratio $\zeta_{S,I} = 0.05$ and for the following mass ratios μ:

$$\boldsymbol{\mu} = [0.01, 0.02, 0.03, 0.05, 0.07, 0.10] \,, \tag{5.28}$$

which are representative of possible engineering applications and, at the same time, allow for reading possible trends. In this sense, in Tables 5.18–5.33 the optimum reference TMD parameters obtained with Den Hartog tuning formulas [26] have been also reported, which have been also assumed for the evaluation of the starting point of the optimisation process, as pointed out in Eq. (5.19). Such additional results have been reported so that to provide an useful reference with respect to the outcomes achieved with the proposed tuning method.

The results related to the optimum frequency ratio f^{opt} are discussed first. This parameter displays, as a basic trend, a decreasing value at increasing mass ratio μ, which is a situation that usually occurs for the frequency ratio, independently on the dynamic excitation.

In particular, f^{opt} generally takes values from about 0.9-1 at $\mu = 0.01$ to about 0.7-0.8 at $\mu = 0.10$, reflecting the trend for f^{opt} obtainable for ideal deterministic loading (see e.g. Chapter 3), as for instance pointed out also by the values coming from Den Hartog's tuning. Within this general statement, several differences can be noted from the range of considered cases, mainly due to the large variability of the whole set of considered cases. In detail, four main trends are recovered, by reading the results at increasing mass ratio, i.e. starting from $\mu = 0.01$, either relatively high or low f^{opt} have been obtained, that then decrease either steeply or smoothly.

In some cases, f^{opt} takes values larger than 1, which is usually recognised as a sort of threshold value by all the tuning methods in the literature; when this situation occurs, the parameter could tend to assume almost constant values, with marked insensitivity with respect to the mass ratio μ. However, such trend appears to occur also for other cases, specifically when f^{opt} settles down to at about 0.8–0.9, which fact denotes an almost fixed location of the minimum region of the objective function.

Other cases display a sudden change of the value of f^{opt} for neighbouring values of μ, which fact denotes a sort of 'knee' on the parameter trend. It is remarkable that, in general, the particular cases of values and trends of the optimum frequency ratio f^{opt} stated above are recovered, especially for the primary structures characterised by the shortest periods, i.e. the stiffer ones, while the buildings with larger periods appear to provide more regular trends.

As previously recovered for the frequency ratio f^{opt}, also the optimum TMD damping ratio ζ_T^{opt} displays a general trend, i.e. the value increases at increasing mass ratio μ. In this sense, also ζ_T^{opt} confirms the typical behaviour obtainable for ideal deterministic loading, such as harmonic or white noise excitations, as confirmed by the numerical values coming from Den Hartog tuning formulas.

In particular, the values assumed by ζ_T^{opt} are from about 0.01-0.09 at $\mu = 0.01$ to about 0.10-0.30 at $\mu = 0.10$. A noticeable variability of ζ_T^{opt} can be easily recognised, even if the trends exhibit higher regularity with respect to those related to the optimum frequency ratio f^{opt}.

The parameter takes quite low values for small mass ratios, which fact denotes a quite lightly damped TMD, whose effect on the dynamic response of the structural system is therefore demanded to the resonance condition of the control device with respect to the primary structure, which is represented by a frequency ratio close to 1.

As explained before, at increasing μ, ζ_T^{opt} increases as well, but it generally reaches values slightly lower than 0.2. In some cases, ζ_T^{opt} takes almost constant values, independently on the value of μ, corresponding to the cases where also the optimum frequency ratio shows negligible changes. This feature indicates a sort of fixed location of the optimum region, and highlights the fact that an optimum TMD for the considered context could be characterised by parameters slightly different than those predictable by known tuning formulas.

Finally, as recovered before for f^{opt}, the cases where ζ_T^{opt} displays usual values and trends are mainly concentrated in correspondence of the primary structures with the shortest periods, especially for $n_s = 1$, $m_{S,i} = 100$ t.

As a general consideration, the results provided here revealed different positive features. First, and most important, it appears that the optimum TMD parameters in the context of a seismic input can be obtained, and usually provide parameters close to those obtainable for ideal excitations, which therefore allow for possible modelling (curve fitting, see Chapter 3) and comparison with existing tuning formulas from the literature.

Table 5.18: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 1$, $m_{S,i} = 100\ t$.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.920645	0.919602	0.901429	0.814898	0.4500	0.4500	0.0217289	0.063985	0.103746	0.164666	0.395735	0.489788
E1934	0.937642	0.921427	0.895968	0.853111	0.810863	0.775014	0.036	0.0702549	0.0889263	0.121757	0.142362	0.166987
I1940	0.969452	0.941223	0.919395	0.879576	0.854648	0.787292	0.0387758	0.0580696	0.0720249	0.0945694	0.114642	0.152941
K1952	0.907136	0.897866	0.884004	0.778507	0.741537	0.689449	0.0642321	0.0871932	0.108546	0.151901	0.166043	0.206891
B1968	0.968161	0.973853	0.970871	0.729458	0.662631	0.621572	0.0211102	0.0484839	0.104511	0.10023	0.147736	0.159259
S1971	1.00677	1.00819	0.90839	0.869496	0.828352	0.782548	0.0329682	0.0510251	0.10885	0.122197	0.143294	0.161718
I1979	0.950637	0.898232	0.885651	0.869959	0.82076	0.4500	0.0264901	0.05355	0.0675506	0.106745	0.167365	0.305462
C1985	0.969058	0.873116	0.706758	0.697192	0.680746	0.4500	0.0665463	0.128006	0.0777076	0.107809	0.130602	0.419076
L1989	0.868738	0.874922	0.879692	0.88594	0.88164	0.4500	0.0181092	0.0406204	0.055992	0.0838671	0.136006	0.30581
N1994	0.969396	0.889538	0.868576	0.853652	0.837003	0.802753	0.0486096	0.071291	0.0673451	0.0823385	0.101579	0.140577
K1995TZ	1.01261	0.984304	0.839557	0.805728	0.745111	0.698109	0.0209108	0.0683007	0.124892	0.127195	0.158723	0.16594
K1995TK	1.09451	0.944079	0.91654	0.883265	0.84285	0.4500	0.0116056	0.166071	0.174435	0.193552	0.223684	0.440592
A2009	0.971906	0.970048	0.961454	0.93639	0.874562	0.822881	0.0317942	0.0488232	0.0659905	0.103356	0.134637	0.140363
C2010A	0.969939	0.949231	0.926814	0.4500	0.770772	0.803178	0.0430597	0.0830568	0.116371	0.130031	0.5000	0.5000
C2010C	0.915577	0.892536	0.839042	0.805899	0.776268	0.718548	0.0557557	0.0801069	0.0969567	0.109988	0.134493	0.174678
N2010	0.912196	0.911665	0.900951	0.791426	0.780403	0.756509	0.0429611	0.0640239	0.0915753	0.10791	0.119987	0.153299
T2011T	0.684597	0.68114	0.615442	0.586669	0.570523	0.542437	0.0227415	0.0433649	0.0708991	0.0822605	0.0918589	0.110844
T2011S	0.92923	0.91413	0.903775	0.872873	0.832452	0.788892	0.0453103	0.067916	0.0857511	0.119387	0.14204	0.163791
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.19: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 1$, $m_{S,i} = 150$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.944376	0.931815	0.918674	0.675738	0.672516	0.664564	0.0373361	0.0582525	0.0820564	0.0737383	0.0873016	0.106583
E1934	0.927315	0.930755	0.928098	0.899874	0.870479	0.784569	0.0342316	0.0537435	0.0720244	0.109444	0.141431	0.213999
I1940	1.00134	0.89582	0.897198	0.89343	0.875025	0.79544	0.057711	0.0584853	0.0690086	0.0923933	0.126109	0.176615
K1952	0.90674	0.900727	0.898923	0.872635	0.834549	0.701541	0.0368268	0.0445301	0.0620276	0.101732	0.133523	0.186432
B1968	0.953192	0.920751	0.914141	0.900853	0.823111	0.782415	0.0343691	0.0477259	0.0538	0.0799846	0.114946	0.120939
S1971	0.945761	0.927698	0.919087	0.904717	0.893799	0.850202	0.045609	0.0665657	0.0813221	0.101614	0.119489	0.182163
I1979	0.887738	0.764146	0.756349	0.740332	0.668631	0.622637	0.0484407	0.0396287	0.0685317	0.110158	0.153763	0.165965
C1985	0.895799	0.89092	0.883632	0.868125	0.839018	0.796725	0.0371034	0.0500494	0.0602156	0.0804044	0.104894	0.128634
L1989	0.832497	0.646782	0.645857	0.644201	0.641268	0.623953	0.0150754	0.0183141	0.029203	0.0653777	0.1054	0.161588
N1994	1.01606	1.00846	0.955743	0.879004	0.828672	0.791747	0.0513081	0.073289	0.114767	0.150825	0.161106	0.168125
K1995TZ	1.0066	0.998278	0.905104	0.874784	0.858589	0.833091	0.0301535	0.0541064	0.0963932	0.0945386	0.105885	0.133336
K1995TK	0.966943	0.910584	0.877648	0.732261	0.684909	0.666135	0.0481307	0.0923891	0.109137	0.152499	0.135399	0.136034
A2009	0.974349	0.952656	0.944362	0.942843	0.919553	0.881102	0.0271275	0.0467152	0.0495036	0.0696121	0.100234	0.116361
C2010A	0.760755	0.762706	0.758034	0.727613	0.688383	0.658787	0.016273	0.0351274	0.0525413	0.0786478	0.093461	0.101423
C2010C	0.983062	0.958396	0.919213	0.890209	0.845189	0.754769	0.0342257	0.0668281	0.0849582	0.109502	0.146644	0.180485
N2010	0.959261	0.962511	0.960579	0.904483	0.869184	0.747981	0.014106	0.026906	0.0535571	0.100897	0.130248	0.16329
T2011T	0.964807	0.873175	0.860823	0.835717	0.796883	0.75345	0.0503305	0.0711582	0.0783013	0.100137	0.117246	0.130532
T2011S	0.957699	0.930198	0.918182	0.898036	0.861087	0.798855	0.0404706	0.0624786	0.0759954	0.106573	0.137496	0.172141
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.20: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 2$, $m_{S,i} = 100$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.936045	0.924999	0.918755	0.91506	0.910296	0.874035	0.0441677	0.0476501	0.0482942	0.0582415	0.074858	0.118124
E1934	0.962918	0.961652	0.953171	0.733522	0.71863	0.714195	0.0179851	0.03792	0.0855113	0.0905771	0.0676085	0.0851254
I1940	0.962743	0.957267	0.934943	0.888195	0.848152	0.769068	0.037163	0.050636	0.074576	0.105144	0.134717	0.163657
K1952	0.907359	0.874336	0.862425	0.840388	0.827736	0.80068	0.0497943	0.0618361	0.0761173	0.0849427	0.104072	0.122617
B1968	1.0039	0.985798	0.982803	0.980309	0.865394	0.766105	0.0467071	0.0528849	0.0559704	0.0910515	0.181663	0.1903
S1971	0.901629	0.905393	0.880486	0.860286	0.855998	0.844797	0.0119925	0.0254574	0.0484794	0.0554537	0.0684019	0.101231
I1979	1.03103	0.99979	0.98227	0.885002	0.866795	0.839412	0.0243734	0.0524753	0.0797021	0.114505	0.116352	0.139134
C1985	0.974551	0.951011	0.932214	0.897977	0.872871	0.8375	0.0499853	0.0665893	0.0837684	0.116246	0.142189	0.171876
L1989	0.947837	0.878732	0.875956	0.868358	0.856568	0.824897	0.0393993	0.0401101	0.0459384	0.0688454	0.0971193	0.146265
N1994	0.986197	0.988904	0.983399	0.955079	0.921165	0.885196	0.049575	0.0639999	0.0779419	0.108587	0.13396	0.164928
K1995TZ	0.912638	0.781942	0.781151	0.778574	0.775393	0.771673	0.0704111	0.0327684	0.044401	0.0599101	0.0703552	0.0822769
K1995TK	0.956914	0.914815	0.901429	0.896823	0.892812	0.879755	0.0350083	0.0545994	0.055661	0.0641976	0.0789307	0.107131
A2009	0.885034	0.884785	0.886475	0.893066	0.889999	0.877369	0.0402432	0.0734201	0.0893694	0.125476	0.138049	0.163969
C2010A	1.00331	0.978982	0.964171	0.930904	0.913177	0.893047	0.0495156	0.0714367	0.0872529	0.103617	0.110949	0.12538
C2010C	0.95631	0.948984	0.931297	0.888552	0.842324	0.791745	0.0337618	0.054228	0.0742132	0.103291	0.128646	0.141582
N2010	0.954188	0.943066	0.926784	0.908828	0.899783	0.861431	0.0161626	0.0417738	0.0565986	0.0757137	0.0933183	0.12984
T2011T	0.977063	0.957161	0.948097	0.925653	0.896587	0.854278	0.0456822	0.0593435	0.0728292	0.10188	0.123728	0.141295
T2011S	0.959449	0.95246	0.925345	0.872886	0.855211	0.847803	0.035875	0.0658335	0.0829759	0.102538	0.101893	0.122384
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.21: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 2$, $m_{S,i} = 150$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.901452	0.890873	0.890585	0.89098	0.892113	0.883075	0.0314232	0.0407267	0.0597757	0.0993126	0.126657	0.151156
E1934	0.974079	0.923641	0.90636	0.897959	0.895162	0.890303	0.0326214	0.0554749	0.0490657	0.0480867	0.0534057	0.0781185
I1940	0.912264	0.902748	0.896824	0.886421	0.876161	0.840542	0.0583334	0.0660358	0.074331	0.0872299	0.101848	0.134584
K1952	1.00895	1.00752	0.97166	0.934025	0.915808	0.886342	0.038072	0.0625792	0.0929927	0.108364	0.131394	0.163939
B1968	0.955916	0.925334	0.878806	0.870131	0.859425	0.849866	0.0383841	0.0567125	0.0522999	0.0592607	0.060273	0.0737211
S1971	1.03338	1.11226	0.946165	0.900795	0.890841	0.866478	0.0462572	0.0424954	0.145384	0.146758	0.159685	0.185755
I1979	1.04313	0.988337	0.957905	0.919819	0.875616	0.813115	0.027892	0.0844491	0.0946508	0.12414	0.145412	0.181543
C1985	0.984398	0.966721	0.956896	0.925784	0.621879	0.61239	0.0498141	0.071016	0.0932697	0.144332	0.0951802	0.1056
L1989	0.988649	0.966135	0.873904	0.776643	0.718331	0.677457	0.0823333	0.119308	0.155689	0.184369	0.185365	0.184284
N1994	0.953034	0.947195	0.94203	0.925115	0.686748	0.619732	0.0521654	0.0679715	0.0811501	0.131842	0.206582	0.193384
K1995TZ	0.980556	0.974957	0.969474	0.967162	0.970764	0.96939	0.0233427	0.0296916	0.0345565	0.0448952	0.0558765	0.0694825
K1995TK	0.937436	0.935642	0.918359	0.818279	0.740264	0.645255	0.0923179	0.12968	0.157705	0.212693	0.231169	0.256777
A2009	0.927148	0.914365	0.901674	0.875656	0.856981	0.823178	0.0690222	0.102014	0.124453	0.152172	0.176148	0.197268
C2010A	1.02346	0.925925	0.895227	0.877488	0.867647	0.858312	0.0568876	0.112165	0.108432	0.114717	0.120317	0.126032
C2010C	0.962533	0.940833	0.921418	0.900351	0.878827	0.845479	0.0484659	0.0632137	0.0765459	0.0925437	0.109081	0.128338
N2010	0.929702	0.920326	0.910991	0.903733	0.900581	0.887317	0.0319341	0.0647119	0.0786928	0.0970963	0.116187	0.14269
T2011T	0.967361	0.970357	0.964431	0.945539	0.934969	0.928246	0.0301782	0.0496452	0.065548	0.0867124	0.0992037	0.112915
T2011S	1.03172	1.02564	1.00329	0.952381	0.912472	0.864649	0.0319573	0.0584146	0.0906085	0.13363	0.161701	0.200398
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.22: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 3$, $m_{S,i} = 100$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	1.0326	1.03322	1.01769	0.981183	0.971374	0.909091	0.0282767	0.03677	0.0617914	0.0839545	0.104586	0.184638
E1934	1.00379	1.01488	1.0251	1.03877	0.997463	0.904191	0.02056	0.0396895	0.050824	0.073282	0.130263	0.177651
I1940	1.00085	0.998825	0.985806	0.941402	0.903043	0.881143	0.0305707	0.0458001	0.0678696	0.103243	0.116053	0.128561
K1952	0.997918	0.976814	0.941677	0.878209	0.807587	0.768186	0.0283732	0.072005	0.095519	0.148984	0.151926	0.176524
B1968	0.979348	0.977551	0.97709	0.979207	0.968682	0.92202	0.0308781	0.0461093	0.0560563	0.0675418	0.093885	0.163041
S1971	0.992249	0.999764	0.955403	0.891815	0.885569	0.879408	0.0129416	0.0317241	0.0817385	0.080935	0.0882088	0.127712
I1979	0.941744	0.938887	0.925727	0.850909	0.794728	0.767214	0.0504947	0.0668709	0.0864431	0.13038	0.129736	0.127664
C1985	0.747875	0.747662	0.744895	0.71635	0.708632	0.700576	0.00769063	0.0192911	0.0376593	0.0598778	0.0664692	0.0782639
L1989	0.938042	0.921173	0.843167	0.824079	0.79365	0.741125	0.0354656	0.0685068	0.0711385	0.084856	0.105952	0.099488
N1994	0.851605	0.759808	0.761446	0.737941	0.706909	0.681364	0.0239004	0.0580176	0.0895473	0.110509	0.118993	0.127788
K1995TZ	1.13656	1.13389	1.12948	1.12066	1.12224	1.00633	0.0154805	0.0214539	0.0280965	0.045361	0.0684132	0.384989
K1995TK	0.894448	0.896656	0.89385	0.831852	0.74829	0.685683	0.0171638	0.0458212	0.0691649	0.120423	0.142975	0.12913
A2009	0.943544	0.942004	0.93233	0.901352	0.877956	0.629543	0.0388362	0.0611746	0.0823328	0.111767	0.14217	0.177777
C2010A	0.975404	0.968326	0.963524	0.953062	0.944815	0.929996	0.032394	0.036912	0.0447132	0.0628601	0.0799272	0.104913
C2010C	0.988364	0.973994	0.959823	0.934914	0.915936	0.889073	0.0473447	0.0660256	0.0804314	0.101917	0.11748	0.142445
N2010	0.99723	1.01958	1.01719	0.989903	0.937749	0.894168	0.0429081	0.0501429	0.0614269	0.105738	0.147569	0.180039
T2011T	0.993258	1.02433	1.03817	1.04173	1.03715	1.02504	0.0429192	0.0656673	0.0679626	0.0783944	0.0947264	0.11931
T2011S	0.961334	0.888242	0.879117	0.868911	0.828609	0.714953	0.047172	0.0692412	0.0737112	0.109578	0.152547	0.198085
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.23: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 3$, $m_{S,i} = 150$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.929715	0.92483	0.926634	0.923253	0.854964	0.786954	0.0214275	0.0243864	0.0354892	0.0811157	0.127025	0.168278
E1934	0.992856	0.96832	0.945677	0.910091	0.86721	0.823824	0.0333618	0.0611859	0.0719463	0.101349	0.123056	0.13204
I1940	1.02455	1.01658	0.979269	0.911907	0.873986	0.798035	0.02935	0.0647684	0.100299	0.13959	0.16627	0.219069
K1952	0.975847	0.939392	0.927119	0.915861	0.898684	0.807738	0.0208407	0.0517189	0.0581288	0.0814127	0.108634	0.155992
B1968	0.923171	0.920158	0.909105	0.871323	0.754802	0.728672	0.0329953	0.0429075	0.0859506	0.118602	0.174166	0.225021
S1971	1.07765	0.835233	0.815707	0.796833	0.780929	0.761225	0.0162251	0.169414	0.158029	0.156267	0.156802	0.158438
I1979	0.944647	0.937588	0.934446	0.921052	0.912671	0.89742	0.0317417	0.0403184	0.0520125	0.0690222	0.0829566	0.106143
C1985	0.93603	0.938051	0.932179	0.906373	0.897216	0.88294	0.0273378	0.03786	0.0514654	0.0628977	0.0678505	0.081817
L1989	1.0136	0.982731	0.933772	0.919355	0.916226	0.906786	0.03376	0.0695636	0.0775525	0.0710474	0.0755133	0.0911379
N1994	0.936947	0.936424	0.914257	0.883441	0.86654	0.844201	0.0199274	0.0603153	0.0841694	0.098236	0.108121	0.120077
K1995TZ	0.900192	0.900483	0.90246	0.904046	0.871739	0.612254	0.0193847	0.0242242	0.030527	0.0558764	0.122532	0.0600416
K1995TK	0.962169	0.893564	0.855335	0.835574	0.831962	0.829726	0.0402085	0.0680281	0.0687901	0.0695136	0.0818924	0.100455
A2009	1.00568	0.798406	0.793404	0.783915	0.774029	0.756354	0.0395171	0.0343981	0.0478262	0.0690196	0.0856945	0.108628
C2010A	0.978593	0.962565	0.959017	0.949957	0.942365	0.948832	0.0507516	0.0801604	0.106106	0.139822	0.161842	0.185018
C2010C	0.964101	0.961833	0.954503	0.912475	0.884236	0.859382	0.0494053	0.0696554	0.0890868	0.121696	0.13811	0.156322
N2010	0.990262	0.892457	0.899332	0.905062	0.904451	0.613318	0.0609496	0.0464428	0.0600207	0.0725695	0.0985445	0.196116
T2011T	0.984736	0.976566	0.96782	0.95823	0.95888	0.961624	0.035369	0.05651	0.0748971	0.103702	0.135018	0.172898
T2011S	0.845723	0.852511	0.851564	0.838824	0.818304	0.781319	0.0278174	0.0532526	0.0801819	0.106182	0.127329	0.158856
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.24: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 5$, $m_{S,i} = 100$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.936088	0.895472	0.860192	0.849589	0.846733	0.838976	0.0628573	0.0840756	0.0785353	0.0730981	0.0743603	0.0917926
E1934	1.0016	1.00133	0.958768	0.922728	0.909315	0.849271	0.0308537	0.0591523	0.0906423	0.103583	0.121913	0.171623
I1940	0.882793	0.883718	0.882658	0.867682	0.832277	0.780189	0.0281339	0.0490221	0.0648403	0.0961327	0.123716	0.142381
K1952	0.936903	0.929417	0.918355	0.905244	0.895335	0.880541	0.0223279	0.0404525	0.0529785	0.0694209	0.0854278	0.111187
B1968	0.995674	0.986106	0.940729	0.891547	0.866563	0.848905	0.029638	0.0492624	0.0820735	0.0991101	0.105277	0.126939
S1971	0.960197	0.971755	0.976273	0.973211	0.952829	0.91543	0.0444295	0.0582123	0.0643304	0.0811749	0.102355	0.128765
I1979	0.968068	0.970793	0.974839	0.9866	0.983996	0.914771	0.0279861	0.0481524	0.0692794	0.109811	0.150886	0.212916
C1985	0.984462	1.00274	1.01736	1.02423	1.01843	0.996451	0.0404105	0.077468	0.0956376	0.123901	0.146584	0.172763
L1989	0.982111	0.986852	1.00761	1.0257	0.998851	0.945966	0.0592152	0.0984003	0.12729	0.151048	0.183532	0.221069
N1994	1.01158	0.947872	0.94707	0.953055	0.956564	0.945961	0.062405	0.0737448	0.0808911	0.0929711	0.103784	0.133931
K1995TZ	0.826967	0.81853	0.813495	0.805244	0.798498	0.791678	0.0296539	0.0359009	0.0404577	0.0466109	0.0510526	0.0601937
K1995TK	1.07174	1.08194	1.08366	1.01286	0.970881	0.942284	0.0240905	0.0376321	0.0589429	0.126373	0.144341	0.159591
A2009	1.04659	1.0329	1.02116	1.0033	0.940479	0.911574	0.0256241	0.0445918	0.0580381	0.0886591	0.121968	0.118164
C2010A	0.990529	0.933437	0.891758	0.840444	0.810374	0.781502	0.0880498	0.126264	0.152327	0.168815	0.178345	0.199114
C2010C	1.01367	1.01944	0.943465	0.888538	0.859556	0.833322	0.0433703	0.0723827	0.123189	0.135894	0.147151	0.15862
N2010	0.787404	0.788321	0.788398	0.788125	0.786212	0.77709	0.0130136	0.0199646	0.0250905	0.0341234	0.045696	0.0662215
T2011T	0.973564	0.957806	0.93323	0.907286	0.875443	0.85787	0.0233233	0.0518969	0.0694577	0.103441	0.124975	0.145259
T2011S	0.965951	0.94342	0.92832	0.886253	0.850702	0.8032	0.0507719	0.0754237	0.0972706	0.132138	0.157023	0.187354
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.25: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 5$, $m_{S,i} = 150$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	1.04006	1.04083	1.0404	0.997554	0.931129	0.90739	0.0206614	0.0344724	0.0551956	0.112812	0.127982	0.121911
E1934	0.922346	0.915993	0.918644	0.923557	0.919793	0.891537	0.0175086	0.0404522	0.0585739	0.082203	0.100817	0.134243
I1940	0.927219	0.90967	0.908197	0.90902	0.905883	0.874297	0.0472684	0.0647859	0.0805229	0.10418	0.122291	0.159636
K1952	0.994321	0.941161	0.939222	0.936744	0.926487	0.877974	0.0763531	0.0946926	0.108101	0.133537	0.159098	0.204835
B1968	1.02411	0.985798	0.973844	0.952164	0.77119	0.748089	0.0422624	0.0775703	0.0957797	0.138768	0.170545	0.142373
S1971	0.956334	0.964351	0.968233	0.974427	0.991407	1.02159	0.0270779	0.0402425	0.0471371	0.0689546	0.0912499	0.106216
I1979	0.953812	0.955626	0.949162	0.909228	0.785418	0.660378	0.0358901	0.0418168	0.0634394	0.112968	0.190529	0.145913
C1985	1.02204	1.02266	1.02781	1.03722	1.04445	1.02419	0.0227671	0.0427868	0.055385	0.087378	0.124464	0.18234
L1989	1.00506	0.970767	0.95971	0.953795	0.949323	0.917314	0.0358339	0.0614077	0.0677953	0.0828989	0.106651	0.174672
N1994	0.988316	0.860946	0.85203	0.837907	0.829202	0.822193	0.0378293	0.0536056	0.0625228	0.0844498	0.106934	0.132839
K1995TZ	1.00868	1.01045	1.01125	1.01248	1.00829	0.987351	0.0310335	0.0399083	0.0447915	0.0533502	0.0664745	0.0905701
K1995TK	0.946924	0.939347	0.915592	0.8963	0.883777	0.861616	0.0546089	0.087378	0.101348	0.113529	0.125798	0.146895
A2009	1.04552	1.02662	1.01365	0.939969	0.887625	0.846428	0.0189555	0.0482874	0.0797753	0.142035	0.157996	0.174066
C2010A	0.968494	0.970385	0.970299	0.943308	0.890994	0.860838	0.0328897	0.0442778	0.0577692	0.0954687	0.113894	0.11912
C2010C	0.983805	0.987522	0.983081	0.957253	0.917736	0.863799	0.0346022	0.0568964	0.0730441	0.102423	0.131186	0.148431
N2010	0.989715	0.992738	0.99257	0.98584	0.981125	0.978435	0.0229908	0.0276665	0.0347985	0.0548071	0.0688064	0.0970917
T2011T	0.996829	1.00029	1.00623	0.995388	0.971	0.910192	0.0268267	0.0473623	0.0629217	0.0961309	0.131726	0.178291
T2011S	0.980728	0.960377	0.935069	0.844291	0.812768	0.779919	0.0405246	0.0658049	0.0953523	0.121899	0.128886	0.144953
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.26: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 10$, $m_{S,i} = 100$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.906249	0.895409	0.881801	0.847534	0.766592	0.735626	0.0409942	0.061303	0.0738121	0.102575	0.115448	0.113243
E1934	1.11283	1.10553	1.08119	1.02908	0.990354	0.926245	0.0131042	0.0356198	0.0782872	0.133381	0.165488	0.221295
I1940	0.894596	0.891794	0.885057	0.86244	0.851632	0.842488	0.0203201	0.034549	0.0506207	0.0752768	0.0966221	0.129738
K1952	0.905533	0.904638	0.906591	0.906228	0.901712	0.889878	0.0283035	0.0434278	0.055648	0.0683702	0.0785874	0.102348
B1968	1.01628	1.09385	1.14453	1.15666	1.17173	0.981812	0.0478102	0.0962147	0.0810448	0.0911624	0.106313	0.357294
S1971	0.8573	0.85607	0.84981	0.752857	0.740244	0.728329	0.0214943	0.039444	0.0626208	0.0858553	0.0908107	0.101548
I1979	1.04149	1.04924	1.05578	1.053	0.987123	0.901601	0.0265978	0.0307926	0.0379572	0.0793615	0.142564	0.165085
C1985	0.94881	0.946746	0.943489	0.928029	0.911354	0.888041	0.0238648	0.0391356	0.0509529	0.0763105	0.0991254	0.132604
L1989	0.984916	1.15818	1.15245	1.14364	1.1324	1.12245	0.0422804	0.0240903	0.0276964	0.0307143	0.0356375	0.0397636
N1994	1.01288	1.01293	1.01053	0.991547	0.956712	0.924312	0.026811	0.0426578	0.0613068	0.0980227	0.129498	0.147582
K1995TZ	0.902713	0.895526	0.893043	0.886868	0.877791	0.862511	0.0333026	0.0575958	0.0801378	0.114973	0.142459	0.179506
K1995TK	0.991097	0.844689	0.844673	0.608323	0.601652	0.593348	0.0358841	0.0290847	0.0366261	0.0434556	0.0524928	0.0579114
A2009	1.03166	1.02472	1.02287	1.03363	1.04679	1.06338	0.0291143	0.0544367	0.0677882	0.0793569	0.0827553	0.100715
C2010A	1.0769	0.983372	0.945569	0.925312	0.91037	0.888129	0.0235578	0.109393	0.116811	0.124194	0.136095	0.154862
C2010C	0.865211	0.866279	0.864107	0.853295	0.842576	0.82928	0.025992	0.0622438	0.0753763	0.0920347	0.104435	0.118901
N2010	0.858006	0.853831	0.845813	0.809467	0.791969	0.772581	0.0186042	0.0333259	0.0565658	0.102707	0.129009	0.161018
T2011T	0.869402	0.857462	0.846291	0.823589	0.806539	0.789199	0.0532272	0.0690096	0.0815444	0.0976359	0.10913	0.126371
T2011S	1.01356	0.989804	0.966751	0.942303	0.879874	0.81258	0.0440041	0.0742129	0.0917754	0.122956	0.164128	0.168806
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.27: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 10$, $m_{S,i} = 150$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.910882	0.910057	0.907252	0.896619	0.882223	0.864213	0.0367583	0.0542833	0.0665753	0.0859395	0.0998442	0.112264
E1934	0.955831	0.931619	0.878312	0.867375	0.866852	0.866281	0.0229479	0.0565274	0.0639286	0.0684057	0.0769091	0.0872556
I1940	1.00962	1.05966	1.06201	0.978976	0.946192	0.899761	0.047773	0.0661894	0.0789285	0.138926	0.158608	0.182518
K1952	0.969098	1.00502	1.02018	0.993457	0.947336	0.927356	0.0856358	0.113414	0.119966	0.154821	0.176668	0.182966
B1968	0.936264	0.911842	0.736405	0.715186	0.512994	0.501807	0.0579527	0.096664	0.0806188	0.116951	0.106037	0.112411
S1971	0.91226	0.91315	0.91483	0.91429	0.909161	0.893909	0.0387563	0.0619153	0.0764475	0.0927566	0.104831	0.1243
I1979	1.00362	1.00163	0.99658	0.994116	0.956739	0.900288	0.0219393	0.0356813	0.0450819	0.0717355	0.122985	0.184427
C1985	0.942834	0.944286	0.933222	0.916711	0.91273	0.913095	0.0490954	0.0697308	0.0873049	0.107543	0.125754	0.148346
L1989	0.781795	0.78377	0.783234	0.772197	0.757376	0.744769	0.0120957	0.0250439	0.036763	0.0572589	0.0677983	0.0826599
N1994	1.03349	1.03859	1.05401	0.878286	0.83374	0.807201	0.0172798	0.0335468	0.0558588	0.190044	0.170853	0.172274
K1995TZ	0.979914	0.98394	0.959803	0.697946	0.658859	0.6383	0.0428403	0.0833355	0.124603	0.178877	0.152347	0.148107
K1995TK	1.06633	1.07181	0.755961	0.754492	0.753152	0.750019	0.0150571	0.0255139	0.0542767	0.0873912	0.099528	0.105286
A2009	0.933238	0.893544	0.871071	0.839567	0.823435	0.805591	0.0396952	0.0524771	0.0596613	0.0680732	0.0764246	0.098101
C2010A	0.956663	0.972041	0.974688	0.969644	0.957171	0.946457	0.049162	0.0680207	0.0792072	0.0999601	0.114249	0.127885
C2010C	1.03447	1.03705	1.02919	1.0111	1.00113	0.988175	0.0479169	0.0517494	0.0600772	0.0734259	0.0811074	0.103593
N2010	0.950458	0.964303	0.963557	0.950505	0.930548	0.795346	0.0593891	0.0829553	0.0935922	0.112691	0.139846	0.212214
T2011T	0.979131	0.983707	0.983361	0.970065	0.951091	0.915328	0.0438346	0.0571295	0.067188	0.0881692	0.107641	0.144378
T2011S	0.946446	0.939754	0.937238	0.935324	0.935331	0.932932	0.0298021	0.0328566	0.039071	0.0565723	0.0700536	0.0853793
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.28: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 15$, $m_{S,i} = 100$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.991207	1.00825	1.01341	1.00812	1.00725	1.01543	0.0476293	0.0682326	0.077574	0.0866273	0.0856158	0.0812205
E1934	1.03011	1.03622	1.03966	1.06126	1.05155	1.0227	0.0163672	0.0303364	0.0461453	0.0702796	0.0927188	0.114954
I1940	0.955671	0.956427	0.959495	0.957945	0.945325	0.922314	0.0352794	0.0496656	0.0609686	0.0776937	0.0961949	0.11997
K1952	1.03848	1.02594	0.983923	0.937233	0.901811	0.81816	0.0198126	0.0579781	0.0964896	0.126719	0.15913	0.220588
B1968	0.905063	0.908281	0.899771	0.862137	0.827761	0.703195	0.0208824	0.0390981	0.0565562	0.0925438	0.11572	0.134997
S1971	0.998088	0.997131	0.970663	0.894744	0.848896	0.805059	0.0681299	0.0965792	0.12618	0.16667	0.183328	0.19888
I1979	0.839627	0.836646	0.831907	0.820013	0.803952	0.783548	0.0390406	0.0533719	0.0605066	0.0706303	0.0780311	0.0800942
C1985	1.04217	1.04019	1.05017	1.0498	0.99446	0.895581	0.0335723	0.0586141	0.0714129	0.103929	0.174646	0.208327
L1989	0.962373	0.962785	0.953342	0.933767	0.926921	0.892879	0.0266667	0.0391376	0.0513278	0.0612468	0.0740529	0.0982027
N1994	1.00367	1.00098	0.969373	0.934103	0.925974	0.913731	0.0225535	0.0335262	0.0597873	0.0619153	0.0665882	0.095484
K1995TZ	0.889096	0.856	0.834576	0.811293	0.795166	0.776664	0.0442453	0.0598909	0.0660023	0.0753896	0.0822569	0.0903934
K1995TK	0.944935	0.945365	0.944374	0.935386	0.92571	0.921579	0.0328148	0.0422237	0.0478054	0.0546051	0.0531697	0.0483808
A2009	0.993702	0.995261	0.994434	0.980731	0.957835	0.932007	0.039588	0.0522935	0.0637576	0.0882782	0.106936	0.124066
C2010A	1.0092	0.961512	0.934956	0.908475	0.880149	0.846889	0.0769876	0.104512	0.118959	0.143832	0.164453	0.179934
C2010C	0.935225	0.926742	0.92265	0.915415	0.901671	0.879288	0.0348505	0.057642	0.0769075	0.111241	0.13755	0.161501
N2010	0.868051	0.869304	0.869249	0.861905	0.846923	0.824957	0.018787	0.0323114	0.0462229	0.0713633	0.0900963	0.105509
T2011T	0.91782	0.917658	0.9165	0.904473	0.890597	0.873908	0.0315917	0.0522838	0.066763	0.0927556	0.113188	0.141413
T2011S	1.14702	1.15986	1.15691	1.13855	1.05863	0.978439	0.024491	0.0368904	0.0469213	0.0850896	0.17571	0.228104
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.29: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 15$, $m_{S,i} = 150$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.874686	0.872473	0.852194	0.832261	0.83127	0.830477	0.013518	0.0258985	0.0413121	0.0551057	0.0871443	0.127865
E1934	0.888055	0.889552	0.890744	0.894885	0.892701	0.745435	0.010785	0.020692	0.0360991	0.0671024	0.0988801	0.152347
I1940	0.974474	0.95681	0.953487	0.938228	0.895583	0.823977	0.061904	0.0724227	0.0935233	0.14083	0.185086	0.217958
K1952	0.945927	0.861956	0.838356	0.824043	0.817536	0.809299	0.0584755	0.076565	0.0746268	0.0780745	0.0851592	0.0974257
B1968	0.978499	0.855863	0.857848	0.849707	0.820256	0.787717	0.0347719	0.0408889	0.0598284	0.0925698	0.117743	0.130328
S1971	0.973794	0.957739	0.941401	0.910879	0.884883	0.854169	0.0428002	0.0609743	0.0747642	0.0942826	0.10703	0.117877
I1979	1.01876	1.00663	0.995654	0.987823	0.988142	0.990028	0.0360354	0.0519338	0.0589639	0.0602324	0.0572288	0.054351
C1985	0.996828	0.986352	0.979809	0.972697	0.975694	0.931292	0.0340055	0.0381571	0.0420164	0.051335	0.074861	0.14667
L1989	0.97488	0.994976	1.00756	1.02082	1.03366	1.0365	0.00324059	0.0499929	0.068937	0.100506	0.117773	0.133864
N1994	0.885628	0.878199	0.875024	0.868137	0.852982	0.817364	0.0343654	0.061028	0.0873856	0.122991	0.147039	0.171825
K1995TZ	0.987623	0.980903	0.974954	0.964731	0.955859	0.943423	0.0361247	0.0537234	0.06368	0.0780635	0.0884118	0.101465
K1995TK	1.1682	1.16792	1.17022	1.17842	1.17669	1.05838	0.0160657	0.0265709	0.0370473	0.0543038	0.0811151	0.230316
A2009	0.984176	0.981079	0.977998	0.970375	0.9643	0.961917	0.0508024	0.0625469	0.0726236	0.0940152	0.113199	0.136004
C2010A	0.974714	0.971528	0.975655	0.984872	0.991874	0.987397	0.0210065	0.0462924	0.058701	0.0685593	0.0805557	0.127673
C2010C	0.986158	0.990611	0.998843	1.01395	1.02708	1.03756	0.0230656	0.0472883	0.0633417	0.0811572	0.0876313	0.0960437
N2010	1.07198	1.02321	1.01907	1.01739	1.01998	1.02827	0.0444565	0.0760691	0.0775298	0.0751652	0.0713327	0.0730199
T2011T	0.991474	0.957965	0.954468	0.940137	0.824729	0.771784	0.0609461	0.0805369	0.0995503	0.147276	0.195393	0.194129
T2011S	0.964349	0.92496	0.908189	0.900177	0.89507	0.886158	0.0586046	0.0777011	0.0803758	0.0862377	0.0931838	0.106415
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.30: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 25$, $m_{S,i} = 100$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.939931	0.826521	0.809932	0.788371	0.769421	0.750173	0.013228	0.0694907	0.0813822	0.111088	0.13349	0.156097
E1934	0.981302	0.979245	0.97979	0.98145	0.930106	0.86475	0.0169047	0.0142654	0.0163538	0.0459779	0.105257	0.105194
I1940	1.03607	0.965417	0.951602	0.938609	0.927757	0.906233	0.0496347	0.0819431	0.0872295	0.0936078	0.104752	0.129363
K1952	1.10046	1.08635	1.0895	1.08783	0.997207	0.966748	0.0380121	0.0561919	0.0604806	0.089503	0.155915	0.155811
B1968	1.02718	0.988204	0.977137	0.964091	0.958075	0.934242	0.0242963	0.055978	0.0647198	0.0925466	0.117785	0.155492
S1971	1.03289	1.04145	1.04783	1.05563	1.05983	1.06316	0.032604	0.0507049	0.0600175	0.0744559	0.0859416	0.0997474
I1979	0.951371	0.80405	0.800335	0.791435	0.781766	0.771812	0.0429224	0.0351896	0.040157	0.0463662	0.0486892	0.0496231
C1985	1.06577	0.940815	0.927271	0.824088	0.786652	0.767792	0.0272665	0.106603	0.120613	0.161011	0.151897	0.149367
L1989	0.917268	0.915713	0.907578	0.857594	0.845574	0.846172	0.0169774	0.0302927	0.046487	0.0601355	0.0612919	0.0787564
N1994	0.998778	0.995999	0.991586	0.982221	0.978377	0.98168	0.0252577	0.0393052	0.0512659	0.0700855	0.0839443	0.0995443
K1995TZ	0.951956	0.930757	0.915035	0.8916	0.874088	0.854452	0.0367625	0.0645064	0.0799123	0.100421	0.116587	0.137053
K1995TK	0.86594	0.864846	0.863429	0.857214	0.847044	0.826076	0.0266625	0.0452862	0.0594914	0.0829134	0.102456	0.125102
A2009	0.968242	0.944308	0.917185	0.87648	0.849124	0.819571	0.026997	0.0560224	0.0739257	0.0951376	0.113081	0.135344
C2010A	1.00432	0.98958	0.988452	0.989581	0.979482	0.954934	0.0439532	0.0417818	0.0390831	0.0445278	0.0639232	0.0944417
C2010C	0.927535	0.924213	0.924274	0.923886	0.915395	0.885282	0.032239	0.0492752	0.0566544	0.0705245	0.0964311	0.145772
N2010	0.943001	0.928838	0.92388	0.91768	0.897741	0.840947	0.0596568	0.0581873	0.059447	0.0685906	0.0943401	0.121479
T2011T	1.04147	0.993899	0.991847	0.983552	0.971446	0.953448	0.0414243	0.0651747	0.0650069	0.0704125	0.0807776	0.0963509
T2011S	1.00363	0.985809	0.983494	0.974621	0.950174	0.89323	0.0437455	0.0648794	0.093102	0.143198	0.182617	0.22737
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.31: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 25$, $m_{S,i} = 150$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.992601	0.971582	0.950909	0.925062	0.911684	0.901492	0.0369583	0.0550948	0.0653114	0.0743308	0.078846	0.0897318
E1934	1.00307	0.995106	0.972244	0.909864	0.899448	0.890849	0.0162797	0.030268	0.0682883	0.0692808	0.0754957	0.10343
I1940	0.980614	0.961494	0.846162	0.822081	0.803338	0.782962	0.0468958	0.107142	0.111149	0.12362	0.135032	0.143763
K1952	1.06989	1.05555	1.05314	0.950323	0.873776	0.858049	0.0113524	0.029747	0.0443776	0.197227	0.185004	0.176457
B1968	0.962803	0.964775	0.959241	0.892566	0.871883	0.855935	0.0193724	0.0452373	0.0756322	0.110294	0.120942	0.137828
S1971	0.851341	0.838063	0.828773	0.814267	0.803128	0.790931	0.000154467	0.0229259	0.0396972	0.0671108	0.0881976	0.113152
I1979	0.999107	0.993928	0.989899	0.986314	0.986239	0.988889	0.0262931	0.0340141	0.03776	0.0409861	0.0431076	0.0484671
C1985	0.959379	0.956726	0.935969	0.913072	0.912946	0.915154	0.020013	0.0333983	0.0526062	0.0579159	0.0680727	0.0794637
L1989	1.00748	1.0227	1.07607	1.1078	0.794216	0.740031	0.0116191	0.0432962	0.0510072	0.044141	0.247586	0.218596
N1994	1.24854	0.933284	0.923502	0.917447	0.916884	0.920651	0.00683942	0.172167	0.184643	0.205571	0.221335	0.238792
K1995TZ	1.01101	1.00418	0.999314	0.991572	0.98379	0.969493	0.00545145	0.0415647	0.0626589	0.0873306	0.102454	0.120374
K1995TK	1.0702	1.0283	1.00805	0.98387	0.963951	0.934584	0.0367884	0.0747715	0.0886981	0.104804	0.117915	0.136347
A2009	0.985815	0.975983	0.968182	0.953441	0.933833	0.892733	0.0250419	0.053513	0.0700614	0.0946187	0.118618	0.150209
C2010A	0.956447	0.940572	0.893682	0.874538	0.864582	0.856626	0.0277925	0.0713449	0.0832118	0.0978947	0.118228	0.155032
C2010C	0.830731	0.835697	0.837682	0.834989	0.831277	0.827711	0.0139303	0.0298387	0.0428648	0.0709675	0.0976275	0.125624
N2010	0.943861	0.922716	0.916796	0.918836	0.923674	0.930191	0.0381597	0.0545801	0.0690579	0.0940344	0.107532	0.116855
T2011T	0.904013	0.89795	0.897711	0.895686	0.892415	0.870719	0.0142034	0.0581179	0.076601	0.105172	0.133669	0.182377
T2011S	0.934158	0.932302	0.927329	0.916209	0.899075	0.885927	0.0361291	0.0498266	0.0575147	0.0760728	0.0886161	0.10215
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.32: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 40$, $m_{S,i} = 100$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.907456	0.877317	0.848997	0.810085	0.786253	0.760725	0.0646236	0.103496	0.125606	0.151502	0.169615	0.190865
E1934	0.778468	0.778907	0.775693	0.765178	0.75943	0.753758	0.005000	0.0155729	0.0327488	0.0687502	0.105425	0.143176
I1940	0.972383	0.990345	1.00394	1.02402	1.03001	1.01767	0.0306529	0.048193	0.0546465	0.0539484	0.0539203	0.0650546
K1952	0.915292	0.868955	0.868513	0.869639	0.861433	0.839496	0.016938	0.0734992	0.107815	0.142807	0.159407	0.174512
B1968	1.10092	1.04321	1.02652	1.01252	1.00733	1.01085	0.0175373	0.0755945	0.084742	0.0931874	0.0966495	0.119384
S1971	1.04389	1.0465	1.05049	1.06018	1.0704	1.07952	0.005000	0.0346906	0.051548	0.0635448	0.0592716	0.0452765
I1979	0.907987	0.82141	0.822308	0.823108	0.823069	0.807642	0.0236191	0.0102298	0.0169544	0.0277208	0.0418395	0.0898808
C1985	0.836361	0.836333	0.835012	0.817537	0.793252	0.774814	0.0180947	0.0294891	0.0438122	0.0861832	0.117139	0.146799
L1989	1.00092	0.996509	0.987926	0.963269	0.9373	0.903634	0.005000	0.0313646	0.0478674	0.069359	0.0853942	0.112002
N1994	0.925581	0.907679	0.894529	0.87233	0.846285	0.796583	0.038575	0.0745779	0.0976237	0.133513	0.163735	0.198008
K1995TZ	1.0034	0.994111	0.983118	0.965621	0.951259	0.932789	0.005000	0.026436	0.0589001	0.102445	0.13245	0.165803
K1995TK	1.00393	1.0055	1.00751	1.01023	1.01016	1.00542	0.0326187	0.049696	0.0613227	0.0810168	0.097604	0.118998
A2009	0.992529	0.983696	0.976662	0.964326	0.950343	0.923915	0.005000	0.0269553	0.049837	0.0800729	0.102742	0.130523
C2010A	0.90041	0.90052	0.908295	0.918923	0.921443	0.776206	0.0226348	0.0596004	0.0912718	0.1215	0.145842	0.231786
C2010C	1.10614	1.09889	1.08734	1.06645	1.0212	0.881326	0.0231044	0.0471123	0.0673609	0.103945	0.170307	0.244243
N2010	0.934317	0.923111	0.914318	0.910091	0.914673	0.919755	0.005000	0.0237832	0.0462743	0.0921793	0.126919	0.163837
T2011T	0.927724	0.901184	0.890333	0.883083	0.874731	0.865693	0.0196207	0.0377875	0.0404993	0.0571701	0.0741921	0.09935
T2011S	1.00207	1.02095	1.01098	0.985343	0.957026	0.925375	0.0602234	0.0894362	0.114272	0.152309	0.178745	0.200684
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

Table 5.33: *Optimum TMD parameters for different values of mass ratio μ, for the structure with $n_S = 40$, $m_{S,i} = 150$ t.*

	f^{opt}						ζ_T^{opt}					
Earthquake	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$	$\mu = 0.05$	$\mu = 0.07$	$\mu = 0.1$
L1933	0.968639	0.955698	0.943906	0.921882	0.900292	0.870429	0.0141075	0.046404	0.0649798	0.0871767	0.102346	0.117867
E1934	0.968431	0.971989	0.970452	0.966795	0.971151	0.95564	0.0208149	0.0327772	0.0398574	0.0469736	0.0547529	0.0796828
I1940	1.2	0.919803	0.914788	0.915793	0.92265	0.942269	0.005000	0.189829	0.20924	0.243787	0.262991	0.273188
K1952	0.935066	0.950311	0.95616	0.965017	0.971438	0.908763	0.0284062	0.0468714	0.0539024	0.0763063	0.101242	0.158386
B1968	0.932132	0.874511	0.883022	0.852131	0.822138	0.810247	0.005000	0.005000	0.0214289	0.0735794	0.086459	0.108967
S1971	0.792956	0.777448	0.763319	0.721697	0.679013	0.641167	0.005000	0.005000	0.005000	0.0396067	0.0546014	0.0635663
I1979	1.02743	1.03415	1.0453	1.05568	0.979121	0.915848	0.0106039	0.0311838	0.0460101	0.0760374	0.134379	0.155227
C1985	1.0527	1.03084	0.994202	0.971066	0.96358	0.960881	0.0339416	0.0670524	0.0851934	0.0892792	0.0859522	0.0771376
L1989	0.954246	0.936382	0.926039	0.915903	0.911864	0.909256	0.032932	0.0585033	0.0710987	0.0843657	0.0949403	0.114003
N1994	0.94866	0.921999	0.904238	0.88197	0.868304	0.853532	0.0316015	0.0639964	0.0804922	0.0972512	0.106949	0.120303
K1995TZ	1.00798	0.995601	0.985217	0.967441	0.953475	0.936211	0.005000	0.005000	0.0104693	0.0581476	0.0910466	0.126568
K1995TK	1.00276	0.998614	0.99568	0.992932	0.988852	0.978554	0.0458443	0.0768521	0.104943	0.152677	0.187235	0.222538
A2009	0.982965	0.967124	0.957641	0.944549	0.935866	0.92713	0.005000	0.0205529	0.0499585	0.0879076	0.112609	0.137977
C2010A	0.859549	0.855924	0.854054	0.853093	0.852453	0.854516	0.0167843	0.0227172	0.0273247	0.0406807	0.0600535	0.0831297
C2010C	0.998288	0.999338	0.932562	0.904089	0.894394	0.889656	0.0170951	0.0392166	0.0743484	0.0706218	0.0732522	0.0833463
N2010	0.882752	0.829933	0.815204	0.790883	0.769232	0.74296	0.0812191	0.0866787	0.10118	0.126996	0.145178	0.163178
T2011T	1.06875	1.0801	1.07076	1.06137	1.00873	0.960681	0.0497093	0.0750689	0.0897698	0.109446	0.152191	0.163874
T2011S	1.00216	1.01173	1.01509	1.01583	1.01177	1.0095	0.0300267	0.0362881	0.0452568	0.0633858	0.0775938	0.0990347
Den Hartog [26]	0.990099	0.980392	0.970874	0.952381	0.934579	0.909091	0.0609333	0.0857493	0.10451	0.133631	0.156629	0.184637

5.6 Seismic response reduction

In this section the results of the tuning process for the range of primary structures and earthquakes considered in this study, in terms of performance of the optimum TMD, will be presented and discussed in detail, specifically for the case of $\mu = 0.02$, $\zeta_{S,I} = 0.05$, which are values that reflect potential contexts of application of the control device.

In this sense, it is important to note that the levels of TMD performance have been recovered for quite a high value of inherent structural damping. Remarkably higher TMD effectiveness would be obtained for lower intrinsic damping.

The main outcomes of such a wide analysis have been condensed in Tables 5.34–5.49, where the effect of the control device is detected through the percentage reduction of several response indexes, namely:

- Peak kinematic response of the primary structure top storey: $\Delta x_{S,n}^{max}$, $\Delta \dot{x}_{S,n}^{max}$, $\Delta \ddot{x}_{S,n}^{max}$;
- RMS kinematic response of the primary structure top storey: $\Delta x_{S,n}^{RMS}$, $\Delta \dot{x}_{S,n}^{RMS}$, $\Delta \ddot{x}_{S,n}^{RMS}$;
- Peak and RMS primary structure kinetic energy: ΔT_{S}^{max}, ΔT_{S}^{RMS}.

The aim of this wide representation lies in the need of comprehension of the global effectiveness of the Tuned Mass Damper in reducing the seismic response, even if optimised just on a given response index. In this sense, it is somewhat expected that a general benefit on the dynamic behaviour of the primary structure shall be obtained, and that the assumed indexes should allow for a comprehensive effective understanding of the actual situation after the addition of the control device. As additional instrument of analysis, in Tables 5.34–5.49 the same values of percentage reduction achieved through Den Hartog tuning formulas have been also reported, in order to assess the further benefit coming from the specific seismic TMD optimisation with respect to the performance of the control device obtained with a benchmark tuning method. Finally, for all the response reduction indexes, the average value (intended as mean value) has been evaluated, so that to provide a useful indication on the overall effectiveness of the optimum TMD for all the considered seismic input signals, for an assumed primary structure.

At a glance, a general reduction of the primary structure response has been obtained, with different levels of performance depending on the considered primary structure and earthquake.

In general, the indexes related to the peak kinematic response exhibit a wide variability, which is expressed not only within the performance for the different seismic input signals, but also among the three indexes of a each earthquake case, i.e. the reduction in terms of displacement, velocity and acceleration. The main indication concerns a reduced peak response, even if with noticeable differences between the different cases. Indeed, the percentage reduction of peak response can take values from almost zero to up to 40%. Moreover, it appears that specific general trend could not be outlined, especially for a considered primary structure. In this sense, since in the present study the peak response has not been involved within the TMD optimisation process, it is reasonable to conclude that the level of peak response abatement is related only to the level of correspondence between the modal parameters and the earthquake signal.

By observing the average values of the peak indexes of response, a higher reduction has been obtained for the primary structures with short and medium natural periods, namely those with $T_{S,I} < 2$ s, while long period primary structures as the 25- and 40-storeys ones exhibit smaller peak response decrease. The comparison with the results achieved through Den Hartog tuning formulas turns out quite interesting and useful, since such tuning method is focused on the reduction of the peak displacement response (for harmonic loading). However, with reference to these results, it appears that the proposed TMD tuning, even if based on an overall response abatement, provides better results with respect to those achieved with Den Hartog's method, for all the three indexes.

The percentage reduction of the RMS kinematic response displays as well large variability of values, which at a glance would not allow to extract general trends. However, these results witness in principle a remarkable response abatement, not only referred to the RMS displacement assumed as objective function within the optimisation process, but also by considering that significantly decreasing RMS velocity and acceleration have been recovered. In general, the RMS kinematic response reduction takes value from 5% to up to 40%, with a mean value of about 20%. Many subcases could be detected within this set of results: in some cases the performance decreases from the displacement to the acceleration, in other cases, viceversa, the acceleration appears to be the most reduced index, other cases again display almost constant abatement for all the three quantities of response. Instead of what previously recovered for the peak response, the average reduction of the RMS displacement takes remarkable values for all the considered primary structures, included long-period buildings such as the 25- and 40-storey frame structures. Same consideration holds for the RMS velocity. On the other hand, the average RMS acceleration abatement seems to decrease starting from the 10-storey primary structures, i.e. at increasing modal periods.

The parallel contest with the results achieved from Den Hartog benchmark tuning again points out a general better performance obtainable with the proposed seismic tuning method, for all the considered RMS indexes and cases. One could note an interesting fact involving the average values, namely that for all the considered primary structures the hierarchy of the response decrease, i.e. the magnitude of the percentage values among the three quantities of response, is equal between peak and RMS response. Such an issue points out remarkable implications about possible connections between structural configuration and seismic response.

The reduction of the peak response in terms of kinetic energy of the primary structure T_S^{max} mainly exhibits considerable values, even if in a situation of wide variability, especially if considering the results related to each primary structure. In this sense, the abatement varies between very small values (about 5%) and very large decreases, such as peaks of about 70%, for some outstanding cases; an approximate mean value of reduction is about 20%. At a glance, one may note that the effect of the TMD on the reduction of T_S^{max} is quite larger for the short- and the medium-period structures than for the long-period buildings (25- and 40-storey frame structures), as already observed for the peak kinematic response.

From an examination of the results for the RMS kinetic energy index T_S^{RMS}, less variability than for T_S^{max} is recovered. Such quantity has been decreased, in the best cases, of about 60%, with an average value of nearby 30%, which is a better result with respect to that obtained for the peak index T_S^{max}, except for the long-period structures, again distinguished by a lower performance. As found before, for the kinetic energy indexes the proposed method also provided a more efficient TMD, with respect to that coming from benchmark tuning formulas.

Table 5.34: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 1$, $m_{S,i} = 100$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.919713	0.0638899	5.05 (3.66)	9.10 (7.51)	5.63 (2.48)	2.58 (1.11)	9.52 (7.07)	11.82 (10.09)	17.38 (14.46)	13.93 (12.40)
E1934	0.921112	0.0703318	-14.24 (-11.98)	9.76 (9.54)	9.42 (14.09)	6.98 (5.76)	15.29 (13.76)	17.57 (16.81)	18.58 (18.17)	22.34 (21.74)
I1940	0.941225	0.0580685	23.02 (15.89)	22.43 (15.86)	28.38 (21.04)	14.70 (13.19)	24.39 (22.41)	26.55 (24.99)	39.82 (29.21)	47.14 (40.60)
K1952	0.89792	0.0870196	6.43 (-0.91)	11.56 (6.53)	9.04 (11.18)	3.25 (2.25)	10.04 (9.14)	12.75 (13.11)	21.79 (12.64)	20.52 (18.53)
B1968	0.974222	0.0500581	-9.40 (-9.07)	7.03 (7.87)	11.34 (11.10)	0.85 (0.68)	12.91 (11.84)	19.20 (17.61)	13.57 (15.11)	21.69 (19.68)
S1971	1.00819	0.0510264	16.02 (11.77)	19.94 (16.33)	24.28 (21.85)	10.88 (10.40)	19.43 (18.03)	22.77 (20.66)	35.91 (29.99)	34.57 (33.46)
I1979	0.898226	0.05352	3.24 (3.85)	5.36 (9.59)	9.29 (11.48)	4.27 (2.50)	14.73 (10.95)	18.84 (15.23)	10.43 (18.25)	18.92 (14.74)
C1985	0.869281	0.129671	6.25 (10.41)	9.53 (12.93)	12.89 (19.77)	1.36 (0.85)	7.10 (7.14)	10.36 (11.80)	18.16 (24.18)	12.86 (11.97)
L1989	0.874935	0.0405821	4.00 (-9.14)	4.75 (-14.48)	6.10 (-15.71)	2.61 (0.75)	9.93 (7.27)	13.48 (12.50)	9.28 (-31.06)	21.95 (-0.17)
N1994	0.889527	0.0712831	12.72 (10.77)	17.96 (13.67)	18.12 (16.90)	11.16 (9.16)	17.43 (16.10)	17.90 (18.12)	32.70 (25.47)	31.08 (28.36)
K1995TZ	0.984203	0.0684151	7.76 (7.72)	12.04 (10.77)	18.11 (16.36)	3.96 (3.90)	15.17 (14.76)	21.94 (21.15)	22.62 (20.38)	31.61 (29.40)
K1995TK	0.943665	0.164683	-2.83 (-2.46)	-1.95 (-1.29)	5.78 (6.62)	0.05 (-0.45)	6.00 (5.03)	8.52 (7.82)	-3.93 (-2.60)	7.85 (6.39)
A2009	0.970043	0.0488281	0.19 (4.48)	-0.95 (4.27)	6.55 (8.71)	24.22 (22.35)	27.33 (25.43)	27.31 (25.75)	-1.91 (8.35)	41.59 (40.91)
C2010A	0.949067	0.0828206	5.21 (4.93)	7.06 (7.59)	10.44 (10.89)	2.82 (2.64)	9.01 (8.80)	13.56 (13.55)	13.63 (14.60)	10.09 (8.85)
C2010C	0.892958	0.0798059	7.59 (4.20)	11.93 (8.58)	10.59 (7.91)	6.82 (4.63)	13.14 (10.95)	15.96 (14.99)	22.43 (16.42)	24.72 (20.63)
N2010	0.911654	0.0640074	3.22 (-0.20)	5.47 (-1.19)	-2.22 (-7.96)	5.36 (3.90)	14.34 (12.23)	14.89 (13.93)	10.65 (-2.39)	20.55 (10.56)
T2011T	0.681163	0.0433526	6.98 (-1.52)	9.68 (-17.85)	9.31 (-21.04)	4.40 (-1.90)	6.51 (1.85)	7.04 (5.90)	18.42 (-38.87)	13.05 (-0.83)
T2011S	0.914064	0.0677918	-1.21 (-0.54)	-3.03 (-3.32)	-2.95 (-2.87)	7.57 (6.06)	15.52 (13.93)	15.77 (15.43)	-6.16 (-6.76)	14.00 (10.60)
Average [%] reduction			4.44 (2.32)	8.76 (5.16)	10.56 (7.38)	6.32 (4.88)	13.77 (12.04)	16.46 (15.52)	16.30 (9.20)	22.69 (18.21)

Table 5.35: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 1$, $m_{S,i} = 150$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.928751	0.0678157	1.81 (-2.19)	6.72 (3.10)	12.78 (12.87)	8.59 (6.89)	16.30 (13.95)	19.41 (17.39)	12.99 (6.09)	27.22 (18.28)
E1934	0.940918	0.0751522	1.63 (9.81)	5.41 (6.70)	16.75 (18.39)	10.41 (8.89)	17.87 (16.12)	20.15 (18.99)	10.53 (12.95)	31.83 (30.01)
I1940	0.896668	0.0670746	-0.02 (-1.45)	14.52 (17.81)	15.32 (22.93)	10.63 (8.40)	15.59 (14.52)	15.62 (16.56)	26.93 (32.44)	30.11 (33.41)
K1952	0.900373	0.0408651	28.52 (18.29)	24.13 (22.99)	19.93 (24.59)	12.53 (8.53)	19.89 (15.25)	21.66 (18.31)	42.44 (40.70)	41.74 (31.30)
B1968	0.980391	0.0857489	-1.89 (-1.89)	16.62 (16.62)	26.95 (26.95)	9.22 (9.22)	21.89 (21.89)	25.45 (25.45)	30.47 (30.47)	35.36 (35.36)
S1971	0.926297	0.0671687	9.40 (16.81)	16.50 (24.72)	4.31 (14.83)	13.30 (11.81)	18.19 (17.27)	18.63 (18.84)	30.28 (43.32)	30.03 (32.32)
I1979	0.900109	0.12465	-1.42 (-1.65)	-0.34 (-0.89)	2.85 (2.29)	2.55 (0.91)	8.17 (6.21)	11.02 (9.90)	-0.68 (-1.80)	7.69 (6.71)
C1985	0.892942	0.0530296	11.62 (7.42)	29.82 (14.28)	30.59 (17.80)	13.34 (8.29)	20.00 (14.70)	22.03 (18.40)	50.75 (26.52)	42.37 (29.40)
L1989	0.722396	0.127725	3.25 (-23.26)	8.57 (-15.23)	8.11 (-7.02)	1.05 (-5.57)	4.03 (-4.47)	5.10 (-1.86)	16.41 (-32.78)	9.11 (-22.96)
N1994	1.00766	0.0740443	6.67 (5.13)	12.39 (9.79)	16.40 (14.95)	10.29 (10.10)	16.61 (15.85)	18.62 (17.36)	23.24 (18.62)	32.49 (28.75)
K1995TZ	0.998208	0.0538141	17.30 (16.98)	27.99 (28.54)	35.68 (31.65)	11.87 (11.50)	22.60 (21.26)	27.73 (25.49)	48.15 (48.93)	44.52 (42.37)
K1995TK	0.948948	0.0866292	1.24 (0.21)	4.90 (3.25)	5.42 (4.18)	3.73 (3.43)	12.72 (12.36)	16.27 (16.27)	9.56 (6.39)	14.83 (14.61)
A2009	0.953816	0.0472419	31.66 (27.18)	32.72 (28.51)	30.09 (22.43)	28.72 (25.71)	30.39 (27.72)	28.29 (26.43)	54.74 (48.89)	56.38 (51.82)
C2010A	0.763433	0.0360203	15.62 (-1.08)	7.18 (-1.27)	0.42 (2.59)	8.62 (0.11)	10.54 (4.31)	10.71 (8.36)	13.85 (-2.56)	22.18 (5.51)
C2010C	0.963104	0.0667677	14.26 (10.76)	10.01 (7.19)	16.37 (12.88)	15.89 (15.27)	22.25 (21.51)	25.09 (24.46)	19.02 (13.86)	40.13 (38.85)
N2010	0.96174	0.0280278	18.63 (11.00)	24.25 (15.53)	26.55 (17.16)	18.66 (15.08)	31.35 (25.49)	31.59 (26.20)	42.62 (28.65)	46.95 (30.99)
T2011T	0.872123	0.0703578	18.10 (22.31)	28.08 (18.17)	27.37 (20.54)	15.09 (12.37)	17.47 (16.68)	18.47 (20.00)	48.27 (33.03)	35.69 (29.78)
T2011S	0.946026	0.0696897	6.90 (5.27)	8.10 (4.65)	10.66 (8.75)	11.28 (10.17)	19.92 (18.60)	20.57 (19.83)	15.55 (9.08)	29.88 (25.22)
Average [%] reduction			10.18 (6.65)	15.42 (11.36)	17.03 (14.93)	11.43 (8.95)	18.10 (15.51)	19.80 (18.13)	27.51 (20.16)	32.14 (25.65)

Table 5.36: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 2$, $m_{S,i} = 100$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.924999	0.0476498	5.49 (4.49)	2.54 (3.46)	7.82 (8.96)	24.76 (20.03)	28.19 (24.34)	28.39 (26.11)	5.00 (7.05)	41.59 (36.58)
E1934	0.961652	0.0379202	9.62 (5.94)	11.41 (5.17)	18.82 (13.60)	15.00 (12.90)	21.61 (18.64)	24.50 (21.36)	21.79 (10.42)	36.50 (31.14)
I1940	0.957267	0.050636	9.60 (10.70)	6.34 (6.12)	5.31 (5.80)	17.36 (15.57)	23.28 (21.14)	24.08 (22.22)	13.83 (13.49)	36.14 (32.35)
K1952	0.874341	0.0618378	17.45 (18.96)	15.92 (12.50)	5.44 (2.92)	13.42 (9.46)	16.68 (14.03)	16.75 (16.26)	29.04 (24.17)	27.36 (25.43)
B1968	0.982152	0.0653077	1.88 (2.90)	12.66 (11.24)	7.43 (10.51)	12.13 (11.57)	22.95 (21.90)	24.64 (23.54)	23.67 (21.20)	35.86 (34.72)
S1971	0.905353	0.0255086	1.52 (0.36)	4.52 (-4.85)	14.66 (-2.73)	22.58 (5.29)	25.55 (7.85)	24.08 (9.14)	9.73 (-7.92)	41.72 (17.06)
I1979	0.99979	0.0524752	-3.40 (-4.54)	-5.14 (-6.25)	-0.66 (-2.21)	16.12 (15.31)	23.33 (21.51)	26.94 (24.27)	-10.56 (-12.89)	25.84 (22.44)
C1985	0.951011	0.0665893	15.57 (16.02)	18.42 (19.20)	20.03 (21.12)	12.47 (11.87)	17.51 (17.13)	19.02 (19.13)	34.09 (35.73)	34.49 (35.11)
L1989	0.878732	0.0401101	-0.52 (6.52)	1.49 (9.40)	1.85 (7.72)	18.54 (13.15)	22.54 (17.83)	23.36 (20.35)	2.97 (17.73)	37.05 (34.63)
N1994	0.988904	0.0639999	3.45 (5.80)	16.28 (18.80)	15.61 (15.80)	16.28 (15.91)	21.23 (20.62)	21.60 (20.91)	32.30 (36.24)	32.57 (31.92)
K1995TZ	0.781938	0.0327414	4.99 (-6.15)	8.62 (-2.78)	5.68 (-3.67)	7.83 (4.12)	9.24 (8.80)	7.65 (11.17)	16.08 (-5.32)	20.83 (5.97)
K1995TK	0.914814	0.054598	20.83 (20.84)	22.98 (22.96)	21.00 (24.62)	17.62 (14.75)	25.02 (22.25)	25.62 (24.20)	40.37 (40.60)	39.44 (39.89)
A2009	0.884789	0.0734193	17.87 (9.44)	17.42 (7.11)	17.83 (12.43)	7.43 (4.16)	7.86 (6.07)	5.49 (5.42)	32.73 (16.31)	21.54 (14.74)
C2010A	0.978982	0.0714367	19.56 (19.03)	18.45 (17.79)	16.37 (16.57)	21.40 (21.17)	23.43 (23.19)	24.24 (24.04)	33.51 (32.46)	37.89 (37.41)
C2010C	0.948987	0.054228	27.55 (20.26)	24.12 (17.39)	16.87 (11.24)	20.28 (18.15)	23.83 (21.81)	24.66 (23.16)	41.79 (31.16)	45.13 (39.25)
N2010	0.943066	0.0417738	-12.00 (-12.73)	25.98 (14.25)	9.61 (-0.33)	21.45 (16.99)	29.26 (23.70)	28.02 (23.49)	45.35 (26.61)	49.30 (37.27)
T2011T	0.957161	0.0593435	-21.28 (-13.45)	-1.35 (4.64)	11.72 (13.20)	23.62 (22.36)	24.20 (23.42)	23.46 (23.29)	-1.42 (10.21)	34.95 (34.81)
T2011S	0.95246	0.0658335	-3.01 (-3.23)	15.40 (16.83)	17.84 (17.63)	12.85 (11.99)	19.86 (18.97)	20.36 (19.93)	26.81 (27.25)	29.87 (29.25)
Average [%] reduction			6.40 (5.62)	12.00 (9.61)	11.85 (9.62)	16.73 (13.60)	21.42 (18.51)	21.82 (19.89)	22.06 (18.03)	34.89 (30.00)

Table 5.37: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 2$, $m_{S,i} = 150$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.890873	0.0407271	3.74 (-13.96)	11.13 (-7.78)	24.32 (10.62)	14.48 (7.95)	14.42 (9.71)	11.64 (9.52)	18.15 (-19.89)	27.47 (9.12)
E1934	0.923641	0.0554748	18.00 (12.63)	18.98 (14.99)	11.46 (8.66)	27.01 (23.76)	29.80 (27.39)	30.38 (29.10)	36.34 (30.16)	47.70 (42.63)
I1940	0.902748	0.0660359	16.09 (7.90)	12.30 (2.86)	15.75 (7.35)	14.69 (12.08)	17.22 (16.12)	16.36 (17.15)	26.36 (9.68)	29.52 (23.64)
K1952	1.00752	0.0625796	16.55 (14.76)	9.52 (7.35)	7.90 (5.83)	18.41 (17.94)	22.86 (21.76)	24.10 (22.50)	18.25 (14.27)	34.93 (32.39)
B1968	0.925334	0.0567125	1.47 (-3.25)	13.60 (4.26)	25.35 (18.60)	17.17 (14.39)	26.27 (22.78)	26.90 (24.15)	26.39 (9.58)	39.10 (35.03)
S1971	1.1115	0.0437293	14.37 (1.59)	10.26 (-2.96)	17.93 (-2.06)	11.89 (10.51)	16.91 (13.11)	19.43 (13.32)	23.02 (-2.90)	28.57 (14.41)
I1979	0.988337	0.0844491	15.15 (14.92)	14.16 (13.82)	14.06 (13.72)	13.41 (13.38)	18.03 (17.83)	19.81 (19.44)	26.43 (25.82)	32.50 (31.97)
C1985	0.966721	0.0710159	9.28 (8.10)	11.78 (11.18)	22.15 (21.91)	10.03 (9.83)	14.93 (14.77)	17.28 (17.29)	22.27 (21.22)	28.01 (27.59)
L1989	0.966135	0.119308	14.95 (14.59)	13.49 (13.52)	12.72 (12.50)	7.56 (7.27)	10.81 (10.80)	12.58 (12.79)	25.36 (25.44)	22.05 (22.84)
N1994	0.947195	0.0679715	10.27 (13.84)	10.32 (14.72)	7.90 (12.43)	10.27 (9.55)	14.49 (14.01)	15.03 (15.08)	22.99 (30.47)	28.37 (30.44)
K1995TZ	0.974957	0.0296916	36.16 (24.05)	37.34 (24.77)	37.35 (25.10)	36.16 (30.37)	42.00 (35.32)	41.72 (35.48)	60.81 (43.61)	66.62 (52.46)
K1995TK	0.935642	0.129681	3.61 (3.96)	3.07 (3.50)	6.07 (6.90)	4.53 (3.95)	8.62 (8.29)	9.42 (9.49)	6.11 (6.93)	10.32 (10.38)
A2009	0.914413	0.101998	8.50 (7.52)	5.37 (4.75)	8.20 (8.68)	6.47 (5.64)	8.65 (8.49)	6.99 (7.47)	11.58 (10.51)	16.52 (15.75)
C2010A	0.925678	0.108814	22.56 (20.80)	23.03 (23.32)	17.23 (21.60)	13.62 (13.03)	13.78 (14.32)	12.91 (14.27)	40.63 (41.83)	24.59 (26.79)
C2010C	0.940833	0.0632137	24.30 (17.41)	26.12 (18.46)	18.35 (12.42)	21.19 (19.49)	22.85 (21.84)	22.31 (22.15)	42.77 (30.46)	39.27 (37.36)
N2010	0.920326	0.064712	-0.67 (10.58)	0.84 (15.56)	1.97 (15.33)	10.55 (8.42)	14.99 (13.27)	13.10 (12.75)	1.45 (28.26)	16.63 (22.48)
T2011T	0.970357	0.0496457	25.95 (21.56)	25.23 (21.66)	27.25 (24.55)	27.94 (25.94)	26.19 (24.73)	23.00 (22.21)	43.94 (38.55)	50.55 (44.75)
T2011S	1.02564	0.0584143	-1.00 (-1.07)	17.94 (9.65)	30.95 (25.82)	12.02 (11.26)	19.87 (18.02)	21.47 (18.91)	27.80 (22.05)	36.41 (31.66)
Average [%] reduction			13.29 (9.77)	14.69 (10.76)	17.05 (13.89)	15.41 (13.60)	19.04 (17.36)	19.13 (17.95)	26.70 (20.34)	32.17 (28.43)

Table 5.38: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 3$, $m_{S,i} = 100$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	1.03322	0.03677	19.96 (14.22)	21.39 (13.77)	24.12 (15.72)	23.64 (20.41)	28.30 (23.54)	28.07 (22.68)	38.12 (25.50)	44.76 (34.30)
E1934	1.01488	0.0396895	36.12 (23.31)	37.20 (23.70)	42.94 (27.64)	31.16 (27.13)	33.51 (29.07)	32.89 (28.58)	60.21 (41.56)	57.18 (47.61)
I1940	0.998825	0.0458001	15.86 (12.66)	14.48 (11.24)	12.45 (9.48)	23.83 (22.12)	28.56 (26.11)	28.56 (25.90)	27.36 (21.67)	35.91 (31.20)
K1952	0.976814	0.072005	10.56 (11.63)	19.69 (20.86)	23.95 (21.76)	15.36 (15.20)	19.16 (18.93)	19.65 (19.46)	35.38 (37.27)	33.84 (33.28)
B1968	0.977551	0.0461093	30.98 (23.31)	45.23 (34.51)	41.17 (31.67)	22.36 (20.58)	30.89 (28.69)	29.82 (28.16)	70.02 (57.03)	56.24 (50.29)
S1971	0.999757	0.0316686	26.07 (20.50)	29.33 (24.64)	26.53 (20.40)	26.56 (23.99)	30.29 (26.97)	29.17 (25.86)	50.91 (43.95)	53.55 (45.91)
I1979	0.938887	0.0668709	5.18 (7.91)	8.35 (11.29)	5.32 (6.82)	14.10 (12.54)	17.22 (16.15)	17.45 (17.20)	15.13 (21.62)	23.60 (26.72)
C1985	0.74759	0.0192923	13.13 (-6.37)	12.21 (-7.56)	9.12 (0.75)	10.13 (0.64)	9.16 (4.47)	7.20 (7.61)	22.94 (-12.62)	17.01 (7.56)
L1989	0.921173	0.0685068	0.17 (7.19)	9.08 (18.17)	12.93 (14.82)	16.59 (13.46)	19.47 (16.71)	20.49 (18.48)	17.79 (33.34)	31.75 (30.40)
N1994	0.759808	0.0580176	4.16 (-3.47)	9.60 (-2.75)	1.70 (0.95)	6.57 (1.42)	7.09 (5.22)	6.03 (7.78)	21.79 (1.21)	16.39 (7.55)
K1995TZ	1.13389	0.0214539	20.21 (-1.01)	18.81 (-4.49)	19.42 (-5.10)	11.31 (3.03)	19.95 (5.82)	23.22 (6.13)	34.51 (-9.67)	37.17 (0.34)
K1995TK	0.896656	0.0458212	6.35 (3.77)	5.01 (3.18)	7.91 (6.74)	10.09 (5.02)	14.63 (9.84)	14.56 (11.94)	9.72 (6.41)	18.64 (10.04)
A2009	0.942004	0.0611746	11.50 (10.47)	9.54 (9.16)	7.57 (7.01)	12.83 (11.48)	16.44 (15.36)	13.88 (13.52)	18.75 (18.18)	25.96 (24.91)
C2010A	0.968326	0.036912	39.84 (37.84)	39.89 (36.86)	36.20 (33.47)	35.26 (30.76)	33.81 (29.93)	30.37 (27.41)	63.40 (60.31)	60.07 (51.79)
C2010C	0.973994	0.0660256	3.62 (8.45)	-7.72 (-3.42)	-21.40 (-18.58)	21.70 (21.24)	23.06 (22.70)	22.11 (21.96)	-16.61 (-7.40)	36.04 (36.54)
N2010	1.01958	0.0501429	2.55 (4.76)	-1.13 (4.46)	3.64 (7.52)	14.10 (13.00)	22.44 (20.20)	21.84 (19.26)	-1.76 (9.04)	37.90 (35.36)
T2011T	1.02433	0.0656673	20.30 (18.35)	16.29 (14.57)	15.75 (13.37)	23.24 (21.95)	20.93 (19.26)	17.15 (15.41)	29.86 (26.85)	37.39 (32.52)
T2011S	0.888242	0.0692412	0.46 (-0.47)	16.13 (12.07)	10.36 (9.40)	9.21 (7.57)	14.11 (13.13)	13.94 (14.29)	30.95 (24.70)	24.78 (20.36)
Average [%] reduction			14.83 (10.73)	16.86 (12.24)	15.54 (11.32)	18.22 (15.09)	21.61 (18.45)	20.91 (18.42)	29.36 (22.16)	36.01 (29.26)

Table 5.39: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 3$, $m_{S,i} = 150$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.92483	0.0243864	12.71 (14.62)	11.79 (13.71)	13.22 (11.07)	25.00 (15.96)	27.58 (18.55)	24.11 (17.23)	19.09 (25.39)	41.20 (31.33)
E1934	0.96832	0.0611859	12.26 (9.99)	16.37 (13.55)	12.82 (10.36)	22.03 (21.12)	24.23 (23.29)	23.35 (22.57)	30.27 (25.49)	41.69 (40.24)
I1940	1.01658	0.0647684	15.54 (13.84)	9.76 (8.46)	10.43 (9.08)	14.28 (13.80)	18.91 (17.79)	19.15 (17.64)	18.34 (15.97)	31.95 (27.53)
K1952	0.939392	0.0517189	36.23 (33.94)	37.45 (30.57)	35.69 (29.39)	25.44 (22.63)	27.83 (25.35)	27.00 (25.27)	60.45 (51.62)	51.24 (45.70)
B1968	0.920158	0.0429075	-1.69 (-5.98)	0.74 (3.73)	2.47 (8.91)	8.94 (6.40)	15.75 (12.50)	14.62 (12.56)	5.30 (10.24)	25.06 (19.92)
S1971	0.835233	0.169414	-0.55 (-7.32)	4.09 (-2.00)	4.98 (-2.78)	5.53 (3.90)	6.47 (7.04)	6.00 (8.18)	8.17 (-6.34)	7.83 (6.13)
I1979	0.937588	0.0403184	30.01 (12.71)	34.43 (16.71)	28.78 (11.18)	29.48 (24.25)	30.65 (26.26)	28.56 (25.60)	56.91 (31.00)	57.92 (38.86)
C1985	0.938051	0.03786	21.46 (15.78)	22.16 (17.64)	22.83 (18.27)	33.60 (26.08)	34.18 (27.66)	33.61 (28.44)	39.26 (32.26)	57.67 (47.32)
L1989	0.982731	0.0695636	19.19 (19.09)	23.00 (21.57)	19.22 (19.05)	23.99 (23.70)	25.65 (25.15)	25.99 (25.35)	41.38 (39.17)	43.42 (41.73)
N1994	0.936424	0.0603153	26.05 (23.48)	25.72 (21.20)	25.61 (25.05)	19.28 (16.02)	20.51 (18.16)	19.91 (18.81)	44.37 (40.88)	40.74 (38.86)
K1995TZ	0.900483	0.0242242	16.09 (0.95)	34.18 (15.01)	27.34 (8.91)	18.37 (6.87)	24.10 (10.85)	24.18 (12.43)	55.93 (29.83)	44.16 (19.49)
K1995TK	0.893564	0.0680281	12.75 (13.34)	10.49 (11.85)	8.17 (9.04)	16.68 (13.84)	19.77 (17.97)	19.15 (18.89)	21.80 (24.47)	29.90 (29.67)
A2009	0.798406	0.0343981	15.17 (3.85)	5.34 (0.06)	-0.05 (1.94)	10.41 (4.64)	10.13 (8.86)	7.47 (10.23)	12.36 (3.50)	17.31 (11.43)
C2010A	0.962565	0.0801604	12.10 (11.83)	2.85 (2.66)	2.23 (2.09)	15.37 (15.26)	12.90 (13.04)	9.06 (9.33)	5.35 (5.04)	17.52 (17.77)
C2010C	0.961833	0.0696554	-0.64 (4.29)	12.54 (8.28)	14.54 (8.90)	17.14 (16.65)	17.76 (17.61)	15.68 (15.91)	23.49 (17.93)	31.59 (31.03)
N2010	0.892457	0.0464428	3.31 (9.70)	9.16 (16.41)	11.40 (17.43)	8.21 (6.13)	13.11 (12.16)	11.79 (12.96)	21.36 (33.83)	23.89 (28.25)
T2011T	0.976566	0.05651	19.11 (16.45)	15.66 (13.15)	12.17 (10.73)	19.33 (18.62)	14.22 (13.88)	7.98 (7.95)	31.20 (26.98)	27.00 (24.75)
T2011S	0.852511	0.0532526	4.15 (5.27)	6.98 (10.96)	-2.56 (3.42)	9.25 (4.42)	11.58 (8.21)	10.57 (9.92)	12.25 (27.56)	14.45 (10.13)
Average [%] reduction			14.07 (10.88)	15.71 (12.42)	13.85 (11.22)	17.91 (14.46)	19.74 (16.91)	18.23 (16.63)	28.18 (24.16)	33.59 (28.34)

Table 5.40: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with* $n_S = 5$, $m_{S,i} = 100$ *t (in brackets the results obtained with Den Hartog tuning formulas [26]:* $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$*).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.895472	0.0840756	21.31 (19.07)	22.96 (12.38)	25.01 (25.32)	14.59 (12.38)	15.99 (15.44)	14.58 (15.62)	43.88 (28.40)	35.81 (30.51)
E1934	1.00133	0.0591523	18.03 (16.17)	18.47 (18.35)	11.92 (12.26)	21.83 (21.19)	23.76 (22.67)	22.44 (21.17)	33.03 (32.76)	38.16 (36.50)
I1940	0.883718	0.0490221	2.46 (-9.99)	5.15 (-3.00)	-1.13 (-4.36)	14.16 (7.44)	15.24 (10.68)	13.46 (11.83)	9.89 (-0.72)	19.06 (2.96)
K1952	0.929417	0.0404525	33.13 (15.71)	31.61 (11.54)	39.00 (19.46)	28.16 (20.82)	28.07 (21.82)	24.91 (20.48)	55.36 (24.88)	54.35 (34.60)
B1968	0.986106	0.0492624	4.81 (4.24)	9.35 (8.02)	7.76 (7.20)	16.89 (16.05)	25.54 (23.89)	26.23 (24.32)	19.80 (17.49)	38.27 (34.73)
S1971	0.971755	0.0582123	16.30 (15.83)	15.59 (15.45)	15.52 (15.74)	24.40 (23.36)	24.72 (24.07)	22.29 (22.14)	27.60 (27.87)	35.29 (34.81)
I1979	0.970793	0.0481524	15.08 (9.85)	9.54 (5.46)	6.25 (3.80)	17.70 (16.54)	18.54 (17.76)	14.90 (14.77)	18.80 (11.04)	29.53 (23.55)
C1985	1.00274	0.077468	11.57 (11.37)	8.37 (8.20)	10.31 (9.88)	18.74 (18.47)	16.70 (16.26)	13.67 (13.18)	16.08 (15.72)	28.44 (27.45)
L1989	0.986852	0.0984003	16.58 (16.18)	16.65 (16.39)	16.70 (17.36)	12.19 (12.13)	11.09 (10.85)	8.69 (8.32)	29.67 (29.12)	28.51 (28.28)
N1994	0.947872	0.0737448	6.20 (8.56)	4.39 (8.09)	5.53 (8.73)	18.54 (17.97)	18.07 (18.36)	16.01 (17.02)	8.53 (15.67)	23.71 (25.37)
K1995TZ	0.81853	0.0359009	21.01 (8.67)	17.28 (5.46)	15.13 (12.43)	18.18 (4.64)	18.07 (7.78)	16.16 (9.92)	30.19 (13.44)	31.13 (7.11)
K1995TK	1.08194	0.0376321	19.09 (12.58)	11.00 (9.30)	16.36 (13.07)	17.41 (11.46)	23.13 (14.37)	23.47 (13.82)	19.58 (15.88)	41.47 (24.32)
A2009	1.0329	0.0445918	20.93 (20.46)	22.59 (21.98)	29.49 (25.01)	26.44 (22.83)	29.42 (24.23)	29.47 (23.35)	48.00 (43.63)	51.94 (46.01)
C2010A	0.933437	0.126264	10.43 (9.88)	0.84 (1.94)	4.76 (4.66)	7.63 (6.99)	6.35 (6.18)	3.38 (3.37)	1.42 (3.65)	11.05 (10.47)
C2010C	1.01944	0.0723827	17.42 (20.27)	18.19 (16.89)	8.18 (6.65)	15.07 (14.62)	17.04 (16.03)	15.36 (14.09)	38.44 (36.39)	34.68 (33.02)
N2010	0.788342	0.0199646	11.13 (9.45)	16.44 (1.95)	17.48 (9.71)	12.92 (0.80)	14.89 (4.28)	12.16 (6.22)	28.55 (12.50)	32.91 (12.96)
T2011T	0.957806	0.0518969	1.55 (-0.33)	5.76 (4.04)	0.62 (-0.28)	18.58 (17.11)	11.48 (10.75)	4.05 (3.88)	9.78 (6.35)	9.34 (7.35)
T2011S	0.94342	0.0754237	-1.54 (2.50)	-3.15 (0.46)	-4.22 (0.05)	14.36 (13.73)	17.59 (17.45)	17.67 (18.15)	-6.58 (0.84)	27.99 (27.96)
Average [%] reduction			13.64 (10.58)	12.83 (9.05)	12.48 (10.37)	17.65 (14.36)	18.65 (15.72)	16.61 (14.54)	24.00 (18.61)	31.76 (24.89)

Table 5.41: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 5$, $m_{S,i} = 150$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	1.04083	0.0344724	8.02 (3.51)	12.45 (11.88)	15.49 (10.62)	28.28 (22.42)	31.10 (23.98)	28.98 (22.00)	23.86 (22.52)	40.22 (29.47)
E1934	0.915993	0.0404522	11.48 (9.33)	12.64 (12.39)	9.77 (9.36)	20.41 (12.67)	18.36 (13.10)	13.85 (11.45)	22.88 (23.29)	29.43 (24.52)
I1940	0.90967	0.0647859	5.82 (3.54)	8.31 (6.42)	11.02 (10.18)	16.85 (14.35)	16.98 (16.03)	14.82 (15.47)	16.30 (13.06)	23.13 (14.69)
K1952	0.941161	0.0946926	10.66 (7.55)	11.43 (9.90)	10.47 (8.00)	12.36 (11.84)	12.34 (12.44)	10.19 (10.74)	21.44 (18.75)	18.00 (17.12)
B1968	0.985798	0.0775703	0.81 (0.48)	9.67 (9.06)	6.03 (5.92)	11.38 (11.33)	17.65 (17.46)	18.21 (17.95)	17.06 (15.99)	28.69 (28.40)
S1971	0.964351	0.0402425	29.47 (22.43)	27.35 (19.28)	1.31 (0.73)	26.25 (23.25)	24.17 (22.21)	17.99 (17.28)	46.39 (34.48)	43.34 (34.82)
I1979	0.955626	0.0418168	-3.11 (-3.45)	6.03 (3.31)	2.52 (1.29)	16.02 (13.79)	19.62 (17.20)	16.69 (14.98)	11.13 (6.32)	27.74 (22.83)
C1985	1.02266	0.0427868	9.41 (7.77)	7.39 (5.97)	5.89 (4.92)	24.94 (21.56)	19.94 (17.26)	13.20 (11.53)	13.62 (11.27)	31.20 (26.90)
L1989	0.970767	0.0614077	27.89 (23.78)	22.59 (18.32)	12.20 (8.34)	21.56 (20.91)	19.77 (19.35)	14.21 (14.07)	42.94 (35.96)	38.52 (35.64)
N1994	0.860946	0.0536056	11.17 (9.97)	8.99 (8.06)	6.38 (7.93)	14.61 (10.25)	12.62 (10.99)	9.16 (9.54)	16.79 (15.65)	17.35 (19.41)
K1995TZ	1.01045	0.0399083	35.41 (25.46)	33.22 (23.48)	31.78 (23.58)	36.99 (32.23)	37.24 (32.13)	35.63 (30.58)	55.74 (41.51)	60.42 (48.79)
K1995TK	0.939347	0.087378	12.52 (9.43)	6.25 (1.90)	0.44 (0.70)	10.08 (9.66)	13.11 (13.23)	11.25 (11.89)	16.15 (8.31)	18.55 (15.77)
A2009	1.02662	0.0482874	6.40 (6.41)	2.62 (2.78)	7.58 (6.91)	18.10 (15.85)	20.21 (17.06)	18.95 (15.59)	5.07 (5.35)	21.41 (19.56)
C2010A	0.970385	0.0442778	18.01 (22.22)	19.57 (20.80)	-4.75 (-2.54)	27.51 (25.05)	23.55 (21.74)	12.45 (11.74)	30.72 (30.47)	42.68 (39.82)
C2010C	0.987522	0.0568964	17.88 (23.55)	31.89 (31.43)	28.48 (24.93)	23.95 (23.01)	24.42 (23.40)	20.43 (19.63)	53.89 (55.97)	44.79 (43.78)
N2010	0.992738	0.0276665	22.79 (13.87)	25.61 (16.40)	30.75 (21.14)	38.35 (32.06)	42.63 (35.50)	41.44 (34.73)	47.10 (32.71)	63.07 (49.79)
T2011T	1.00029	0.0473623	3.30 (4.11)	1.70 (1.92)	0.46 (0.83)	22.59 (20.94)	13.79 (12.86)	4.02 (3.82)	3.14 (3.65)	13.81 (13.16)
T2011S	0.960377	0.0658049	9.54 (9.30)	11.57 (10.96)	6.09 (5.92)	17.21 (16.56)	20.62 (20.04)	21.16 (20.84)	20.82 (19.82)	29.16 (30.89)
Average [%] reduction			13.19 (11.07)	14.40 (11.90)	10.11 (8.26)	21.53 (18.76)	21.56 (19.22)	17.92 (16.32)	25.83 (21.95)	32.86 (28.63)

Table 5.42: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with* $n_S = 10$, $m_{S,i} = 100$ *t (in brackets the results obtained with Den Hartog tuning formulas [26]:* $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$*).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.895409	0.061303	9.55 (6.22)	3.90 (5.16)	0.33 (0.17)	14.66 (9.03)	16.08 (11.82)	13.26 (11.22)	7.65 (10.82)	24.34 (20.07)
E1934	1.10553	0.0356198	-2.11 (-0.77)	19.27 (19.90)	14.93 (12.74)	14.47 (11.60)	16.06 (10.91)	12.80 (7.54)	28.88 (29.45)	28.56 (20.55)
I1940	0.891794	0.034549	5.51 (5.84)	0.60 (1.15)	0.32 (1.12)	19.19 (7.51)	17.12 (8.55)	12.45 (7.67)	0.87 (2.06)	26.33 (12.37)
K1952	0.904638	0.0434278	28.09 (7.53)	20.85 (13.58)	23.22 (9.36)	21.62 (14.73)	19.56 (15.48)	14.16 (12.93)	38.30 (25.81)	41.76 (28.32)
B1968	1.09385	0.0962147	-0.52 (1.86)	-1.32 (0.86)	6.52 (7.66)	5.20 (4.57)	10.00 (8.17)	9.95 (7.27)	-1.56 (2.45)	19.18 (11.46)
S1971	0.85607	0.039444	0.64 (-2.94)	3.98 (-1.09)	7.50 (-1.63)	10.74 (-1.19)	11.46 (1.74)	7.19 (2.76)	8.42 (-1.23)	20.27 (0.61)
I1979	1.04924	0.0307926	27.02 (9.07)	30.26 (12.82)	31.81 (13.41)	30.58 (22.60)	32.84 (23.68)	30.46 (21.63)	55.51 (31.67)	56.42 (35.15)
C1985	0.946746	0.0391356	0.82 (0.64)	-0.03 (0.25)	0.27 (0.50)	24.20 (19.91)	14.50 (12.48)	4.65 (4.22)	-1.34 (-0.66)	23.21 (18.01)
L1989	1.15818	0.0240903	23.63 (18.30)	21.08 (8.66)	14.27 (9.63)	17.77 (11.35)	16.66 (8.29)	9.50 (3.72)	37.18 (15.75)	34.07 (14.76)
N1994	1.01293	0.0426578	12.25 (8.31)	12.67 (8.91)	8.16 (5.71)	25.31 (22.73)	21.19 (18.94)	12.13 (10.90)	23.17 (16.88)	31.22 (24.21)
K1995TZ	0.895526	0.0575958	6.41 (5.34)	7.57 (8.32)	6.17 (6.16)	9.51 (5.70)	8.48 (6.31)	4.91 (4.36)	15.57 (17.03)	13.30 (15.25)
K1995TK	0.844689	0.0290847	6.69 (3.75)	6.30 (1.69)	8.55 (5.22)	10.14 (2.37)	13.39 (6.75)	11.80 (8.60)	11.95 (3.90)	18.93 (10.30)
A2009	1.02472	0.0544367	6.76 (6.27)	3.71 (3.43)	3.34 (3.00)	24.62 (22.14)	22.66 (20.02)	16.11 (14.09)	6.97 (6.45)	20.51 (18.58)
C2010A	0.983372	0.109393	10.14 (7.24)	7.93 (5.56)	-3.70 (-4.63)	12.25 (11.91)	10.19 (9.90)	4.08 (3.92)	-7.29 (-10.87)	14.50 (13.48)
C2010C	0.866279	0.0622438	20.63 (1.57)	13.84 (-1.96)	14.15 (19.60)	13.32 (9.05)	11.51 (10.39)	7.42 (8.43)	22.10 (-1.43)	20.18 (10.83)
N2010	0.853831	0.0333259	7.11 (-0.69)	9.73 (5.05)	6.21 (-0.12)	10.29 (-0.84)	10.36 (1.32)	7.70 (2.50)	17.70 (8.97)	18.23 (3.53)
T2011T	0.857462	0.0690096	9.66 (8.23)	3.27 (4.24)	0.57 (1.90)	11.09 (7.04)	8.15 (6.82)	1.70 (1.73)	6.31 (9.39)	9.31 (8.50)
T2011S	0.989804	0.0742129	15.17 (14.43)	13.33 (12.92)	3.13 (3.24)	18.25 (18.12)	19.46 (19.14)	18.27 (17.86)	28.57 (27.89)	34.16 (33.18)
Average [%] reduction			10.41 (5.57)	9.83 (6.08)	8.10 (5.17)	16.29 (11.02)	15.54 (11.15)	11.03 (8.41)	16.61 (10.80)	25.25 (16.62)

Table 5.43: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 10$, $m_{S,i} = 150$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.910057	0.0542833	15.95 (10.77)	13.66 (9.45)	13.75 (10.04)	20.80 (15.89)	20.01 (17.21)	16.34 (15.73)	25.46 (18.52)	34.61 (28.73)
E1934	0.931619	0.0565274	7.68 (7.38)	7.72 (7.20)	4.34 (4.77)	22.91 (19.43)	20.75 (18.27)	13.19 (12.12)	16.96 (17.95)	31.15 (29.30)
I1940	1.05966	0.0661894	21.30 (22.61)	18.68 (19.73)	22.54 (21.62)	17.93 (16.58)	18.81 (16.24)	16.06 (13.01)	36.68 (37.53)	31.73 (30.09)
K1952	1.00502	0.113414	3.01 (3.13)	16.04 (17.42)	7.45 (7.45)	10.47 (10.00)	10.54 (9.58)	7.56 (6.48)	30.30 (30.98)	17.84 (15.15)
B1968	0.911842	0.096664	2.48 (2.25)	3.17 (3.43)	-4.64 (-8.47)	3.78 (2.76)	8.57 (7.64)	9.75 (9.41)	5.83 (6.47)	9.28 (6.23)
S1971	0.91315	0.0619153	3.55 (3.14)	11.12 (9.19)	14.51 (5.93)	17.86 (14.66)	17.54 (16.21)	13.52 (13.99)	20.81 (18.17)	31.78 (30.23)
I1979	1.00163	0.0356813	13.28 (12.61)	26.15 (22.07)	16.65 (10.38)	30.82 (26.94)	31.77 (27.58)	26.95 (23.45)	37.61 (35.82)	45.64 (37.07)
C1985	0.944286	0.0697308	6.95 (4.74)	8.57 (8.20)	0.97 (1.25)	14.82 (13.92)	7.60 (7.50)	1.69 (1.77)	17.01 (16.07)	16.74 (16.08)
L1989	0.78377	0.0250439	7.09 (7.44)	4.45 (4.57)	-1.09 (-2.38)	11.28 (2.74)	6.79 (3.11)	1.74 (1.19)	9.13 (-5.35)	12.94 (0.58)
N1994	1.03859	0.0335468	7.63 (0.03)	13.29 (6.54)	0.44 (-1.67)	17.01 (12.70)	13.98 (10.65)	5.73 (4.50)	27.11 (13.61)	25.05 (14.56)
K1995TZ	0.98394	0.0833355	3.62 (3.60)	2.43 (2.41)	2.75 (2.73)	9.11 (9.11)	10.97 (10.94)	7.67 (7.63)	5.27 (5.22)	11.39 (11.33)
K1995TK	1.07181	0.0255139	5.08 (7.19)	7.05 (8.91)	8.77 (8.78)	7.02 (2.33)	13.65 (6.68)	18.18 (9.58)	16.97 (19.51)	24.64 (14.45)
A2009	0.893544	0.0524771	1.63 (1.66)	2.41 (2.98)	-0.33 (0.29)	19.28 (13.24)	16.05 (12.11)	7.87 (6.45)	3.70 (4.87)	9.09 (8.46)
C2010A	0.972041	0.0680207	14.03 (12.86)	5.61 (4.55)	1.41 (1.15)	19.06 (18.74)	14.89 (14.88)	4.79 (4.88)	2.62 (5.83)	19.11 (19.59)
C2010C	1.03705	0.0517494	3.70 (6.97)	8.27 (7.49)	-2.54 (-3.62)	25.99 (23.49)	25.38 (21.81)	19.57 (16.17)	15.15 (13.63)	40.01 (33.48)
N2010	0.964303	0.0829553	-6.72 (-7.16)	-8.52 (-8.59)	-5.44 (-5.28)	14.39 (14.26)	16.26 (16.47)	15.47 (16.00)	-18.05 (-18.50)	15.54 (15.18)
T2011T	0.983707	0.0571295	13.91 (11.71)	13.48 (11.48)	-0.78 (-0.73)	24.07 (23.09)	21.93 (21.10)	5.84 (5.67)	9.91 (8.63)	22.47 (21.36)
T2011S	0.939754	0.0328566	34.02 (23.57)	30.87 (27.54)	8.80 (7.17)	35.11 (27.66)	32.57 (26.87)	27.06 (23.47)	52.76 (47.34)	54.22 (45.93)
Average [%] reduction			8.79 (7.47)	10.25 (9.14)	4.86 (3.30)	17.87 (14.86)	17.12 (14.71)	12.17 (10.64)	17.51 (15.35)	25.18 (20.99)

Table 5.44: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 15$, $m_{S,i} = 100$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	1.00825	0.0682327	20.58 (17.72)	17.89 (16.78)	6.10 (5.93)	22.61 (21.94)	21.73 (20.71)	17.23 (16.17)	32.90 (30.83)	34.65 (31.68)
E1934	1.03622	0.0303364	15.20 (13.19)	19.19 (13.01)	11.71 (9.04)	33.57 (25.71)	29.98 (22.99)	18.22 (14.24)	36.94 (26.03)	45.75 (36.16)
I1940	0.956427	0.0496657	15.83 (15.46)	9.17 (8.70)	1.81 (1.73)	25.43 (23.25)	22.75 (21.52)	15.45 (15.17)	17.19 (16.67)	31.04 (30.07)
K1952	1.02594	0.0579781	13.73 (14.03)	-1.11 (-0.90)	0.33 (0.53)	15.28 (14.25)	16.39 (14.74)	10.84 (9.53)	-2.14 (-1.65)	21.55 (21.01)
B1968	0.908281	0.0390982	3.21 (2.79)	8.88 (8.35)	5.10 (5.19)	20.03 (12.87)	24.36 (17.31)	24.70 (19.17)	16.61 (15.79)	26.61 (22.71)
S1971	0.997131	0.0965792	5.69 (5.97)	4.53 (4.65)	6.49 (6.56)	12.44 (12.26)	13.84 (13.42)	11.23 (10.69)	9.20 (9.27)	17.53 (17.68)
I1979	0.836646	0.0533721	3.31 (3.96)	4.85 (-0.37)	2.88 (4.33)	14.43 (7.29)	13.14 (9.74)	9.70 (9.54)	8.82 (8.64)	22.51 (13.59)
C1985	1.04019	0.0586145	16.67 (13.92)	1.55 (1.81)	1.16 (0.85)	15.88 (13.55)	9.47 (7.88)	1.88 (1.56)	3.60 (3.74)	15.13 (12.72)
L1989	0.962785	0.0391376	24.13 (20.29)	21.95 (17.93)	16.15 (16.39)	34.21 (29.40)	31.31 (27.37)	17.47 (15.70)	43.04 (38.53)	43.64 (37.73)
N1994	1.00098	0.0335262	13.68 (8.75)	10.26 (6.70)	2.44 (1.42)	32.44 (28.86)	29.22 (25.49)	13.94 (12.17)	21.69 (14.28)	41.81 (33.70)
K1995TZ	0.855999	0.0598906	7.04 (6.67)	4.18 (3.30)	3.45 (5.02)	16.05 (8.09)	15.93 (10.57)	11.04 (9.07)	8.48 (7.02)	19.00 (16.54)
K1995TK	0.945365	0.0422236	23.65 (16.61)	35.91 (27.02)	36.16 (28.89)	33.79 (28.17)	33.75 (29.33)	32.07 (29.12)	58.40 (47.77)	56.57 (48.70)
A2009	0.995261	0.0522925	1.99 (1.94)	0.76 (0.76)	1.06 (1.09)	27.21 (25.67)	24.03 (22.59)	12.84 (12.12)	1.27 (1.30)	24.48 (22.60)
C2010A	0.961498	0.10454	10.33 (10.55)	6.79 (6.20)	1.27 (1.28)	10.71 (10.53)	7.95 (7.99)	1.55 (1.57)	8.63 (8.76)	9.57 (8.98)
C2010C	0.926742	0.0576418	22.13 (16.57)	17.19 (12.75)	-4.47 (-6.55)	17.12 (14.61)	13.17 (12.08)	6.35 (6.29)	35.56 (28.01)	27.28 (23.79)
N2010	0.869304	0.0323114	8.74 (8.80)	6.07 (6.29)	4.22 (3.73)	18.55 (6.23)	17.12 (8.41)	13.31 (8.90)	11.46 (12.36)	24.80 (25.56)
T2011T	0.917658	0.0522836	17.05 (12.62)	8.48 (-2.70)	5.39 (3.72)	17.60 (13.26)	14.16 (11.94)	2.14 (2.02)	23.77 (14.85)	16.66 (7.40)
T2011S	1.15983	0.0368739	9.93 (10.27)	8.84 (8.04)	-2.03 (-2.90)	12.70 (7.05)	13.86 (6.02)	11.52 (3.91)	11.13 (8.75)	27.05 (10.54)
Average [%] reduction			12.94 (11.12)	10.30 (7.68)	5.51 (4.79)	21.11 (16.83)	19.57 (16.12)	12.86 (10.94)	19.25 (16.16)	28.09 (23.40)

Table 5.45: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 15$, $m_{S,i} = 150$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.872473	0.0258985	11.38 (-0.22)	9.16 (6.22)	3.22 (3.21)	15.48 (-0.77)	11.66 (-0.05)	5.16 (0.02)	16.25 (12.58)	17.08 (4.82)
E1934	0.889552	0.020692	5.49 (-8.02)	5.51 (1.36)	1.05 (1.43)	15.69 (3.70)	10.74 (4.00)	3.30 (1.76)	17.37 (14.55)	11.37 (1.90)
I1940	0.95681	0.0724227	-9.53 (-10.19)	0.92 (0.84)	1.14 (1.18)	13.43 (13.07)	11.87 (11.89)	6.26 (6.46)	-0.31 (-0.70)	11.64 (10.88)
K1952	0.861956	0.076565	7.77 (-0.90)	1.51 (1.39)	-0.54 (-3.82)	14.40 (11.40)	13.55 (12.93)	8.38 (9.27)	2.70 (2.64)	15.99 (8.91)
B1968	0.855863	0.0408889	3.67 (4.21)	2.70 (3.50)	3.16 (5.33)	16.13 (13.61)	16.06 (16.59)	14.27 (17.57)	5.09 (7.37)	22.79 (23.43)
S1971	0.957739	0.0609743	5.53 (5.43)	4.63 (4.57)	18.54 (20.23)	24.04 (22.74)	24.32 (23.43)	19.82 (19.50)	8.58 (8.72)	27.42 (26.21)
I1979	1.00663	0.0519338	23.48 (19.84)	18.91 (16.20)	13.80 (11.37)	31.65 (29.44)	31.11 (28.40)	27.05 (24.39)	36.27 (31.76)	46.91 (42.17)
C1985	0.986352	0.0381571	22.67 (21.94)	-0.25 (0.44)	-0.06 (0.12)	29.07 (25.78)	20.23 (18.06)	3.92 (3.55)	-0.92 (0.35)	21.76 (19.28)
L1989	0.994976	0.0499929	4.71 (5.98)	-0.26 (0.75)	0.53 (0.45)	21.50 (20.33)	16.91 (16.27)	6.51 (6.46)	-0.87 (1.17)	8.80 (10.14)
N1994	0.878199	0.061028	10.97 (11.00)	2.65 (1.76)	0.92 (0.12)	11.44 (6.52)	8.32 (6.23)	2.63 (2.51)	12.87 (11.33)	17.42 (18.19)
K1995TZ	0.980903	0.0537234	13.18 (11.82)	3.67 (3.56)	2.02 (1.83)	29.14 (27.65)	28.56 (27.21)	23.04 (22.17)	6.96 (6.85)	33.75 (31.61)
K1995TK	1.16792	0.0265709	20.17 (4.22)	24.38 (5.14)	16.16 (-4.46)	15.02 (4.87)	17.38 (4.75)	16.98 (3.72)	43.37 (7.92)	37.76 (9.84)
A2009	0.981079	0.0625469	2.86 (2.82)	1.02 (1.02)	0.72 (0.77)	20.33 (19.83)	15.74 (15.49)	5.92 (5.91)	1.54 (1.55)	10.78 (10.60)
C2010A	0.971528	0.0462924	25.63 (24.72)	0.63 (0.61)	-2.37 (-1.75)	28.19 (25.97)	19.61 (18.48)	2.68 (2.61)	6.24 (6.19)	22.34 (22.67)
C2010C	0.990611	0.0472883	14.01 (21.53)	21.76 (18.67)	9.23 (10.40)	27.71 (25.88)	20.56 (19.50)	7.90 (7.67)	35.80 (30.95)	35.27 (33.76)
N2010	1.02321	0.0760691	10.21 (8.71)	4.13 (2.59)	2.85 (2.23)	18.72 (18.04)	19.38 (17.73)	17.29 (15.06)	7.87 (4.73)	31.72 (25.06)
T2011T	0.957965	0.0805369	-0.31 (2.28)	9.11 (10.54)	-0.42 (-0.17)	13.49 (13.28)	11.94 (12.05)	1.45 (1.50)	1.74 (3.56)	9.75 (11.49)
T2011S	0.92496	0.0777011	-5.85 (-6.53)	7.67 (6.17)	0.87 (1.90)	17.15 (16.11)	14.06 (14.15)	7.90 (8.44)	16.58 (13.86)	17.46 (17.68)
Average [%] reduction			9.22 (6.59)	6.55 (4.74)	3.93 (2.80)	20.14 (16.53)	17.33 (14.84)	10.03 (8.81)	12.06 (9.19)	22.22 (18.26)

Table 5.46: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 25$, $m_{S,i} = 100$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.826521	0.0694907	10.57 (11.07)	8.26 (11.85)	3.33 (7.47)	8.46 (5.57)	6.91 (5.84)	2.92 (2.95)	15.74 (23.77)	12.88 (17.28)
E1934	0.979245	0.0142654	42.04 (25.49)	39.12 (21.33)	0.82 (0.64)	47.52 (35.82)	42.37 (32.55)	18.75 (15.12)	61.48 (39.22)	61.62 (48.45)
I1940	0.965417	0.0819431	-1.16 (-1.52)	-1.42 (-1.67)	0.01 (0.04)	17.22 (17.10)	16.52 (16.73)	9.28 (9.56)	-3.17 (-3.71)	22.83 (22.25)
K1952	1.08635	0.0561919	23.55 (20.29)	13.95 (12.92)	0.70 (0.62)	16.51 (13.12)	16.67 (11.84)	10.65 (6.82)	34.23 (29.47)	35.86 (24.68)
B1968	0.988204	0.055978	4.90 (4.64)	6.38 (5.97)	1.27 (1.20)	24.27 (23.25)	24.54 (23.38)	22.48 (21.42)	11.74 (11.24)	28.15 (27.13)
S1971	1.04145	0.0507049	2.48 (2.35)	1.18 (1.12)	9.43 (7.03)	24.90 (21.29)	22.22 (18.50)	14.03 (11.49)	1.74 (1.74)	20.30 (17.24)
I1979	0.80405	0.0351891	3.06 (2.66)	3.23 (3.93)	-0.56 (-7.27)	12.65 (6.85)	8.50 (6.37)	3.17 (3.00)	5.74 (7.60)	9.49 (6.99)
C1985	0.940815	0.106603	5.79 (5.74)	4.15 (3.80)	-0.07 (-0.31)	10.95 (10.42)	7.65 (7.74)	0.73 (0.76)	0.43 (-0.82)	6.89 (6.91)
L1989	0.915713	0.0302927	13.19 (12.18)	2.61 (0.63)	1.73 (1.29)	22.55 (12.22)	16.26 (9.84)	2.63 (1.79)	6.92 (12.32)	16.49 (14.57)
N1994	0.995999	0.0393052	-2.57 (-2.06)	5.36 (4.47)	0.22 (0.18)	33.79 (30.22)	28.18 (25.33)	9.20 (8.42)	11.12 (9.12)	30.52 (26.47)
K1995TZ	0.930757	0.0645064	3.62 (3.97)	4.37 (5.36)	0.74 (0.90)	17.41 (15.68)	15.10 (14.40)	9.47 (9.57)	5.57 (6.35)	11.67 (12.93)
K1995TK	0.864846	0.0452862	9.27 (8.52)	6.77 (5.64)	5.81 (6.99)	15.14 (7.67)	12.81 (9.10)	8.40 (7.85)	13.03 (11.89)	25.00 (22.83)
A2009	0.944308	0.0560224	0.52 (0.48)	1.16 (1.27)	0.32 (0.36)	17.49 (15.63)	11.67 (10.73)	2.12 (2.02)	0.67 (0.81)	4.24 (4.28)
C2010A	0.98958	0.0417818	32.57 (29.11)	-3.16 (0.90)	-1.12 (-0.71)	31.67 (28.33)	23.47 (21.10)	2.09 (1.91)	16.21 (22.44)	30.54 (27.87)
C2010C	0.924213	0.0492752	15.44 (15.91)	12.77 (8.72)	2.72 (6.31)	15.88 (12.35)	8.53 (7.28)	1.30 (1.23)	24.51 (17.58)	20.15 (17.36)
N2010	0.928838	0.0581873	14.36 (9.44)	18.75 (15.23)	0.97 (1.14)	21.81 (18.95)	21.61 (20.18)	17.40 (17.34)	34.59 (29.23)	31.90 (29.05)
T2011T	0.993899	0.0651747	24.85 (24.36)	17.14 (15.61)	3.02 (2.64)	24.64 (24.00)	22.81 (21.97)	3.11 (2.99)	25.70 (23.48)	37.59 (35.64)
T2011S	0.985809	0.0648794	-2.12 (-2.06)	-1.04 (-0.91)	-0.24 (0.30)	15.30 (15.05)	10.78 (10.66)	3.92 (3.93)	-2.71 (-2.41)	7.17 (7.25)
Average [%] reduction			11.13 (9.48)	7.75 (6.45)	1.62 (1.60)	21.01 (17.42)	17.59 (15.20)	7.87 (7.12)	14.64 (13.30)	22.96 (20.51)

Table 5.47: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 25$, $m_{S,i} = 150$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.971582	0.0550948	18.99 (16.33)	10.21 (9.92)	1.33 (1.39)	26.34 (24.94)	25.57 (24.29)	14.43 (13.83)	28.62 (25.13)	32.29 (29.98)
E1934	0.995106	0.030268	23.52 (20.37)	20.86 (15.54)	1.37 (1.30)	30.76 (27.22)	27.55 (24.14)	9.37 (8.31)	33.29 (25.44)	43.72 (37.30)
I1940	0.961494	0.107142	5.46 (5.63)	9.32 (9.94)	3.27 (3.44)	11.13 (11.01)	11.13 (11.26)	5.56 (5.69)	17.19 (17.32)	17.72 (18.35)
K1952	1.05555	0.029747	3.32 (2.89)	8.82 (7.88)	0.33 (0.32)	15.11 (10.06)	14.57 (9.40)	5.90 (3.78)	10.82 (9.02)	18.39 (15.10)
B1968	0.964775	0.0452373	3.45 (3.24)	1.83 (1.83)	0.57 (0.61)	23.43 (21.32)	21.86 (20.26)	17.20 (16.38)	7.35 (7.46)	21.80 (20.67)
S1971	0.838063	0.0229259	2.64 (3.08)	1.00 (1.26)	0.41 (0.51)	9.97 (-0.22)	5.74 (0.56)	1.28 (0.27)	1.48 (1.99)	3.30 (4.07)
I1979	0.993928	0.0340141	14.90 (14.82)	7.35 (7.31)	4.99 (4.07)	40.50 (34.82)	35.67 (30.80)	20.03 (17.60)	16.87 (16.84)	43.04 (37.91)
C1985	0.956726	0.0333983	27.37 (18.41)	-4.62 (-2.60)	0.46 (-0.02)	36.01 (29.32)	27.02 (22.58)	2.37 (2.07)	8.19 (11.16)	32.07 (25.62)
L1989	1.0227	0.0432962	3.93 (3.90)	5.68 (5.03)	0.92 (0.81)	18.35 (15.43)	16.31 (14.02)	2.42 (2.15)	-0.25 (-0.32)	13.63 (10.63)
N1994	0.933284	0.172167	4.24 (5.50)	2.94 (4.10)	0.46 (0.53)	5.37 (3.77)	3.39 (2.35)	0.43 (0.25)	6.44 (8.96)	6.77 (7.94)
K1995TZ	1.00418	0.0415647	2.03 (2.03)	0.96 (1.05)	-0.02 (0.04)	23.53 (21.53)	20.09 (18.19)	10.99 (10.09)	1.32 (1.64)	18.62 (17.12)
K1995TK	1.0283	0.0747715	10.71 (8.91)	8.15 (7.16)	3.06 (2.52)	18.61 (17.86)	18.64 (16.89)	14.52 (12.54)	21.16 (18.28)	33.21 (29.94)
A2009	0.975983	0.053513	0.69 (0.68)	0.70 (0.80)	-0.09 (0.00)	22.29 (21.18)	16.48 (15.80)	2.44 (2.39)	2.69 (2.83)	8.14 (7.74)
C2010A	0.940572	0.0713449	0.55 (1.24)	1.01 (-1.78)	-0.95 (-0.53)	14.30 (13.07)	8.42 (7.99)	0.32 (0.31)	14.65 (16.14)	8.18 (6.91)
C2010C	0.835697	0.0298387	-3.29 (-6.45)	7.36 (-10.74)	0.55 (0.34)	12.07 (6.03)	5.67 (4.13)	0.54 (0.56)	8.62 (-12.00)	10.42 (-6.89)
N2010	0.922716	0.0545801	10.03 (8.54)	12.67 (12.30)	-0.07 (-0.27)	21.54 (18.62)	18.94 (17.60)	13.11 (13.13)	22.88 (22.77)	24.33 (23.52)
T2011T	0.89795	0.0581179	1.62 (5.34)	-0.01 (3.67)	-0.25 (-1.18)	12.24 (8.12)	8.92 (7.29)	0.48 (0.47)	-8.33 (-8.58)	5.09 (9.16)
T2011S	0.932302	0.0498266	-3.25 (-2.31)	-1.88 (-1.55)	0.41 (0.73)	22.67 (18.84)	15.56 (13.83)	4.65 (4.43)	-5.20 (-4.41)	13.92 (13.18)
Average [%] reduction			7.05 (6.23)	5.13 (3.95)	0.93 (0.81)	20.23 (16.83)	16.75 (14.52)	7.00 (6.35)	10.43 (8.87)	19.70 (17.12)

Table 5.48: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 40$, $m_{S,i} = 100$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.877317	0.103496	-5.76 (-7.40)	-2.68 (-3.04)	-0.14 (-0.12)	7.27 (5.37)	6.47 (5.81)	2.01 (2.04)	-10.93 (-17.90)	2.95 (0.90)
E1934	0.778907	0.0155729	5.87 (5.69)	3.16 (2.58)	-1.71 (2.48)	7.19 (0.22)	3.45 (1.15)	0.27 (0.27)	4.82 (4.65)	7.41 (-0.79)
I1940	0.990345	0.048193	5.64 (5.17)	5.45 (5.18)	2.60 (2.45)	30.29 (28.08)	26.44 (24.88)	12.85 (12.35)	8.54 (8.34)	33.16 (30.60)
K1952	0.868955	0.0734992	11.49 (11.93)	6.37 (5.97)	0.07 (0.08)	8.56 (4.27)	6.23 (4.32)	1.56 (1.49)	11.59 (11.44)	12.03 (11.86)
B1968	1.04321	0.0755945	1.15 (1.26)	2.70 (2.54)	0.73 (0.70)	14.84 (14.07)	14.69 (12.98)	10.50 (8.77)	5.49 (5.23)	18.22 (16.30)
S1971	1.0465	0.0346906	0.56 (0.52)	1.12 (1.07)	3.17 (2.49)	21.22 (17.27)	16.49 (13.06)	4.28 (3.35)	1.67 (1.66)	13.46 (10.71)
I1979	0.82141	0.0102298	1.18 (0.99)	0.28 (0.16)	-0.51 (-2.15)	12.86 (2.43)	6.70 (1.80)	1.19 (0.34)	0.57 (0.35)	5.87 (3.61)
C1985	0.836333	0.0294891	5.87 (1.38)	-2.21 (-0.88)	0.19 (0.54)	10.76 (-1.14)	4.72 (0.08)	0.12 (0.02)	-4.18 (-2.27)	2.42 (-1.14)
L1989	0.996509	0.0313646	17.33 (14.92)	2.37 (2.13)	0.10 (0.14)	33.62 (28.98)	32.98 (27.83)	7.08 (6.07)	3.94 (3.60)	31.63 (28.75)
N1994	0.907679	0.0745779	-2.99 (-4.05)	-1.70 (-1.63)	-0.14 (-0.26)	9.66 (8.07)	4.97 (4.61)	0.28 (0.29)	-3.13 (-3.16)	-0.12 (-0.61)
K1995TZ	0.994111	0.026436	4.91 (4.55)	0.11 (0.20)	-0.04 (0.04)	15.50 (14.02)	9.69 (8.94)	2.27 (2.20)	-0.32 (-0.15)	4.01 (4.14)
K1995TK	1.0055	0.049696	8.19 (8.76)	-2.30 (-1.69)	-1.32 (-1.08)	26.55 (24.54)	20.97 (19.37)	10.32 (9.62)	0.67 (1.93)	20.99 (19.90)
A2009	0.983696	0.0269553	0.64 (0.67)	0.55 (0.71)	0.30 (0.28)	25.20 (21.93)	16.98 (15.00)	1.50 (1.38)	0.38 (0.40)	6.04 (5.67)
C2010A	0.90052	0.0596004	1.89 (-6.08)	-3.34 (1.03)	0.61 (0.76)	9.27 (5.31)	4.61 (3.53)	0.08 (0.08)	9.30 (11.06)	2.66 (1.96)
C2010C	1.09889	0.0471123	20.50 (9.90)	17.12 (6.31)	-0.18 (-2.69)	13.23 (9.52)	10.04 (6.33)	0.85 (0.49)	19.31 (-6.75)	20.10 (6.74)
N2010	0.923111	0.0237832	-4.22 (2.03)	-0.20 (1.06)	0.98 (1.09)	20.54 (13.21)	15.76 (10.80)	7.05 (5.29)	3.34 (2.40)	7.10 (9.56)
T2011T	0.901184	0.0377875	23.93 (11.26)	-3.82 (-7.33)	0.39 (-0.03)	26.49 (13.26)	20.46 (11.49)	0.87 (0.56)	-5.77 (-11.61)	20.95 (12.06)
T2011S	1.02095	0.0894362	5.04 (5.14)	2.60 (2.39)	-0.13 (-0.11)	13.40 (13.03)	8.11 (7.59)	1.49 (1.34)	2.11 (2.17)	7.96 (7.41)
Average [%] reduction			5.62 (3.70)	1.42 (0.93)	0.28 (0.26)	17.03 (12.36)	12.76 (9.98)	3.59 (3.11)	2.63 (0.63)	12.05 (9.31)

Table 5.49: *Percentage reduction [%] of the primary structure seismic response obtained with the proposed TMD tuning method, for the primary structure with $n_S = 40$, $m_{S,i} = 150$ t (in brackets the results obtained with Den Hartog tuning formulas [26]: $f(\mu = 0.02) = 0.980392$, $\zeta_T(\mu = 0.02) = 0.0857493$).*

Earthquake	f^{opt}	ζ_T^{opt}	$\Delta x_{S,n}^{max}$	$\Delta \dot{x}_{S,n}^{max}$	$\Delta \ddot{x}_{S,n}^{max}$	$\Delta x_{S,n}^{RMS}$	$\Delta \dot{x}_{S,n}^{RMS}$	$\Delta \ddot{x}_{S,n}^{RMS}$	ΔT_S^{max}	ΔT_S^{RMS}
L1933	0.955698	0.046404	4.42 (3.09)	4.02 (3.21)	-0.23 (-0.16)	23.20 (20.81)	22.69 (21.04)	9.19 (8.80)	6.78 (5.05)	25.05 (22.29)
E1934	0.971989	0.0327772	22.00 (16.87)	10.57 (7.99)	2.81 (2.21)	37.60 (31.72)	32.11 (27.70)	5.80 (5.19)	3.74 (3.96)	38.05 (33.66)
I1940	0.919803	0.189829	3.81 (4.28)	2.00 (2.17)	0.48 (0.33)	3.99 (2.21)	2.82 (1.47)	0.57 (0.20)	2.92 (2.83)	3.11 (2.15)
K1952	0.950311	0.0468714	7.18 (6.71)	3.06 (3.86)	0.03 (0.04)	20.55 (18.42)	17.45 (16.59)	4.81 (4.81)	6.85 (6.69)	23.16 (22.86)
B1968	0.874511	0.01	1.11 (1.63)	0.97 (1.10)	-0.20 (0.11)	19.74 (10.74)	14.99 (8.39)	7.24 (4.41)	1.58 (2.01)	16.37 (11.08)
S1971	0.777448	0.01	0.44 (0.58)	0.17 (0.58)	-1.06 (-0.57)	8.67 (-6.19)	4.86 (-2.91)	0.53 (-0.27)	0.29 (0.40)	1.87 (0.63)
I1979	1.03415	0.0311838	8.54 (8.07)	0.61 (0.57)	1.43 (1.20)	26.73 (20.76)	18.93 (14.91)	4.11 (3.34)	1.09 (1.01)	12.59 (10.60)
C1985	1.03084	0.0670524	12.13 (13.37)	6.06 (7.30)	-0.41 (-0.40)	22.01 (21.01)	16.20 (14.70)	0.59 (0.51)	10.21 (11.89)	16.12 (15.90)
L1989	0.936382	0.0585033	10.31 (9.46)	6.31 (6.52)	-0.31 (-0.29)	20.32 (18.54)	17.02 (16.45)	2.06 (2.14)	2.23 (2.63)	17.16 (17.16)
N1994	0.921999	0.0639964	4.38 (4.16)	0.23 (0.04)	0.09 (0.11)	16.02 (14.38)	9.62 (9.39)	0.42 (0.45)	0.46 (0.16)	5.15 (4.98)
K1995TZ	0.995601	0.01	3.33 (3.37)	-0.04 (0.05)	-0.14 (-0.08)	14.89 (12.06)	7.49 (6.38)	0.94 (0.91)	-0.27 (-0.24)	1.92 (1.90)
K1995TK	0.998614	0.0768521	4.92 (4.78)	0.53 (0.56)	-0.34 (-0.30)	12.24 (12.10)	6.94 (6.82)	1.76 (1.74)	4.16 (4.03)	6.88 (6.58)
A2009	0.967124	0.0205529	3.13 (3.07)	0.20 (0.22)	0.12 (0.15)	18.70 (16.45)	10.30 (9.51)	0.41 (0.42)	0.19 (0.22)	2.91 (2.85)
C2010A	0.855924	0.0227172	22.69 (-0.63)	6.25 (0.27)	-0.11 (-0.43)	21.72 (0.38)	10.79 (1.21)	0.13 (0.03)	-7.20 (-12.29)	13.67 (-0.64)
C2010C	0.999338	0.0392166	12.71 (10.80)	6.20 (3.26)	0.07 (0.07)	28.41 (26.37)	21.92 (20.04)	1.34 (1.23)	2.48 (5.64)	21.95 (20.19)
N2010	0.829933	0.0866787	10.36 (1.83)	1.44 (2.03)	0.48 (0.97)	7.76 (5.00)	5.88 (5.35)	2.14 (2.46)	2.13 (3.19)	9.05 (5.50)
T2011T	1.0801	0.0750689	18.57 (13.87)	3.08 (1.25)	-0.12 (-0.20)	16.32 (14.21)	13.87 (10.88)	0.47 (0.34)	4.39 (1.60)	23.40 (16.39)
T2011S	1.01173	0.0362881	1.35 (0.93)	-0.04 (0.21)	-0.02 (0.05)	28.78 (25.00)	15.51 (13.62)	1.67 (1.50)	-0.37 (-0.31)	5.89 (4.96)
Average [%] reduction			8.41 (5.90)	2.87 (2.29)	0.14 (0.16)	19.31 (14.66)	13.86 (11.20)	2.45 (2.12)	2.31 (2.14)	13.57 (11.06)

5.7 Analysis of the numerical results: TMD performance and structural characteristics

5.7.1 TMD performance at assumed earthquake

The bar charts in Figs. 5.41–5.58 represent the percentage reduction of the response index assumed as objective function, i.e. the RMS displacement of the top storey of the primary structure, which values have been taken from Tables 5.34–5.49 and gathered here for each earthquake. Such a further and selected representation of the obtained results in terms of TMD performance has been provided here with the aim of achieving a better comprehension of the whole scenario proposed in this study.

The average values of these performance indexes have been collected for each floor mass value and for the complete set of primary structures, as overall mean performance. At first sight, it appears that the global mean value of the average response reduction is included within a range from 15% to 20%, with the extremes cases represented by the lowest value of 13.8% for the K1995TK earthquake and the highest value of 22.8% in the case of the E1934 earthquake. This provides a first information on the attended performance of the TMD device.

Within the wide range of cases considered for this study, various different situations are recovered, and related features are discussed in the following. First, a qualitative survey on the average percentage reductions related to the two floor masses point out that the related values are slightly higher for $m_{S,i} = 150$ t, even if several cases exhibit instead larger reduction for $m_{S,i} = 100$ t. Also, these average values display a small dispersion from the global mean value. Hence, it appears that the performance of the TMD is independent of the floor mass. This fact is somewhat expected, since modal parameters could instead condition significantly the effectiveness of the control device.

Actually, more detailed considerations can be extracted from these outcomes. The main impression is that of quite varied values, which could not be stated by general trends, from both points of view of number of floors and floor masses. However, it appears that the 1-storey primary structure is characterised by the lowest level of TMD effectiveness, especially for $m_{S,i} = 100$ t, which denotes the building with the smallest modal period. In this sense, a noticeable case is instead represented by that of the 1-storey frame building, when subjected to the A2009 earthquake, which response is reduced of about 24% for $m_{S,i} = 100$ t and about 29% for $m_{S,i} = 150$ t. It is remarkable the fact that, for the same earthquake event, the following primary structure, i.e. the 2-storey building, displays instead quite low values of response reduction.

The cases of reduction larger than 20% are recovered for the structures with n_s>2, and in general for the primary structure with large modal periods; also these cases are characterised by an apparently casual distribution among the different earthquakes. Same considerations could be extended to the cases of response abatement larger than 30%, even if this latter range of cases is much restricted.

The best performance is detected for some outstanding cases, where the decrease of the considered response index reaches values higher than 35%, which fact denotes a very positive combination of structural context, seismic signal characteristics and TMD setting. Finally, after a comprehensive study on the outcomes in Figs. 5.41–5.58, it appears that the basis for a wider analysis has been provided. Nevertheless, further contributions are required in order to outline a global interpretation of the results of this research work. In this sense, a significative effort is represented in the following section, where complementary outcomes will be displayed and discussed in detail.

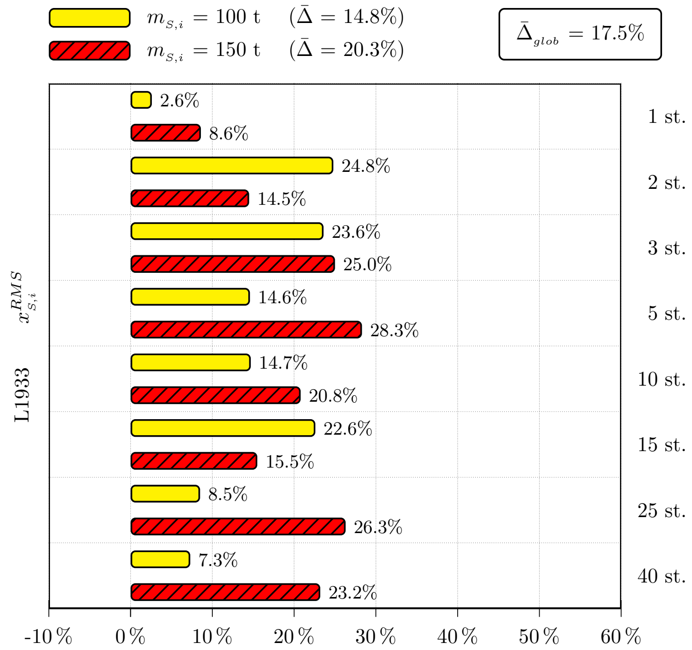


Figure 5.41: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures, subjected to the Long Beach 1933 earthquake.*

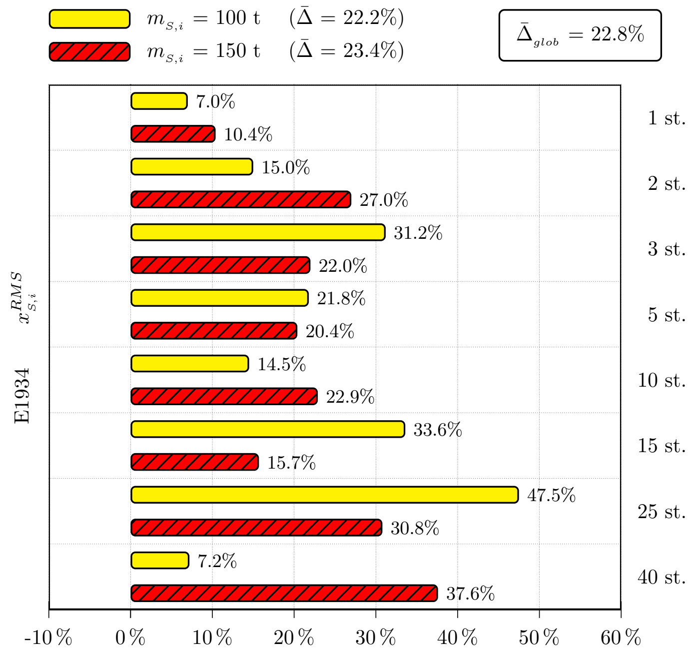


Figure 5.42: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the El Centro 1934 earthquake.*

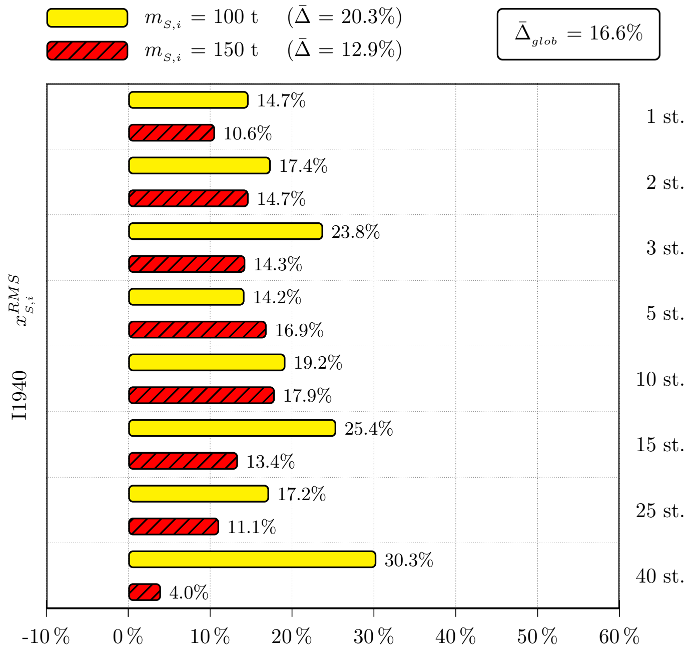


Figure 5.43: *Percentage reduction [%] of* $x^{RMS}_{S,n}$ *for the considered primary structures subjected to the Imperial Valley 1940 earthquake.*

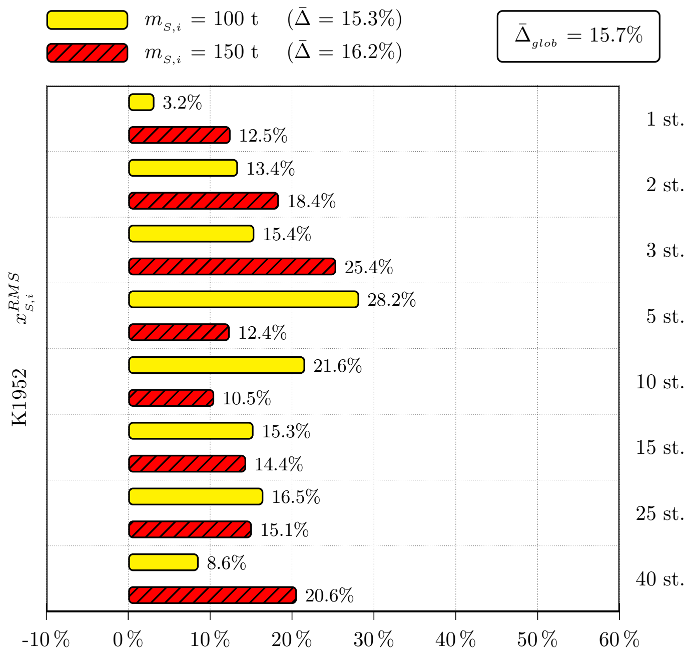


Figure 5.44: *Percentage reduction [%] of* $x^{RMS}_{S,n}$ *for the considered primary structures subjected to the Kern County 1952 earthquake.*

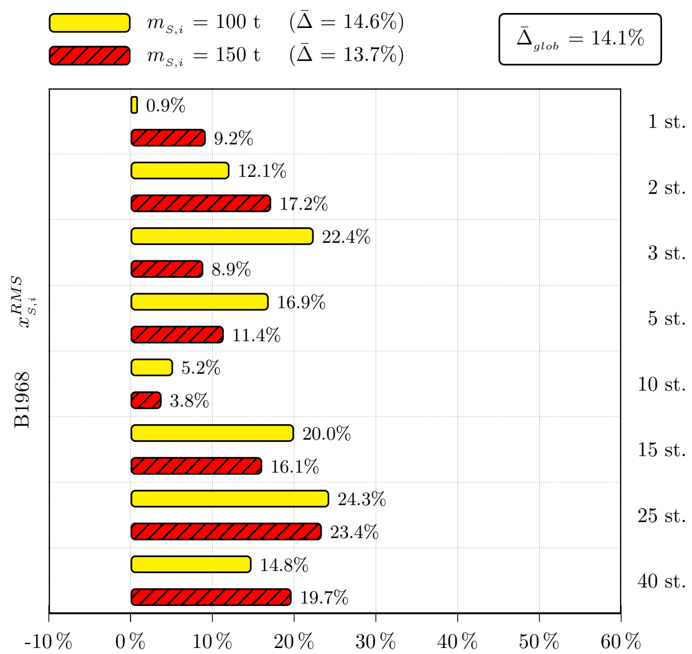


Figure 5.45: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the Borrego Mountain 1968 earthquake.*

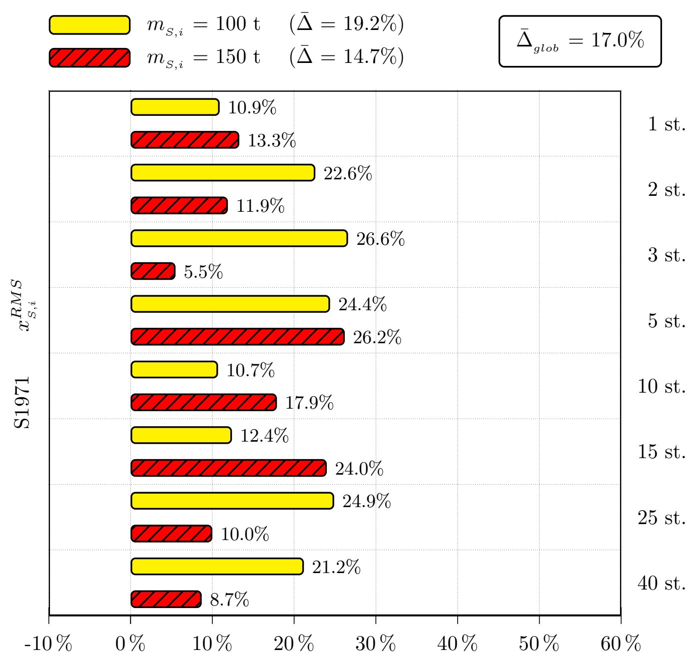


Figure 5.46: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the San Fernando 1971 earthquake.*

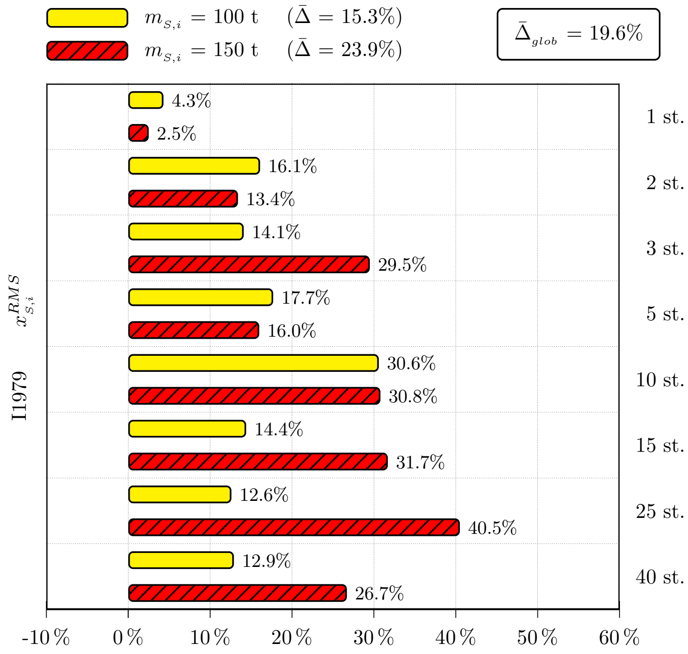


Figure 5.47: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the Imperial Valley 1979 earthquake.*

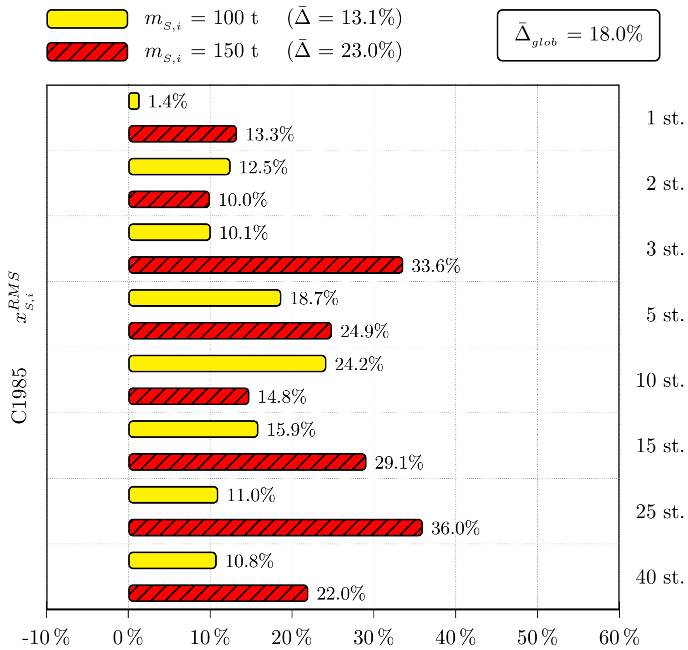


Figure 5.48: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the Chile 1985 earthquake.*

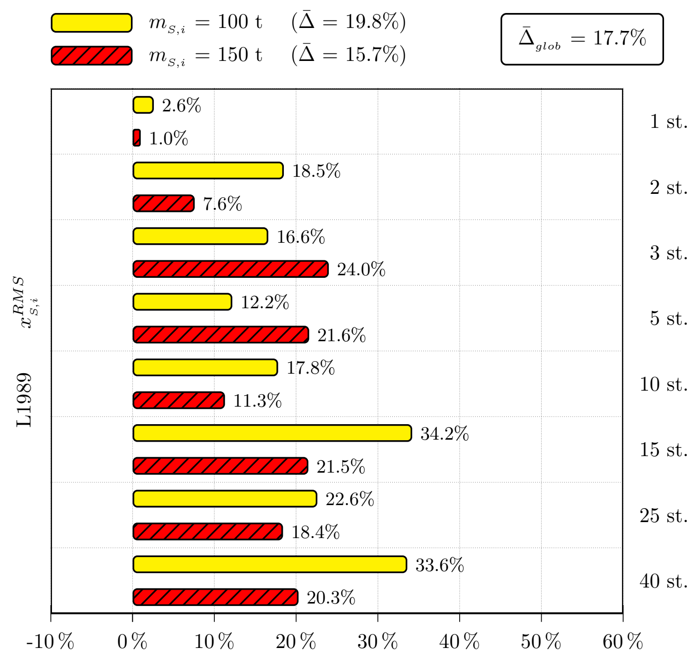


Figure 5.49: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the Loma Prieta 1989 earthquake.*

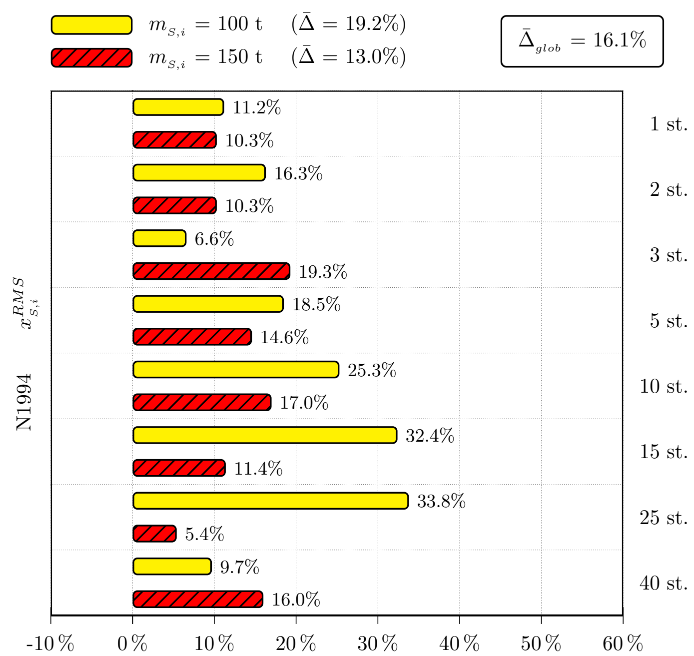


Figure 5.50: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the Northridge 1994 earthquake.*

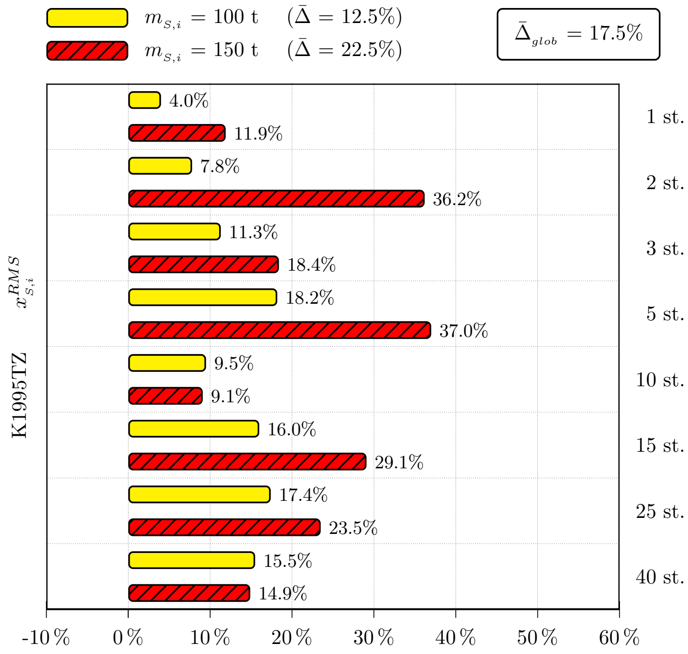


Figure 5.51: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the Kobe 1995 earthquake (Takarazuka station).*

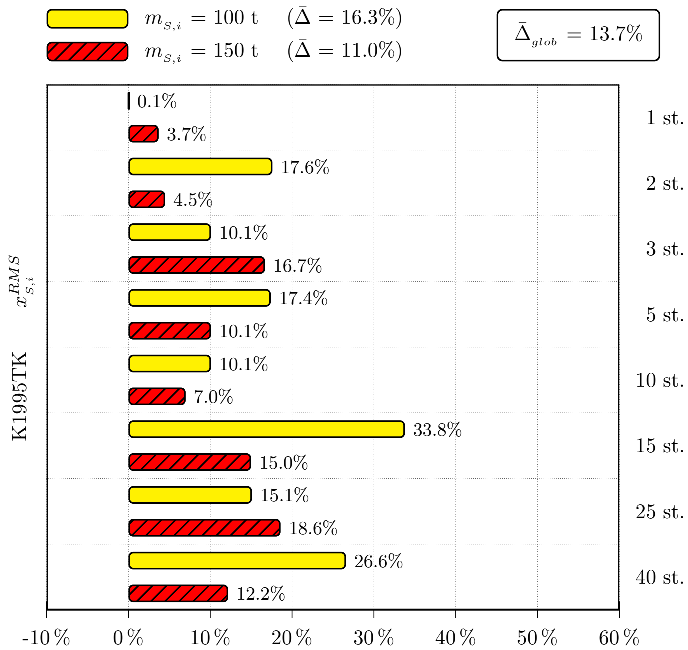


Figure 5.52: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the Kobe 1995 earthquake (Takatori station).*

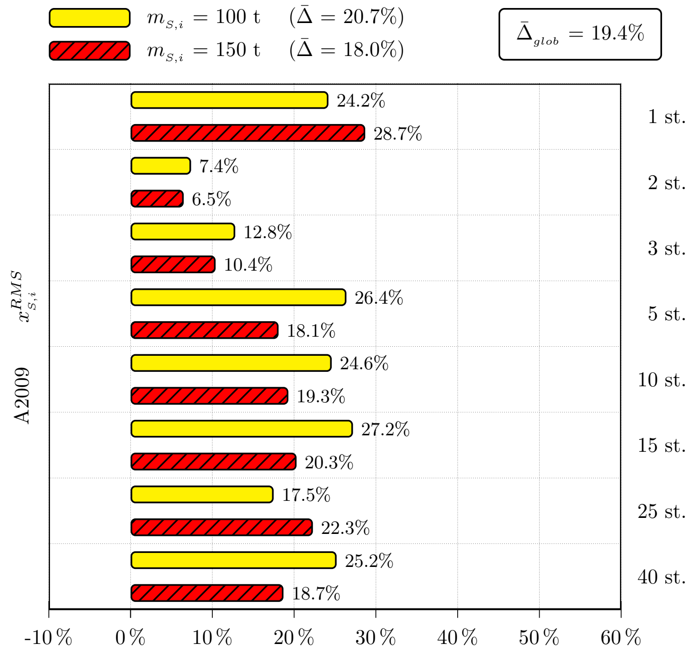


Figure 5.53: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the L'Aquila 2009 earthquake.*

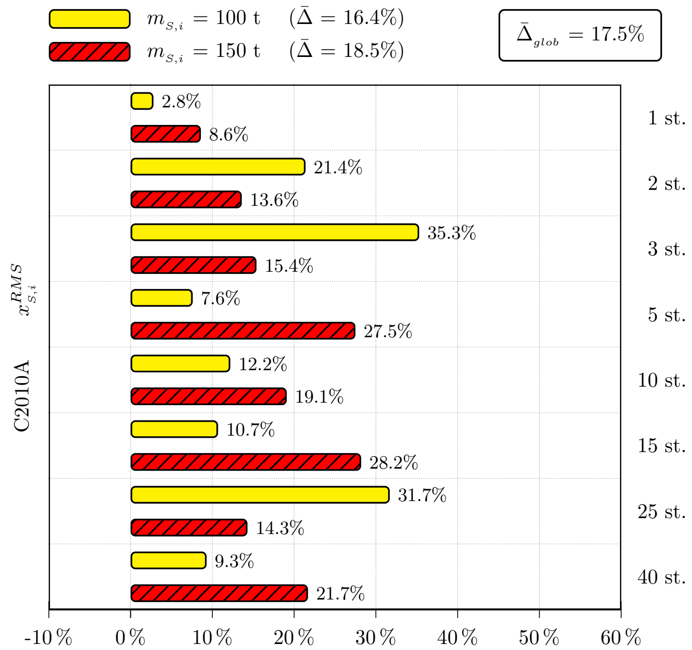


Figure 5.54: *Percentage reduction [%] of $x_{S,n}^{RMS}$ for the considered primary structures subjected to the Chile 2010 earthquake (Angol station).*

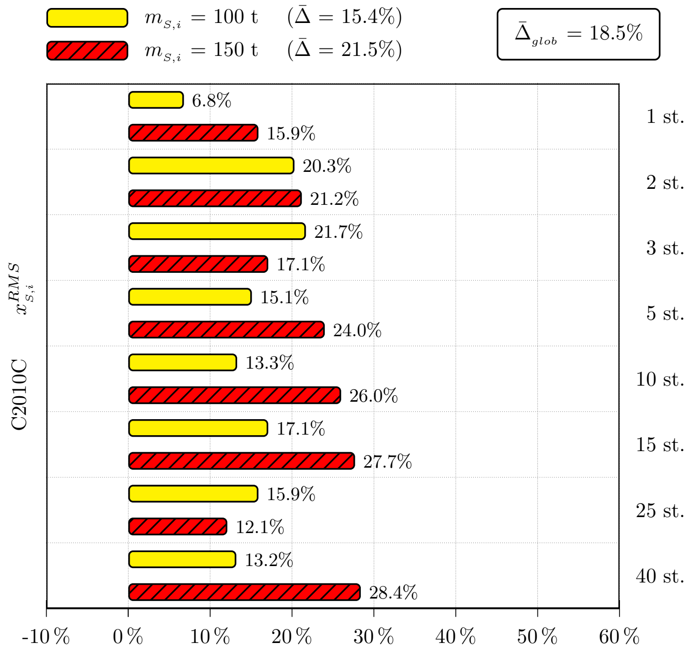


Figure 5.55: *Percentage reduction [%] of $x^{RMS}_{S,n}$ for the considered primary structures subjected to the Chile 2010 earthquake (Concepcion San Pedro station).*

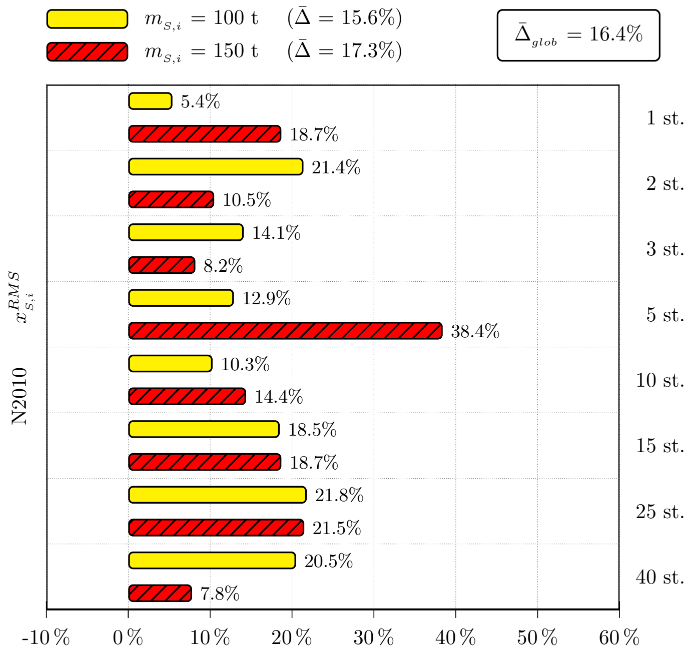


Figure 5.56: *Percentage reduction [%] of $x^{RMS}_{S,n}$ for the considered primary structures subjected to the New Zealand 2010 earthquake.*

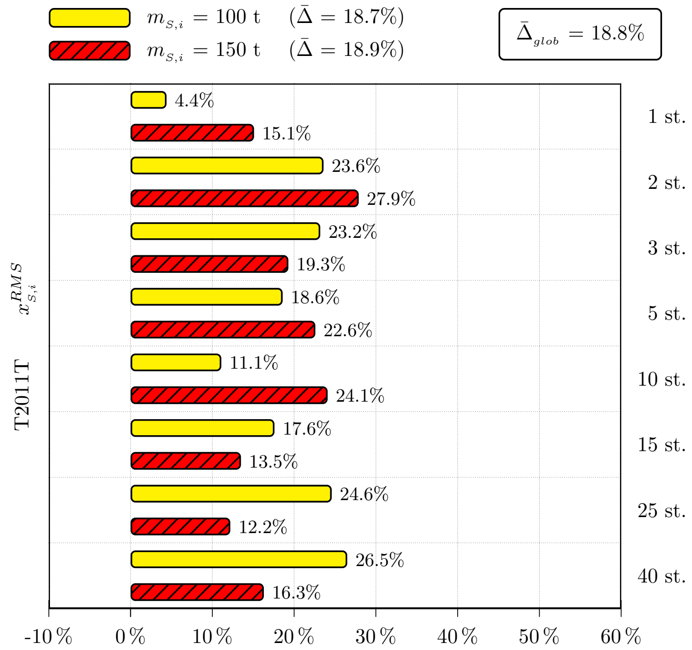


Figure 5.57: *Percentage reduction [%] of $x^{RMS}_{S,n}$ for the considered primary structures subjected to the Tohoku 2011 earthquake (Tsukidate station).*

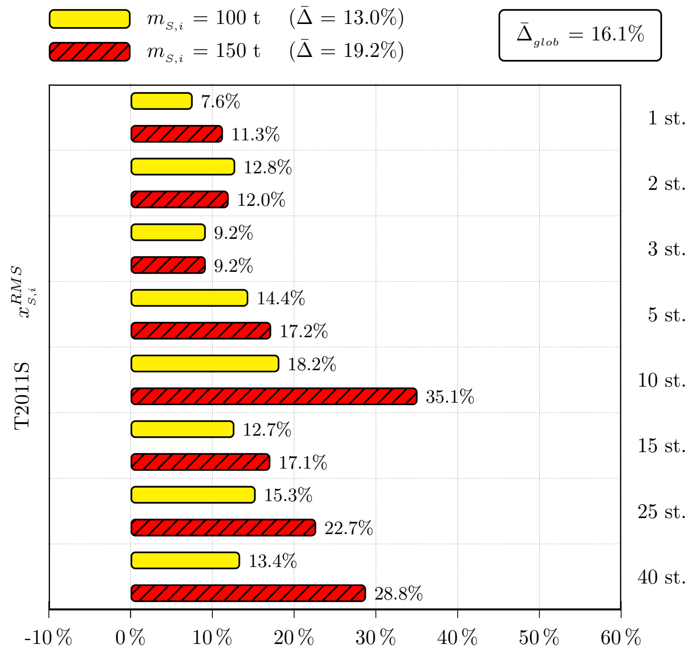


Figure 5.58: *Percentage reduction [%] of $x^{RMS}_{S,n}$ for the considered primary structures subjected to the Tohoku 2011 earthquake (Sendai station).*

5.7.2 Comparison based on modal analysis

The present study on the effectiveness of TMDs for seismic applications will be completed in this section, whose reported outcomes are in a sense complementary to those displayed and commented previously. In Tables 5.50–5.67 several indexes have been reported, which represent modal quantities and response spectra values computed at a pre- and a post-tuning stage.

The modes of vibration considered for this analysis are those characterised by effective modal mass $M_{me,S,i}$>5 %, which has been fixed *a priori* as threshold for the modes considered to be significant within the purposes of the present analysis.

As a direct consequence of this assumption, the contextualisation into the present primary structures leads to consider only the first 2 modes for the pre-tuning analysis and three modes for the post-tuning analysis, as could be immediately read out of Tables 5.50–5.67. Moreover, since the TMD has been tuned with reference to the first mode of vibration, the proposed selection allows for an immediate inspection of the effect of the TMD addition on the modal properties.

In particular, such quantities are reported in terms of:

- Effective modal mass $M_{me,S,i}$;
- Modal periods $\mathrm{T}_{S,i}$;
- Frequency amplitude spectrum FAS($\mathrm{T}_{S,i}$);
- Displacement response spectrum $\mathrm{S}_d(\mathrm{T}_{S,i})$;
- Pseudo-velocity response spectrum $\mathrm{S}_{pv}(\mathrm{T}_{S,i})$;
- Pseudo-acceleration response spectrum $\mathrm{S}_{pa}(\mathrm{T}_{S,i})$.

The effective modal mass represents a fundamental index for the definition of the hierarchy of the vibration modes. The modal periods turn out fundamental to locate the dynamic behaviour with respect to the seismic frequency content and the response spectra. Indeed, all remaining quantities have been evaluated in correspondence of the modal periods, through a linear interpolation method with neighbours values. In this sense, the goodness of the seismic signal record and of the response spectra ensures high level of accuracy in the evaluation of these indexes, which is fundamental for the reliability of the whole investigation, since many quantities exhibit extremely variable shapes, e.g. the pseudo-acceleration response spectrum at short periods.

The FAS index, which express the signal amplitude within the frequency content of the seismic ground acceleration, is an input quantity and its analysis aims at checking the existence of possible connections between seismic signal characteristics and TMD effectiveness. Finally, the three response spectra represent the most usual and complete representation of the earthquake response, and therefore significant indications from their values are somewhat expected.

All these indexes have been gathered, for each earthquake, for all the primary structures, following the scheme adopted for the bar charts in the previous section. Hence, it is possible to develop a crossed comparison between these two sets of results, which would assist in finding possible connections between the structural and dynamic quantities and the TMD performance.

The modification in the modal properties after the insertion of the TMD will be commented first. In this sense, the main direct consequence is a split of the first mode of vibration (in general, that on which the TMD is tuned), which generates two modes, characterised by periods one slightly lower and one slightly higher than that of the reference mode. The further modes are just negligibly affected by the insertion of the TMD.

The hierarchy of the new two modes obtained from the split of the original first mode exhibits different cases. The situation that often occurs concerns two modes with effective modal mass that are quite different from each other (usually the ratio of effective modal mass between the two modes is about 2, but in some cases is even higher), and therefore the presence of a dominant mode with a subordinate mode that could however have a noticeable role within the structural behaviour, since its effective modal mass is usually larger than 25%. In general, it appears that it could not be established if the new dominant mode is that with shorter or larger period, even if some buildings exhibit main trends in this sense, e.g. the 1-storey building mainly (but not only) displays, after a global survey, a dominant second mode, which is that with the shorter period.

In other cases, which however represent a smaller group with respect to the previous case, the new modes show approximately the equal amount of effective modal mass, and thus it may be supposed that their role within the structural dynamic behaviour is equally relevant.

As pointed out before with the bar chart, the insertion of the TMD generally brings benefit since it allows a reduction of the seismic response of about 15%–20%. In Tables 5.50–5.67 it has been noted that a shared feature among the most favourable cases is the modal property discussed just above, i.e. the equal relevance of the two new modes of vibration generated by the addition of the TMD. Indeed, it appears that a remarkable correspondence holds between this property and a good performance of the TMD. In detail, such noticeable feature actually appears to represent a sort of good starting point, but then the TMD effectiveness should be further supported by the reduction of the values related to the response spectra. When this favorable situation occurs, the level of effectiveness of the TMD usually reaches its highest values.

In other cases, where the modes display equal relevance but the control index (FAS and response spectra) are less reduced, the TMD performance turn out generally significant, even if not outstanding. This is the case when the effectiveness takes values around the average performance.

As a sort of confirmation of this statement, it appears from the available data that the less positive cases are detected in correspondence of a dominant new mode. As explained before, the 1-storey buildings are those much characterised by this feature, and also the primary structures that exhibit the lower benefit from the TMD addition. Also different primary structures, with higher number of floors, do not display remarkable response abatements when the two split modes are unbalanced from the point of view of the modal masses, i.e. when one of the modes exhibit an effective modal mass much higher than that of the other.

Table 5.50: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Long Beach 1933 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	33.31	0.1171	0.1322	0.1885	0.1001	0.0009872	0.001324	0.005386	0.006401	0.2886	0.3037
			66.69		0.1127		0.07944		0.0008801		0.004985		0.2774
	150	100.00	36.92	0.1434	0.1604	0.04795	0.2557	0.001871	0.003295	0.008317	0.01315	0.3629	0.515
			63.08		0.1376		0.09579		0.001518		0.007053		0.3217
2	100	94.72	30.86	0.1894	0.2131	0.3558	0.5315	0.005225	0.005786	0.01768	0.01743	0.5865	0.515
		5.28	63.96	0.07236	0.1822	0.08882	0.276	0.0004058	0.005316	0.003599	0.01871	0.3139	0.6461
			5.18		0.07232		0.0887		0.0004057		0.0036		0.3142
	150	94.72	22.70	0.232	0.2686	0.2034	0.5084	0.005873	0.00972	0.0162	0.02319	0.4381	0.5425
		5.28	72.12	0.08862	0.2251	0.1356	0.1418	0.000503	0.005415	0.003633	0.01541	0.2578	0.4303
			5.18		0.08858		0.1355		0.0005026		0.003632		0.2579
3	100	91.41	62.15	0.2631	0.2787	0.4696	0.1151	0.009546	0.009829	0.02325	0.0226	0.5555	0.5096
		7.49	29.41	0.09389	0.2406	0.08891	0.2414	0.0005711	0.007182	0.003883	0.01912	0.2596	0.4991
			7.36		0.0938		0.0922		0.0005697		0.003878		0.2595
	150	91.41	29.17	0.3222	0.3626	0.07613	0.3643	0.01101	0.01278	0.02189	0.02258	0.427	0.3914
		7.49	62.37	0.115	0.3099	0.06487	0.1268	0.0009356	0.01116	0.005193	0.02308	0.2832	0.468
			7.37		0.1149		0.05991		0.0009334		0.005185		0.2829
5	100	87.95	21.64	0.4113	0.4736	0.3955	0.8589	0.01865	0.02714	0.02905	0.03672	0.4436	0.4871
		8.72	66.48	0.1409	0.3985	0.07897	0.6984	0.001678	0.01509	0.007612	0.02425	0.3388	0.3823
			8.59		0.1408		0.07067		0.001668		0.007576		0.3376
	150	87.95	61.35	0.5038	0.5324	0.7242	0.3538	0.02594	0.02727	0.033	0.03282	0.4116	0.3873
		8.72	26.79	0.1726	0.4587	0.1488	0.4571	0.004619	0.02429	0.01712	0.03393	0.6225	0.4649
			8.58		0.1724		0.1592		0.004598		0.01706		0.6213

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$\mathrm{T}_{S,i}$ [s]		FAS($\mathrm{T}_{S,i}$) [m/s]		$\mathrm{S}_d(\mathrm{T}_{S,i})$ [m]		$\mathrm{S}_{pv}(\mathrm{T}_{S,i})$ [m/s]		$\mathrm{S}_{pa}(\mathrm{T}_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	20.28	0.7834	0.9039	0.4196	0.711	0.04601	0.07451	0.03763	0.05281	0.3018	0.3671
	100	9.14	64.70	0.2631	0.7594	0.4696	0.5218	0.009546	0.03944	0.02325	0.03327	0.5555	0.2753
			9.02		0.2628		0.4695		0.009538		0.02325		0.5561
		84.79	23.20	0.9594	1.093	0.6503	0.8119	0.08914	0.1028	0.05953	0.06025	0.3898	0.3463
	150	9.14	61.78	0.3222	0.9267	0.07613	0.721	0.01101	0.08037	0.02189	0.05557	0.427	0.3767
			9.02		0.3219		0.05		0.01101		0.02191		0.4279
15		83.62	49.55	1.156	1.237	0.7078	0.5869	0.09367	0.07042	0.05193	0.03648	0.2823	0.1853
	100	9.16	34.28	0.3866	1.073	0.1674	0.9217	0.0133	0.1039	0.02204	0.06201	0.3582	0.363
			9.03		0.3861		0.1664		0.01328		0.02204		0.3588
		83.62	16.23	1.416	1.668	0.1248	0.3627	0.05711	0.07912	0.02585	0.03038	0.1147	0.1144
	150	9.16	67.59	0.4735	1.379	0.8602	0.06381	0.02713	0.05632	0.03671	0.02617	0.4871	0.1193
			9.04		0.473		0.8644		0.02707		0.03667		0.487
25		82.63	10.83	1.901	2.348	0.2029	0.5657	0.09406	0.1405	0.0317	0.03834	0.1048	0.1026
	100	9.13	72.00	0.6345	1.864	0.08917	0.203	0.02444	0.09712	0.02469	0.03338	0.2446	0.1125
			9.02		0.6339		0.08115		0.02451		0.02478		0.2457
		82.63	38.27	2.328	2.539	0.5782	0.4531	0.1384	0.1428	0.0381	0.03604	0.1028	0.08919
	150	9.13	44.58	0.777	2.202	0.5134	0.4411	0.04428	0.1169	0.03651	0.034	0.2952	0.09703
			9.01		0.7761		0.5269		0.04404		0.03635		0.2942
40		82.05	16.60	3.019	3.542	0.2406	0.3938	0.1157	0.1758	0.02455	0.03179	0.0511	0.05639
	100	9.10	65.66	1.007	2.938	0.9193	0.281	0.09963	0.1148	0.0634	0.02504	0.3957	0.05355
			8.98		1.006		0.9109		0.09948		0.06336		0.3958
		82.05	33.46	3.697	4.073	0.426	0.3898	0.1959	0.2342	0.03394	0.03685	0.05768	0.05685
	150	9.10	48.81	1.233	3.52	0.5943	0.3891	0.07156	0.1727	0.03718	0.03144	0.1895	0.05612
			8.97		1.232		0.5969		0.07196		0.03743		0.191

Table 5.51: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the El Centro 1934 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	33.83	0.1171	0.132	0.06055	0.1186	0.00101	0.001394	0.005523	0.006752	0.2968	0.321
			66.17		0.1127		0.1607		0.0009651		0.005486		0.3063
	150	100.00	36.60	0.1434	0.1605	0.351	0.2291	0.001721	0.002086	0.007683	0.008319	0.3368	0.3254
			63.40		0.1376		0.13		0.001574		0.007313		0.3336
2	100	94.72	42.06	0.1894	0.2077	0.1654	0.2107	0.004001	0.005242	0.01352	0.01616	0.4483	0.4886
		5.28	52.76	0.07236	0.1798	0.05483	0.2462	0.0003447	0.003051	0.003037	0.01087	0.2634	0.38
			5.18		0.07232		0.05091		0.0003442		0.003035		0.2633
	150	94.72	30.86	0.232	0.261	0.8682	0.9559	0.008095	0.009738	0.02235	0.02393	0.6053	0.5766
		5.28	63.96	0.08862	0.2231	0.113	0.1964	0.0006204	0.007306	0.00447	0.02096	0.3162	0.5902
			5.18		0.08857		0.126		0.0006193		0.004464		0.316
3	100	91.41	57.05	0.2631	0.2806	1.161	0.4033	0.009315	0.007372	0.02273	0.01683	0.5442	0.377
		7.49	34.50	0.09389	0.2432	0.1247	0.4418	0.0007111	0.009007	0.004839	0.02371	0.3238	0.612
			7.36		0.0938		0.09354		0.0007097		0.004834		0.3238
	150	91.41	42.36	0.3222	0.3517	0.6398	0.3401	0.01015	0.01214	0.02016	0.02212	0.3928	0.3952
		7.49	49.19	0.115	0.3049	0.1287	0.2335	0.0009885	0.008314	0.005506	0.01747	0.3013	0.36
			7.36		0.1149		0.1089		0.0009874		0.005505		0.3015
5	100	87.95	50.34	0.4113	0.4415	0.5602	0.5253	0.01812	0.0188	0.02822	0.02727	0.431	0.3882
		8.72	37.79	0.1409	0.3833	0.05085	0.7905	0.001669	0.01588	0.007588	0.02653	0.3383	0.4347
			8.58		0.1408		0.07192		0.001666		0.007581		0.3384
	150	87.95	25.63	0.5038	0.571	0.2712	0.6381	0.02355	0.01995	0.02995	0.02239	0.3736	0.2464
		8.72	62.49	0.1726	0.4858	0.2625	0.8186	0.002909	0.02286	0.0108	0.03015	0.3933	0.39
			8.59		0.1724		0.2572		0.002906		0.0108		0.3936

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	70.49	0.7834	0.8152	0.3982	0.2662	0.02899	0.03155	0.02371	0.0248	0.1902	0.1911
	100	9.14	14.51	0.2631	0.6825	1.161	0.571	0.009315	0.03672	0.02273	0.03447	0.5442	0.3174
			9.00		0.2627		1.136		0.009396		0.02296		0.5504
		84.79	28.28	0.9594	1.074	0.3254	0.575	0.03829	0.04087	0.02557	0.02438	0.1675	0.1426
	150	9.14	56.71	0.3222	0.9209	0.6398	0.2935	0.01015	0.03801	0.02016	0.02645	0.3928	0.1805
			9.01		0.3219		0.6292		0.01008		0.02004		0.391
15		83.62	56.95	1.156	1.224	0.4411	0.2625	0.03388	0.02877	0.01878	0.01506	0.1021	0.07736
	100	9.16	26.88	0.3866	1.056	0.6038	0.5422	0.01646	0.04148	0.02726	0.02517	0.4429	0.1498
			9.03		0.3861		0.6314		0.01637		0.02714		0.4416
		83.62	18.93	1.416	1.642	0.1492	0.138	0.0277	0.04059	0.01254	0.01584	0.05564	0.0606
	150	9.16	64.88	0.4735	1.374	0.4075	0.1953	0.02216	0.02576	0.02999	0.01201	0.398	0.05493
			9.04		0.473		0.4468		0.02213		0.02997		0.3981
25		82.63	40.51	1.901	2.064	0.6268	0.1575	0.06133	0.06039	0.02067	0.01875	0.06833	0.05706
	100	9.13	42.33	0.6345	1.792	0.8384	0.1475	0.03057	0.04385	0.03086	0.01568	0.3056	0.05498
			9.01		0.6337		0.8508		0.03039		0.03072		0.3046
		82.63	45.17	2.328	2.507	0.468	0.1277	0.05423	0.06001	0.01492	0.01534	0.04027	0.03844
	150	9.13	37.68	0.777	2.178	0.6835	0.1526	0.02895	0.05964	0.02388	0.01755	0.1931	0.05064
			9.01		0.7761		0.6699		0.02901		0.02395		0.1939
40		82.05	7.50	3.019	3.938	0.1001	0.2609	0.05213	0.09378	0.01106	0.01526	0.02303	0.02435
	100	9.10	74.75	1.007	2.976	0.6751	0.119	0.03829	0.05169	0.02437	0.01113	0.1521	0.0235
			8.99		1.006		0.6813		0.03823		0.02435		0.1521
		82.05	38.09	3.697	4.032	0.3611	0.1586	0.1015	0.08469	0.01759	0.01346	0.0299	0.02098
	150	9.10	44.18	1.233	3.496	0.2702	0.2397	0.02784	0.08846	0.01447	0.01621	0.07372	0.02913
			8.97		1.232		0.269		0.02798		0.01456		0.07426

Table 5.52: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Imperial Valley 1940 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		$FAS(T_{S,i})$ [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	39.90	0.1171	0.1301	0.08368	0.2355	0.002426	0.00291	0.01321	0.01434	0.7066	0.6926
			60.10		0.112		0.16		0.001996		0.01137		0.6362
	150	100.00	27.18	0.1434	0.1652	0.1602	0.4274	0.002758	0.004675	0.01233	0.01811	0.541	0.6884
			72.82		0.1389		0.4467		0.002739		0.01265		0.5728
2	100	94.72	40.34	0.1894	0.2083	0.2694	1.008	0.005639	0.00681	0.01907	0.02094	0.6325	0.6316
		5.28	54.48	0.07236	0.1802	0.1752	0.298	0.0006343	0.005208	0.005613	0.01852	0.4887	0.6458
			5.18		0.07232		0.1928		0.0006339		0.005613		0.4889
	150	94.72	25.39	0.232	0.2657	0.55	0.8687	0.01073	0.014	0.02961	0.03379	0.8015	0.7999
		5.28	69.43	0.08862	0.2244	0.0524	0.4374	0.0009666	0.009163	0.006969	0.0261	0.4934	0.7296
			5.18		0.08858		0.05339		0.0009653		0.006962		0.4932
3	100	91.41	51.91	0.2631	0.2827	1.448	0.7001	0.01413	0.01402	0.03444	0.03179	0.8233	0.7069
		7.49	39.64	0.09389	0.2455	0.1786	0.4081	0.001097	0.01316	0.007464	0.03432	0.4995	0.8782
			7.36		0.0938		0.1681		0.001095		0.007457		0.4996
	150	91.41	56.97	0.3222	0.3437	0.3622	0.3294	0.01723	0.01788	0.03427	0.03333	0.6684	0.6094
		7.49	34.58	0.115	0.2979	0.4296	0.7853	0.00225	0.01521	0.01246	0.03271	0.6778	0.6899
			7.36		0.1149		0.4531		0.002241		0.01242		0.6763
5	100	87.95	19.08	0.4113	0.4795	0.5889	0.8924	0.02436	0.04461	0.03795	0.05961	0.5796	0.781
		8.72	69.04	0.1409	0.3997	0.4844	0.9412	0.00273	0.02326	0.01241	0.03728	0.5537	0.5861
			8.59		0.1408		0.5308		0.002728		0.01242		0.5544
	150	87.95	24.20	0.5038	0.574	0.6615	1.74	0.0519	0.0715	0.06598	0.07981	0.8228	0.8737
		8.72	63.92	0.1726	0.4866	0.1617	1.079	0.005058	0.04673	0.01878	0.06151	0.6843	0.7941
			8.59		0.1724		0.1715		0.005055		0.01879		0.6851

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	19.63	0.7834	0.9068	1.127	1.163	0.08326	0.1085	0.0681	0.07665	0.5462	0.5311
	100	9.14	65.35	0.2631	0.76	1.448	0.7044	0.01413	0.08052	0.03444	0.06788	0.8233	0.5612
			9.02		0.2628		1.278		0.01415		0.0345		0.8256
		84.79	63.03	0.9594	1.009	1.348	0.5461	0.1187	0.1261	0.07925	0.0801	0.519	0.499
	150	9.14	21.97	0.3222	0.8633	0.3622	2.463	0.01723	0.105	0.03427	0.07794	0.6684	0.5673
			9.00		0.3218		0.2597		0.01722		0.03429		0.6697
15		83.62	34.38	1.156	1.272	1.047	0.7282	0.1119	0.1034	0.06201	0.05209	0.3371	0.2573
	100	9.16	49.44	0.3866	1.1	0.2811	0.853	0.02343	0.1122	0.03883	0.06535	0.6312	0.3733
			9.04		0.3862		0.4084		0.0234		0.03882		0.6317
		83.62	34.49	1.416	1.558	0.6263	0.5373	0.08765	0.1151	0.03967	0.04735	0.1761	0.191
	150	9.16	49.34	0.4735	1.347	1.598	0.3402	0.04457	0.08755	0.06032	0.04165	0.8006	0.1943
			9.04		0.4729		1.587		0.04457		0.06038		0.8023
25		82.63	36.48	1.901	2.081	0.6071	0.8464	0.1515	0.1923	0.05107	0.0592	0.1688	0.1788
	100	9.13	46.36	0.6345	1.803	1.12	1.3	0.07512	0.1401	0.07587	0.04981	0.7514	0.1736
			9.01		0.6337		1.04		0.07511		0.07594		0.753
		82.63	35.35	2.328	2.555	0.5024	0.6281	0.2534	0.2682	0.06973	0.06727	0.1882	0.1655
	150	9.13	47.49	0.777	2.211	1.006	0.7922	0.08258	0.2249	0.06809	0.06518	0.5506	0.1852
			9.01		0.7762		0.9934		0.08248		0.06809		0.5512
40		82.05	43.44	3.019	3.259	0.8389	0.6064	0.2472	0.243	0.05247	0.04777	0.1092	0.09211
	100	9.10	38.83	1.007	2.83	0.6176	1.173	0.1259	0.2689	0.08011	0.06086	0.4999	0.1351
			8.97		1.006		0.6685		0.1257		0.08011		0.5006
		82.05	24.44	3.697	4.181	0.2734	0.4281	0.197	0.1739	0.03414	0.02666	0.05802	0.04007
	150	9.10	57.83	1.233	3.562	1.16	0.2959	0.1116	0.2145	0.05801	0.03858	0.2956	0.06804
			8.98		1.232		1.147		0.1119		0.05819		0.2968

Table 5.53: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Kern County 1952 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$\mathrm{T}_{S,i}$ [s]		FAS($\mathrm{T}_{S,i}$) [m/s]		$\mathrm{S}_d(\mathrm{T}_{S,i})$ [m]		$\mathrm{S}_{pv}(\mathrm{T}_{S,i})$ [m/s]		$\mathrm{S}_{pa}(\mathrm{T}_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	27.66	0.1171	0.1346	0.07132	0.2388	0.0008232	0.00122	0.004485	0.00578	0.2399	0.2688
			72.34		0.1134		0.08891		0.0007225		0.00406		0.2241
	150	100.00	28.34	0.1434	0.1645	0.224	0.1369	0.001592	0.002492	0.00709	0.009704	0.31	0.3709
			71.66		0.1388		0.1844		0.001388		0.006395		0.2892
2	100	94.72	21.15	0.1894	0.2209	0.1615	0.6404	0.003568	0.004958	0.01206	0.01437	0.3999	0.4088
		5.28	73.67	0.07236	0.1841	0.01825	0.5031	0.0002472	0.003184	0.002183	0.01106	0.1897	0.3768
			5.18		0.07232		0.01704		0.000247		0.002182		0.1897
	150	94.72	56.93	0.232	0.2484	0.5607	0.2939	0.005453	0.005321	0.01507	0.01373	0.4085	0.3474
		5.28	37.90	0.08862	0.2155	0.03187	0.5913	0.00042	0.004819	0.003026	0.01433	0.2141	0.4179
			5.18		0.08856		0.03323		0.0004191		0.003022		0.2139
3	100	91.41	45.83	0.2631	0.2854	0.4859	0.2992	0.006255	0.007144	0.01523	0.01604	0.3639	0.3531
		7.49	45.72	0.09389	0.2477	0.07057	0.3397	0.0005051	0.005312	0.003427	0.01374	0.2287	0.3486
			7.36		0.09381		0.07774		0.0005037		0.003421		0.2285
	150	91.41	33.27	0.3222	0.3586	0.7515	0.9674	0.01261	0.01163	0.02506	0.02079	0.4887	0.3645
		7.49	58.28	0.115	0.3084	0.1401	0.2167	0.0007662	0.009841	0.004244	0.02044	0.231	0.4165
			7.37		0.1149		0.1224		0.0007636		0.004233		0.2306
5	100	87.95	28.91	0.4113	0.4614	0.2893	0.3915	0.01971	0.02495	0.03069	0.03465	0.4686	0.4721
		8.72	59.22	0.1409	0.3951	0.2652	0.8411	0.001479	0.01461	0.006716	0.02367	0.2992	0.3762
			8.59		0.1408		0.2756		0.001473		0.006695		0.2986
	150	87.95	32.42	0.5038	0.5597	0.6531	0.4205	0.02107	0.02206	0.02679	0.02525	0.3342	0.2835
		8.72	55.71	0.1726	0.4818	0.2098	0.8542	0.002683	0.02129	0.009956	0.02832	0.3625	0.3694
			8.59		0.1724		0.2048		0.002679		0.009949		0.3626

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$\mathrm{T}_{S,i}$ [s]		FAS($\mathrm{T}_{S,i}$) [m/s]		$\mathrm{S}_d(\mathrm{T}_{S,i})$ [m]		$\mathrm{S}_{pv}(\mathrm{T}_{S,i})$ [m/s]		$\mathrm{S}_{pa}(\mathrm{T}_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	22.08	0.7834	0.8967	0.6052	0.6765	0.03965	0.04806	0.03243	0.03434	0.2601	0.2407
	100	9.14	62.91	0.2631	0.7577	0.4859	0.5348	0.006255	0.03354	0.01523	0.02836	0.3639	0.2351
			9.02		0.2628		0.4597		0.006247		0.01523		0.3641
		84.79	49.24	0.9594	1.028	0.2305	0.2943	0.03805	0.0386	0.02541	0.02405	0.1664	0.1469
	150	9.14	35.76	0.3222	0.8928	0.7515	0.8431	0.01261	0.04906	0.02506	0.03521	0.4887	0.2479
			9.01		0.3218		0.8059		0.01256		0.025		0.488
15		83.62	54.35	1.156	1.228	0.4562	0.2107	0.04922	0.04981	0.02728	0.02598	0.1483	0.1329
	100	9.16	29.48	0.3866	1.063	0.3001	0.2914	0.01277	0.03883	0.02115	0.02341	0.3436	0.1384
			9.03		0.3861		0.2861		0.01269		0.02105		0.3424
		83.62	14.83	1.416	1.685	0.4183	0.8661	0.06035	0.1085	0.02732	0.04125	0.1212	0.1538
	150	9.16	68.99	0.4735	1.381	0.6165	0.5119	0.02224	0.05785	0.0301	0.02684	0.3996	0.1221
			9.04		0.4731		0.6292		0.02229		0.0302		0.4013
25		82.63	65.98	1.901	1.986	0.4175	0.2712	0.09023	0.08341	0.03041	0.02691	0.1005	0.08515
	100	9.13	16.87	0.6345	1.68	0.7111	0.8781	0.03004	0.1087	0.03033	0.04146	0.3004	0.1551
			9.00		0.6335		0.7042		0.02997		0.03031		0.3006
		82.63	60.52	2.328	2.45	0.264	0.2014	0.0826	0.08679	0.02273	0.02269	0.06135	0.05819
	150	9.13	22.33	0.777	2.101	0.2962	0.2675	0.03826	0.07675	0.03155	0.0234	0.2551	0.06999
			9.00		0.776		0.2929		0.03802		0.03139		0.2542
40		82.05	15.40	3.019	3.571	0.1909	0.2587	0.1048	0.1093	0.02225	0.01961	0.04631	0.03451
	100	9.10	66.86	1.007	2.942	0.6827	0.2085	0.03844	0.1026	0.02447	0.02235	0.1527	0.04772
			8.98		1.006		0.6721		0.03844		0.02449		0.153
		82.05	31.99	3.697	4.087	0.3407	0.2644	0.1111	0.1064	0.01926	0.01668	0.03273	0.02564
	150	9.10	50.28	1.233	3.527	0.2164	0.2228	0.04945	0.1102	0.0257	0.02002	0.131	0.03567
			8.97		1.232		0.2148		0.04957		0.02579		0.1315

***Table 5.54:** Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Borrego Mountain 1968 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		$FAS(T_{S,i})$ [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	51.18	0.1171	0.1274	0.01369	0.06874	0.0005044	0.0006535	0.002754	0.003279	0.1477	0.1615
			48.82		0.1105		0.07503		0.0004356		0.002524		0.1434
	150	100.00	33.64	0.1434	0.1618	0.03013	0.1252	0.001063	0.001453	0.004726	0.005755	0.2063	0.2236
			66.36		0.138		0.02262		0.0008814		0.004082		0.1855
2	100	94.72	50.15	0.1894	0.2048	0.1193	0.1863	0.001526	0.001994	0.00516	0.006236	0.1711	0.1914
		5.28	44.68	0.07236	0.1779	0.003871	0.06561	0.0001911	0.0014	0.001684	0.00505	0.1461	0.1788
			5.18		0.07231		0.003937		0.0001908		0.001683		0.1461
	150	94.72	30.63	0.232	0.2611	0.1504	0.0751	0.00281	0.004806	0.007749	0.01179	0.2097	0.2838
		5.28	64.19	0.08862	0.2232	0.01032	0.06823	0.0002866	0.002255	0.00207	0.00646	0.1468	0.1816
			5.18		0.08857		0.005948		0.0002864		0.002069		0.1468
3	100	91.41	45.16	0.2631	0.2857	0.275	0.137	0.004825	0.004585	0.01175	0.01028	0.2808	0.2262
		7.49	46.39	0.09389	0.248	0.01994	0.2131	0.000316	0.003917	0.002153	0.01011	0.1442	0.256
			7.36		0.09381		0.02265		0.0003156		0.002152		0.1442
	150	91.41	28.56	0.3222	0.3633	0.1819	0.09664	0.004505	0.007001	0.008954	0.01234	0.1745	0.2134
		7.49	62.98	0.115	0.3101	0.02894	0.1003	0.0004825	0.004385	0.002681	0.009059	0.1463	0.1836
			7.37		0.1149		0.02175		0.0004816		0.002678		0.1463
5	100	87.95	45.38	0.4113	0.445	0.4266	0.09089	0.008813	0.008977	0.01373	0.01293	0.2097	0.1825
		8.72	42.74	0.1409	0.3865	0.05719	0.1326	0.0009636	0.008335	0.004372	0.01382	0.1947	0.2246
			8.58		0.1408		0.04765		0.0009574		0.004351		0.194
	150	87.95	47.04	0.5038	0.5435	0.379	0.2764	0.009631	0.009552	0.01225	0.01126	0.1528	0.1301
		8.72	41.09	0.1726	0.4721	0.11	0.3651	0.001475	0.009483	0.005485	0.01287	0.2001	0.1713
			8.58		0.1724		0.08365		0.001478		0.005501		0.2009

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	68.92	0.7834	0.817	0.1564	0.1316	0.02369	0.02675	0.01937	0.02097	0.1554	0.1613
	100	9.14	16.08	0.2631	0.6883	0.275	0.5881	0.004825	0.02345	0.01175	0.02183	0.2808	0.1993
			9.00		0.2627		0.2726		0.004821		0.01176		0.2814
		84.79	24.52	0.9594	1.088	0.1763	0.2463	0.04039	0.06352	0.02697	0.03741	0.1767	0.2161
	150	9.14	60.47	0.3222	0.9252	0.1819	0.2844	0.004505	0.03694	0.008954	0.02558	0.1745	0.1737
			9.01		0.3219		0.1881		0.004484		0.00892		0.174
15		83.62	22.46	1.156	1.319	0.7468	0.4792	0.07705	0.1029	0.04271	0.04999	0.2322	0.2381
	100	9.16	61.36	0.3866	1.117	0.1337	0.2241	0.008339	0.06971	0.01382	0.03999	0.2246	0.2249
			9.04		0.3862		0.13		0.008325		0.01381		0.2247
		83.62	14.03	1.416	1.696	0.7577	0.8604	0.1163	0.145	0.05264	0.05477	0.2336	0.2029
	150	9.16	69.78	0.4735	1.383	0.3599	0.4708	0.009514	0.1123	0.01287	0.05202	0.1709	0.2364
			9.04		0.4731		0.3738		0.009505		0.01287		0.171
25		82.63	43.15	1.901	2.054	0.9462	0.9723	0.1634	0.1721	0.05509	0.05368	0.1821	0.1642
	100	9.13	39.69	0.6345	1.784	0.6797	0.6662	0.01893	0.1532	0.01911	0.05501	0.1892	0.1937
			9.01		0.6337		0.6863		0.01881		0.01902		0.1885
		82.63	36.29	2.328	2.55	0.7109	0.3132	0.1842	0.1852	0.05068	0.04655	0.1368	0.1147
	150	9.13	46.55	0.777	2.208	0.2213	0.5629	0.02342	0.1803	0.01931	0.0523	0.1562	0.1488
			9.01		0.7762		0.2401		0.02344		0.01935		0.1566
40		82.05	57.43	3.019	3.189	0.1549	0.3857	0.1739	0.1767	0.03691	0.03549	0.07682	0.06992
	100	9.10	24.85	1.007	2.746	0.2952	1.186	0.04514	0.1801	0.02873	0.04201	0.1793	0.09612
			8.97		1.005		0.2944		0.04494		0.02864		0.179
		82.05	16.18	3.697	4.35	0.525	0.2987	0.1705	0.1611	0.02954	0.02372	0.0502	0.03427
	150	9.10	66.08	1.233	3.6	0.2286	0.3655	0.08925	0.173	0.04637	0.03079	0.2363	0.05373
			8.98		1.232		0.2252		0.08907		0.04632		0.2363

Table 5.55: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the San Fernando 1971 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	63.21	0.1171	0.1252	0.3659	1.014	0.006265	0.007947	0.0341	0.04054	1.822	2.032
			36.79		0.1086		0.3612		0.004457		0.02627		1.52
	150	100.00	35.67	0.1434	0.1609	1.167	1.221	0.009615	0.01103	0.04293	0.04391	1.881	1.715
			64.33		0.1377		0.3801		0.009126		0.04246		1.94
2	100	94.72	25.87	0.1894	0.2165	0.5764	2.95	0.01778	0.02775	0.06009	0.08211	1.992	2.384
		5.28	68.94	0.07236	0.1832	0.2872	0.5426	0.002155	0.01533	0.01903	0.05349	1.654	1.832
			5.18		0.07232		0.3167		0.002153		0.01903		1.655
	150	94.72	80.54	0.232	0.2411	2.185	1.106	0.02445	0.02349	0.06756	0.0624	1.832	1.625
		5.28	14.30	0.08862	0.2009	0.4467	1.078	0.003233	0.02382	0.02333	0.07593	1.653	2.374
			5.17		0.08855		0.4236		0.003227		0.0233		1.653
3	100	91.41	52.27	0.2631	0.2825	1.334	0.4875	0.0354	0.03709	0.08615	0.08416	2.057	1.873
		7.49	39.28	0.09389	0.2453	0.655	0.7606	0.003507	0.02537	0.02392	0.06618	1.605	1.694
			7.36		0.0938		0.5887		0.003503		0.02392		1.606
	150	91.41	13.42	0.3222	0.3934	0.9147	3.476	0.04407	0.1102	0.08759	0.1795	1.708	2.869
		7.49	78.12	0.115	0.3156	0.8519	2.334	0.005765	0.04252	0.03189	0.08633	1.733	1.719
			7.37		0.1149		0.8338		0.005748		0.03182		1.73
5	100	87.95	41.15	0.4113	0.4484	2.309	2.386	0.1036	0.09654	0.1613	0.138	2.465	1.934
		8.72	46.97	0.1409	0.3889	1.494	3.37	0.009315	0.1106	0.04234	0.1822	1.888	2.944
			8.59		0.1408		1.381		0.009297		0.0423		1.888
	150	87.95	38.86	0.5038	0.5516	1.793	1.143	0.09835	0.08125	0.1251	0.09441	1.561	1.076
		8.72	49.26	0.1726	0.4778	0.868	2.217	0.01237	0.09775	0.04587	0.1311	1.669	1.724
			8.59		0.1724		0.9188		0.01233		0.04577		1.667

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10	100	84.79	14.28	0.7834	0.9382	0.8814	2.363	0.1392	0.25	0.1139	0.1708	0.9133	1.144
		9.14	70.70	0.2631	0.7651	1.334	1.294	0.0354	0.1257	0.08615	0.1053	2.057	0.8644
			9.02		0.2629		1.227		0.03529		0.08597		2.054
	150	84.79	23.87	0.9594	1.09	2.263	2.843	0.2644	0.3557	0.1766	0.209	1.156	1.204
		9.14	61.12	0.3222	0.9259	0.9147	2.498	0.04407	0.2427	0.08759	0.1679	1.708	1.139
			9.02		0.3219		0.9806		0.04393		0.08741		1.706
15	100	83.62	46.35	1.156	1.243	2.006	2.161	0.3896	0.4193	0.216	0.2161	1.174	1.092
		9.16	37.48	0.3866	1.08	3.317	3.038	0.1095	0.3491	0.1815	0.2071	2.951	1.205
			9.03		0.3861		3.292		0.1093		0.1814		2.952
	150	83.62	34.75	1.416	1.557	3.106	3.009	0.4411	0.4718	0.1997	0.1942	0.8862	0.7835
		9.16	49.07	0.4735	1.346	1.953	2.844	0.0966	0.4309	0.1307	0.205	1.735	0.9569
			9.04		0.4729		1.901		0.09646		0.1307		1.736
25	100	82.63	57.45	1.901	2.009	2.079	1.281	0.4733	0.4692	0.1595	0.1496	0.5272	0.4679
		9.13	25.40	0.6345	1.731	1.394	2.837	0.06611	0.4716	0.06676	0.1745	0.6611	0.6335
			9.00		0.6336		1.375		0.06594		0.06668		0.6612
	150	82.63	11.90	2.328	2.841	0.7386	1.238	0.4307	0.4529	0.1185	0.1021	0.3199	0.2259
		9.13	70.93	0.777	2.28	1.024	0.5733	0.1345	0.4388	0.1109	0.1233	0.8969	0.3398
			9.02		0.7764		1.039		0.1341		0.1106		0.8951
40	100	82.05	58.16	3.019	3.186	0.9968	0.7257	0.4636	0.47	0.0984	0.09453	0.2048	0.1864
		9.10	24.12	1.007	2.741	2.891	1.344	0.3011	0.4455	0.1916	0.1041	1.196	0.2388
			8.97		1.005		2.86		0.3001		0.1913		1.195
	150	82.05	7.42	3.697	4.831	0.6042	1.334	0.4749	0.7868	0.08229	0.1043	0.1398	0.1357
		9.10	74.83	1.233	3.645	2.147	0.5583	0.417	0.4752	0.2167	0.08354	1.104	0.144
			8.99		1.232		2.146		0.4169		0.2168		1.105

Table 5.56: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Imperial Valley 1979 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	27.75	0.1171	0.1346	0.1719	0.47	0.003762	0.0052	0.02051	0.0247	1.098	1.152
			72.25		0.1134		0.09489		0.003367		0.01894		1.047
	150	100.00	10.18	0.1434	0.1902	0.3244	0.4298	0.006082	0.01111	0.02713	0.03742	1.188	1.236
			89.82		0.1415		0.1261		0.005878		0.02659		1.181
2	100	94.72	54.68	0.1894	0.2034	0.4264	0.8914	0.01105	0.01386	0.03736	0.04361	1.239	1.346
		5.28	40.15	0.07236	0.1766	0.09517	0.1493	0.0008519	0.009955	0.007527	0.03605	0.6543	1.281
			5.18		0.07231		0.09123		0.000851		0.007524		0.6545
	150	94.72	52.29	0.232	0.25	0.9324	0.5243	0.02249	0.023	0.06213	0.05895	1.683	1.482
		5.28	42.53	0.08862	0.2171	0.08389	0.8227	0.001512	0.01906	0.01088	0.0561	0.7695	1.621
			5.18		0.08857		0.08692		0.001508		0.01087		0.7686
3	100	91.41	33.07	0.2631	0.2929	0.6264	1.121	0.0217	0.01979	0.05287	0.04329	1.263	0.9286
		7.49	58.47	0.09389	0.2519	0.1397	0.6415	0.001767	0.02276	0.01202	0.05792	0.8034	1.446
			7.37		0.09381		0.1205		0.001764		0.012		0.8031
	150	91.41	32.66	0.3222	0.3591	1.825	0.9319	0.03354	0.03857	0.06663	0.06883	1.298	1.205
		7.49	58.89	0.115	0.3086	0.08628	1.393	0.003539	0.02458	0.01963	0.051	1.069	1.038
			7.37		0.1149		0.08937		0.003529		0.01959		1.068
5	100	87.95	40.85	0.4113	0.4486	0.671	0.6495	0.03805	0.03957	0.05926	0.05652	0.9052	0.7917
		8.72	47.28	0.1409	0.389	0.2793	0.6455	0.005817	0.03142	0.02643	0.05174	1.178	0.8358
			8.59		0.1408		0.3202		0.005801		0.02639		1.178
	150	87.95	36.22	0.5038	0.5547	0.9754	0.5708	0.03782	0.047	0.04809	0.0543	0.5998	0.6152
		8.72	51.91	0.1726	0.4795	0.3271	0.4465	0.009185	0.03701	0.03404	0.04946	1.238	0.6481
			8.59		0.1724		0.3121		0.009151		0.03395		1.236

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		T$_{S,i}$ [s]		FAS(T$_{S,i}$) [m/s]		S$_d$(T$_{S,i}$) [m]		S$_{pv}$(T$_{S,i}$) [m/s]		S$_{pa}$(T$_{S,i}$) [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10	100	84.79	60.84	0.7834	0.8259	1.969	0.4984	0.1357	0.1066	0.111	0.08267	0.8904	0.6291
		9.14	24.16	0.2631	0.7097	0.6264	2.16	0.0217	0.1469	0.05287	0.1326	1.263	1.174
			9.00		0.2627		0.568		0.0217		0.05293		1.267
	150	84.79	48.40	0.9594	1.03	1.69	1.095	0.1623	0.163	0.1084	0.1014	0.7098	0.6188
		9.14	36.59	0.3222	0.8941	1.825	1.358	0.03354	0.1397	0.06663	0.1001	1.298	0.7032
			9.01		0.3218		1.728		0.03327		0.06618		1.291
15	100	83.62	11.92	1.156	1.412	1.45	2.461	0.184	0.2925	0.102	0.1327	0.5544	0.5903
		9.16	71.89	0.3866	1.132	0.4934	1.596	0.03148	0.1812	0.05218	0.1025	0.8483	0.5691
			9.05		0.3863		0.4735		0.03148		0.05223		0.85
	150	83.62	49.10	1.416	1.516	2.424	1.928	0.2911	0.2481	0.1317	0.1049	0.5848	0.4347
		9.16	34.73	0.4735	1.316	0.199	2.753	0.03685	0.2976	0.04987	0.1449	0.6619	0.6919
			9.03		0.4729		0.1627		0.03683		0.04991		0.6632
25	100	82.63	9.08	1.901	2.408	0.9058	1.408	0.1952	0.2423	0.06578	0.06446	0.2174	0.1682
		9.13	73.75	0.6345	1.869	1.635	0.6164	0.1071	0.1937	0.1082	0.0664	1.071	0.2232
			9.02		0.634		1.608		0.1067		0.1078		1.069
	150	82.63	44.82	2.328	2.509	1.618	0.887	0.2623	0.2224	0.07219	0.05681	0.1948	0.1423
		9.13	38.02	0.777	2.179	2.148	1.227	0.1403	0.2549	0.1157	0.07495	0.9356	0.2161
			9.01		0.7761		2.175		0.1409		0.1164		0.9423
40	100	82.05	10.31	3.019	3.751	0.3617	0.5994	0.2364	0.2475	0.05017	0.04228	0.1044	0.07083
		9.10	71.94	1.007	2.963	0.6643	0.312	0.1579	0.2358	0.1005	0.051	0.6272	0.1082
			8.98		1.006		0.6469		0.1577		0.1005		0.6275
	150	82.05	55.31	3.697	3.918	0.6774	0.3546	0.243	0.2475	0.04212	0.04048	0.07157	0.06493
		9.10	26.96	1.233	3.383	1.519	0.5718	0.2547	0.2331	0.1324	0.04415	0.6744	0.08201
			8.97		1.231		1.483		0.2534		0.1318		0.6726

Table 5.57: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Chile 1985 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	22.44	0.1171	0.1377	0.3077	1.555	0.004203	0.006556	0.02292	0.03044	1.227	1.387
			77.56		0.114		0.2611		0.003834		0.02145		1.179
	150	100.00	26.07	0.1434	0.1659	1.527	0.792	0.007505	0.01228	0.03345	0.04732	1.463	1.789
			73.93		0.1391		1.859		0.00676		0.03111		1.404
2	100	94.72	39.24	0.1894	0.2088	2.483	1.496	0.01871	0.01806	0.06325	0.05542	2.098	1.668
		5.28	55.58	0.07236	0.1804	0.198	2.198	0.001214	0.0167	0.01071	0.0593	0.9298	2.065
			5.18		0.07232		0.1769		0.001213		0.0107		0.9298
	150	94.72	43.06	0.232	0.2538	0.8221	2.915	0.02156	0.02541	0.05952	0.06408	1.612	1.585
		5.28	51.77	0.08862	0.2199	0.3667	2.568	0.001942	0.01984	0.01401	0.0578	0.9922	1.651
			5.18		0.08857		0.3892		0.001939		0.01399		0.9919
3	100	91.41	7.03	0.2631	0.3564	1.505	4.253	0.02845	0.1019	0.06927	0.1831	1.654	3.229
		7.49	84.50	0.09389	0.2599	0.1658	2.555	0.002265	0.02758	0.01539	0.068	1.028	1.644
			7.38		0.09384		0.2434		0.002262		0.01538		1.028
	150	91.41	32.79	0.3222	0.359	6.364	4.263	0.08523	0.1029	0.1695	0.1836	3.304	3.213
		7.49	58.76	0.115	0.3086	0.7396	6.004	0.003954	0.069	0.02192	0.1432	1.195	2.913
			7.37		0.1149		0.9483		0.003942		0.02188		1.193
5	100	87.95	50.80	0.4113	0.4412	3.99	2.642	0.1094	0.1045	0.1705	0.1517	2.604	2.16
		8.72	37.33	0.1409	0.383	1.209	2.714	0.007059	0.1056	0.03206	0.1766	1.429	2.897
			8.58		0.1408		1.044		0.007031		0.03197		1.427
	150	87.95	56.68	0.5038	0.5358	3.014	0.9	0.0921	0.08071	0.1172	0.09653	1.462	1.132
		8.72	31.45	0.1726	0.4638	1.088	2.047	0.01426	0.1038	0.05284	0.1434	1.922	1.944
			8.58		0.1724		1.815		0.0142		0.05267		1.918

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	32.20	0.7834	0.8679	1.687	0.789	0.07547	0.06876	0.06172	0.05076	0.4951	0.3676
	100	9.14	52.79	0.2631	0.7481	1.505	1.575	0.02845	0.07846	0.06927	0.0672	1.654	0.5644
			9.01		0.2628		2.193		0.02838		0.06916		1.653
		84.79	31.53	0.9594	1.065	0.8057	0.1099	0.08049	0.08868	0.05375	0.05336	0.352	0.3149
	150	9.14	53.46	0.3222	0.9171	6.364	1.253	0.08523	0.07143	0.1695	0.0499	3.304	0.3419
			9.01		0.3219		8.765		0.08502		0.1692		3.303
15		83.62	57.87	1.156	1.222	0.3881	0.3532	0.07663	0.07241	0.04248	0.03796	0.231	0.1952
	100	9.16	25.96	0.3866	1.053	4.683	0.6802	0.1064	0.08976	0.1764	0.0546	2.867	0.3257
			9.03		0.3861		4.636		0.1063		0.1764		2.872
		83.62	43.15	1.416	1.531	0.5663	0.4354	0.07821	0.0864	0.0354	0.03616	0.1571	0.1484
	150	9.16	40.67	0.4735	1.33	2.928	0.5636	0.1012	0.07016	0.1369	0.03381	1.818	0.1598
			9.03		0.4729		2.573		0.1013		0.1373		1.825
25		82.63	29.71	1.901	2.115	0.5369	0.721	0.1018	0.1112	0.0343	0.03367	0.1134	0.1
	100	9.13	53.13	0.6345	1.819	1.607	0.144	0.07641	0.09498	0.07715	0.03345	0.764	0.1155
			9.01		0.6338		1.655		0.07614		0.07696		0.7629
		82.63	34.01	2.328	2.563	0.9809	0.3582	0.1695	0.1579	0.04665	0.03948	0.1259	0.09679
	150	9.13	48.83	0.777	2.215	1.009	0.439	0.07562	0.1348	0.06236	0.03898	0.5043	0.1106
			9.01		0.7762		0.8877		0.07572		0.06251		0.5061
40		82.05	11.64	3.019	3.691	0.1891	0.4151	0.1253	0.217	0.02659	0.03767	0.05533	0.06413
	100	9.10	70.62	1.007	2.957	0.561	0.2267	0.08971	0.1227	0.05709	0.02658	0.3563	0.05648
			8.98		1.006		0.5255		0.08958		0.05706		0.3564
		82.05	54.51	3.697	3.922	0.3996	0.4374	0.2175	0.203	0.03768	0.03315	0.06404	0.05311
	150	9.10	27.77	1.233	3.39	0.279	0.6828	0.07119	0.1732	0.03699	0.03274	0.1885	0.06068
			8.97		1.231		0.2901		0.07134		0.03712		0.1894

Table 5.58: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Loma Prieta 1989 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	22.78	0.1171	0.1375	0.3685	0.2925	0.004386	0.005428	0.02395	0.02533	1.284	1.16
			77.22		0.114		0.08093		0.0041		0.02299		1.266
	150	100.00	3.46	0.1434	0.2877	0.6214	2.595	0.006227	0.04155	0.02768	0.09245	1.208	2.017
			96.54		0.1429		0.672		0.00611		0.02726		1.194
2	100	94.72	20.39	0.1894	0.2218	0.4894	2.404	0.01388	0.02002	0.04695	0.05783	1.557	1.639
		5.28	74.43	0.07236	0.1843	0.12	0.6181	0.001308	0.01333	0.01156	0.04635	1.006	1.581
			5.18		0.07232		0.1191		0.001307		0.01156		1.006
	150	94.72	43.67	0.232	0.2536	1.084	0.9997	0.02069	0.02467	0.05714	0.06228	1.548	1.542
		5.28	51.15	0.08862	0.2198	0.09513	1.54	0.002167	0.01994	0.01561	0.05814	1.104	1.663
			5.18		0.08857		0.09722		0.002163		0.01559		1.104
3	100	91.41	28.19	0.2631	0.2969	0.8164	0.7579	0.02857	0.05084	0.06949	0.1096	1.658	2.316
		7.49	63.35	0.09389	0.2533	0.0956	1.081	0.00264	0.02458	0.0179	0.06211	1.194	1.54
			7.37		0.09382		0.1062		0.002633		0.01787		1.193
	150	91.41	46.75	0.3222	0.349	3.039	2.358	0.06843	0.07314	0.1361	0.1343	2.653	2.419
		7.49	44.80	0.115	0.303	0.04814	2.389	0.004195	0.05665	0.02331	0.1197	1.272	2.48
			7.36		0.1149		0.04135		0.004186		0.02328		1.272
5	100	87.95	45.81	0.4113	0.4447	1.115	1.246	0.0652	0.0569	0.1016	0.08196	1.553	1.158
		8.72	42.32	0.1409	0.3862	0.4875	0.8965	0.005618	0.07564	0.02549	0.1255	1.135	2.043
			8.58		0.1408		0.4701		0.00558		0.02535		1.13
	150	87.95	40.85	0.5038	0.5495	1.714	1.433	0.07146	0.05714	0.09088	0.06663	1.134	0.762
		8.72	47.28	0.1726	0.4765	0.943	1.451	0.01427	0.05825	0.05308	0.07835	1.937	1.034
			8.59		0.1724		0.9509		0.0143		0.05326		1.946

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		$FAS(T_{S,i})$ [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
		84.79	75.60	0.7834	0.8092	0.9087	0.6425	0.07333	0.06945	0.05998	0.05499	0.4812	0.427
	100	9.14	9.42	0.2631	0.6565	0.8164	1.822	0.02857	0.09452	0.06949	0.09224	1.658	0.8828
10			8.99		0.2626		0.7344		0.02833		0.06904		1.65
		84.79	8.06	0.9594	1.244	0.1883	0.9931	0.0591	0.1261	0.03947	0.06496	0.2585	0.3281
	150	9.14	76.92	0.3222	0.9452	3.039	0.1301	0.06843	0.05804	0.1361	0.03934	2.653	0.2615
			9.02		0.322		3.044		0.06834		0.136		2.653
		83.62	36.20	1.156	1.267	1.232	0.6307	0.1219	0.127	0.06756	0.06422	0.3673	0.3184
	100	9.16	47.63	0.3866	1.097	0.8596	0.8925	0.07559	0.1178	0.1253	0.06878	2.037	0.3939
15			9.04		0.3862		0.903		0.07565		0.1256		2.044
		83.62	45.72	1.416	1.524	0.6313	0.2464	0.09714	0.08842	0.04397	0.03716	0.1952	0.1532
	150	9.16	38.11	0.4735	1.324	1.675	0.9588	0.05893	0.1198	0.07977	0.05798	1.059	0.2752
			9.03		0.4729		1.659		0.05907		0.08005		1.064
		82.63	23.73	1.901	2.156	0.4467	0.3275	0.1199	0.1114	0.0404	0.03311	0.1335	0.09649
	100	9.13	59.11	0.6345	1.833	0.6845	0.3086	0.0829	0.1196	0.0837	0.04179	0.8288	0.1432
25			9.01		0.6338		0.6444		0.08249		0.08337		0.8264
		82.63	52.84	2.328	2.477	0.5071	0.2027	0.1114	0.1117	0.03066	0.02888	0.08273	0.07326
	150	9.13	30.01	0.777	2.145	0.8953	0.3649	0.07512	0.1138	0.06195	0.034	0.501	0.0996
			9.00		0.776		0.8696		0.07535		0.06222		0.5038
		82.05	45.23	3.019	3.249	0.916	0.723	0.1913	0.2136	0.04059	0.04213	0.08449	0.08147
	100	9.10	37.04	1.007	2.822	0.7131	0.5742	0.07814	0.1489	0.04972	0.03381	0.3102	0.07529
40			8.97		1.006		0.7035		0.07745		0.04934		0.3083
		82.05	28.35	3.697	4.128	0.5484	0.4588	0.2798	0.2554	0.04849	0.03964	0.0824	0.06034
	150	9.10	53.91	1.233	3.544	1.168	0.5641	0.1248	0.2688	0.06486	0.0486	0.3305	0.08616
			8.98		1.232		1.189		0.1246		0.06483		0.3307

Table 5.59: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Northridge 1994 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	25.76	0.1171	0.1356	0.3581	1.787	0.01095	0.01378	0.05976	0.06504	3.203	3.015
			74.24		0.1136		0.5314		0.01008		0.05665		3.129
	150	100.00	63.30	0.1434	0.1533	1.34	1.752	0.01497	0.01788	0.06685	0.07445	2.931	3.044
			36.70		0.133		1.203		0.0134		0.06453		3.051
2	100	94.72	51.15	0.1894	0.2045	1.7	1.272	0.02595	0.03195	0.08783	0.1	2.915	3.073
		5.28	43.68	0.07236	0.1776	0.5557	1.859	0.003422	0.02879	0.02999	0.1038	2.587	3.675
			5.18		0.07231		0.687		0.003413		0.02993		2.584
	150	94.72	37.41	0.232	0.2567	2.672	1.592	0.03638	0.04225	0.1005	0.1053	2.725	2.576
		5.28	57.41	0.08862	0.2214	1.552	1.558	0.006861	0.03691	0.04945	0.1068	3.499	3.033
			5.18		0.08857		1.53		0.006849		0.04939		3.497
3	100	91.41	7.56	0.2631	0.3514	2.199	5.675	0.04578	0.1294	0.1114	0.236	2.661	4.22
		7.49	83.98	0.09389	0.2597	0.8902	2.785	0.007556	0.04416	0.05153	0.1089	3.455	2.635
			7.38		0.09384		0.9971		0.007551		0.05152		3.457
	150	91.41	32.32	0.3222	0.3594	2.991	1.671	0.1274	0.1325	0.2534	0.2361	4.941	4.128
		7.49	59.22	0.115	0.3087	0.9512	5.411	0.01043	0.111	0.05789	0.2302	3.159	4.682
			7.37		0.1149		0.8953		0.0104		0.05781		3.157
5	100	87.95	33.95	0.4113	0.4553	5.049	5.379	0.1372	0.1581	0.2138	0.2225	3.266	3.071
		8.72	54.18	0.1409	0.3926	1.332	3.36	0.01455	0.1477	0.06614	0.241	2.95	3.858
			8.59		0.1408		1.318		0.01453		0.0661		2.951
	150	87.95	15.58	0.5038	0.6003	3.929	5.132	0.1852	0.2349	0.2355	0.2507	2.938	2.624
		8.72	72.53	0.1726	0.4916	1.951	1.544	0.0277	0.1814	0.1028	0.2364	3.744	3.021
			8.59		0.1725		1.952		0.02766		0.1028		3.745

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
		84.79	51.66	0.7834	0.8366	2.777	1.044	0.2738	0.2524	0.224	0.1933	1.797	1.452
	100	9.14	33.34	0.2631	0.7256	2.199	2.194	0.04578	0.245	0.1114	0.2163	2.661	1.873
10			9.01		0.2627		2.353		0.04563		0.1112		2.659
		84.79	58.38	0.9594	1.015	1.558	0.7074	0.225	0.1916	0.1503	0.121	0.9842	0.749
	150	9.14	26.62	0.3222	0.8751	2.991	1.566	0.1274	0.2565	0.2534	0.1878	4.941	1.348
			9.00		0.3218		3.191		0.1271		0.2529		4.939
		83.62	47.48	1.156	1.241	2.719	1.642	0.225	0.2404	0.1247	0.1241	0.678	0.6282
	100	9.16	36.35	0.3866	1.078	4.681	1.095	0.1419	0.2189	0.2351	0.1301	3.82	0.7588
15			9.03		0.3861		4.866		0.1412		0.2341		3.809
		83.62	17.08	1.416	1.659	1.057	1.741	0.2812	0.3022	0.1273	0.1167	0.5649	0.4419
	150	9.16	66.73	0.4735	1.377	2.668	0.9009	0.1711	0.2727	0.2316	0.1269	3.073	0.5787
			9.04		0.473		2.714		0.1709		0.2315		3.075
		82.63	45.43	1.901	2.046	2.05	1.11	0.329	0.3911	0.1109	0.1225	0.3665	0.376
	100	9.13	37.42	0.6345	1.777	3.697	1.636	0.2187	0.3155	0.2208	0.1137	2.188	0.4021
25			9.01		0.6337		3.7		0.2191		0.2216		2.197
		82.63	27.80	2.328	2.605	0.4991	0.6597	0.387	0.2978	0.1065	0.07325	0.2874	0.1767
	150	9.13	55.04	0.777	2.234	2.761	0.7351	0.2745	0.4084	0.2264	0.1171	1.83	0.3295
			9.01		0.7762		2.759		0.2745		0.2266		1.834
		82.05	21.88	3.019	3.448	0.4428	0.565	0.2666	0.3059	0.05659	0.05685	0.1178	0.1036
	100	9.10	60.39	1.007	2.918	0.8298	0.4714	0.1926	0.2616	0.1226	0.05744	0.7649	0.1237
40			8.98		1.006		0.8451		0.1928		0.1229		0.7675
		82.05	24.93	3.697	4.173	0.5053	0.6602	0.3161	0.3578	0.05478	0.05493	0.0931	0.08271
	150	9.10	57.34	1.233	3.56	1.72	0.5382	0.2395	0.3121	0.1244	0.05617	0.6341	0.09914
			8.98		1.232		1.732		0.2393		0.1245		0.635

Table 5.60: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Kobe 1995 earthquake (Takarazuka station), with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	54.92	0.1171	0.1266	0.4024	0.1695	0.003345	0.004246	0.01823	0.02143	0.9763	1.062
			45.08		0.11		0.1876		0.002652		0.01545		0.8827
	150	100.00	59.85	0.1434	0.154	0.6179	0.3901	0.008888	0.01107	0.03957	0.04604	1.729	1.879
			40.15		0.1337		0.1272		0.005829		0.02768		1.291
2	100	94.72	9.46	0.1894	0.246	0.6944	1.204	0.01432	0.02826	0.04843	0.07372	1.607	1.886
		5.28	85.35	0.07236	0.1867	0.04909	0.5902	0.00105	0.01405	0.009268	0.04823	0.8052	1.624
			5.19		0.07233		0.05324		0.001049		0.009265		0.8053
	150	94.72	46.47	0.232	0.2523	1.711	0.801	0.02874	0.02656	0.07936	0.06752	2.15	1.683
		5.28	48.35	0.08862	0.219	0.135	0.7955	0.001597	0.02207	0.01153	0.06454	0.8167	1.852
			5.18		0.08857		0.1275		0.001595		0.01152		0.8165
3	100	91.41	79.71	0.2631	0.2725	0.3815	0.3627	0.02407	0.02342	0.05867	0.05507	1.403	1.27
		7.49	11.85	0.09389	0.2243	0.12	1.082	0.001822	0.025	0.01239	0.07127	0.8288	1.993
			7.35		0.09377		0.1207		0.001817		0.01237		0.8286
	150	91.41	23.41	0.3222	0.3699	0.6329	0.9832	0.03674	0.05853	0.07298	0.1014	1.422	1.722
		7.49	68.13	0.115	0.3119	0.2017	0.4718	0.003141	0.02901	0.01742	0.05953	0.9487	1.198
			7.37		0.1149		0.1944		0.003133		0.01738		0.9476
5	100	87.95	10.96	0.4113	0.5124	1.436	3.232	0.07963	0.132	0.124	0.1651	1.894	2.025
		8.72	77.16	0.1409	0.4038	0.7028	1.47	0.008213	0.07513	0.03728	0.1192	1.661	1.854
			8.60		0.1408		0.6843		0.008185		0.03719		1.658
	150	87.95	53.11	0.5038	0.5385	3.72	1.943	0.136	0.1034	0.1729	0.1231	2.158	1.437
		8.72	35.02	0.1726	0.4671	0.7535	3.488	0.01313	0.1349	0.04877	0.185	1.777	2.489
			8.58		0.1724		0.7205		0.01313		0.0488		1.78

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$\mathrm{T}_{S,i}$ [s]		FAS($\mathrm{T}_{S,i}$) [m/s]		$\mathrm{S}_d(\mathrm{T}_{S,i})$ [m]		$\mathrm{S}_{pv}(\mathrm{T}_{S,i})$ [m/s]		$\mathrm{S}_{pa}(\mathrm{T}_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10	100	84.79	20.31	0.7834	0.9038	0.7293	1.16	0.1462	0.1866	0.1196	0.1323	0.9591	0.9198
		9.14	64.67	0.2631	0.7593	0.3815	0.8771	0.02407	0.131	0.05867	0.1105	1.403	0.9143
			9.02		0.2628		0.388		0.0241		0.05879		1.407
	150	84.79	43.14	0.9594	1.039	1.005	0.962	0.1954	0.2342	0.1305	0.1444	0.8547	0.8734
		9.14	41.85	0.3222	0.9021	0.6329	1.166	0.03674	0.1863	0.07298	0.1323	1.422	0.9217
			9.01		0.3218		0.6505		0.03643		0.07247		1.414
15	100	83.62	14.04	1.156	1.384	1.509	2.918	0.2875	0.3679	0.1594	0.1703	0.8664	0.7728
		9.16	69.77	0.3866	1.129	0.7202	1.485	0.06528	0.2794	0.1082	0.1586	1.758	0.8825
			9.04		0.3863		0.7262		0.06514		0.108		1.757
	150	83.62	41.54	1.416	1.536	2.959	2.254	0.3781	0.4056	0.1712	0.1692	0.7597	0.6925
		9.16	42.28	0.4735	1.333	3.508	2.845	0.1387	0.3456	0.1877	0.1661	2.492	0.7829
			9.03		0.4729		3.506		0.1385		0.1876		2.493
25	100	82.63	27.18	1.901	2.131	1.425	1.531	0.3973	0.3603	0.1339	0.1083	0.4426	0.3194
		9.13	55.66	0.6345	1.825	1.431	1.321	0.1254	0.4069	0.1266	0.1428	1.254	0.4916
			9.01		0.6338		1.427		0.1252		0.1266		1.255
	150	82.63	47.78	2.328	2.496	1.336	1.056	0.3277	0.3247	0.09019	0.08334	0.2434	0.2098
		9.13	35.07	0.777	2.167	0.766	1.521	0.1425	0.3542	0.1175	0.1047	0.9499	0.3036
			9.00		0.7761		0.7716		0.1419		0.1171		0.9483
40	100	82.05	44.54	3.019	3.253	0.6456	0.5446	0.2956	0.2834	0.06274	0.05582	0.1306	0.1078
		9.10	37.73	1.007	2.825	0.9855	0.7484	0.2096	0.2992	0.1334	0.06787	0.8324	0.1509
			8.97		1.006		0.9864		0.2086		0.1329		0.8302
	150	82.05	44.97	3.697	3.981	0.4674	0.4366	0.238	0.2104	0.04124	0.03386	0.07009	0.05344
		9.10	37.30	1.233	3.457	2.307	0.4935	0.307	0.2652	0.1595	0.04915	0.8128	0.08932
			8.97		1.232		2.285		0.3066		0.1595		0.8138

Table 5.61: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Kobe 1995 earthquake (Takatori station), with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	40.84	0.1171	0.1298	0.1197	0.3542	0.004088	0.005356	0.02233	0.02643	1.198	1.279
			59.16		0.1119		0.5828		0.003693		0.02112		1.186
	150	100.00	30.84	0.1434	0.1631	0.2808	0.69	0.00782	0.01158	0.03484	0.04535	1.524	1.744
			69.16		0.1384		0.4929		0.006855		0.03167		1.436
2	100	94.72	28.42	0.1894	0.2147	1.032	1.522	0.02044	0.02132	0.06912	0.0636	2.292	1.861
		5.28	66.40	0.07236	0.1826	0.03902	0.9405	0.0009904	0.01898	0.008729	0.06655	0.7571	2.289
			5.18		0.07232		0.03579		0.0009891		0.008723		0.757
	150	94.72	34.67	0.232	0.2584	0.7722	0.8512	0.02519	0.03077	0.06956	0.07629	1.884	1.855
		5.28	60.15	0.08862	0.2221	0.07348	0.9791	0.001736	0.02303	0.01251	0.0664	0.8853	1.878
			5.18		0.08857		0.07985		0.001733		0.01249		0.8847
3	100	91.41	22.62	0.2631	0.303	1.578	0.4431	0.03191	0.04929	0.0777	0.1042	1.856	2.16
		7.49	68.93	0.09389	0.2549	0.1954	0.9993	0.002187	0.02995	0.0148	0.07526	0.9853	1.855
			7.37		0.09382		0.2038		0.002181		0.01477		0.9838
	150	91.41	21.94	0.3222	0.3723	1.6	2.322	0.0557	0.07413	0.1107	0.1276	2.16	2.154
		7.49	69.60	0.115	0.3124	0.2061	0.7251	0.00393	0.05275	0.02185	0.1082	1.193	2.176
			7.37		0.1149		0.2489		0.003924		0.02183		1.193
5	100	87.95	69.77	0.4113	0.43	1.29	0.7944	0.07452	0.06805	0.1161	0.1014	1.774	1.482
		8.72	18.36	0.1409	0.3644	0.4693	2.307	0.007317	0.07185	0.03323	0.1263	1.481	2.179
			8.58		0.1407		0.4493		0.007279		0.03311		1.477
	150	87.95	31.60	0.5038	0.5608	0.8371	1.344	0.06863	0.08487	0.08727	0.09695	1.088	1.086
		8.72	56.52	0.1726	0.4823	0.5042	1.942	0.01495	0.06834	0.05536	0.0908	2.011	1.183
			8.59		0.1724		0.6044		0.01488		0.05513		2.005

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$\mathrm{T}_{S,i}$ [s]		FAS($\mathrm{T}_{S,i}$) [m/s]		$\mathrm{S}_d(\mathrm{T}_{S,i})$ [m]		$\mathrm{S}_{pv}(\mathrm{T}_{S,i})$ [m/s]		$\mathrm{S}_{pa}(\mathrm{T}_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10	100	84.79	12.95	0.7834	0.9491	1.885	2.681	0.1707	0.3124	0.1396	0.2109	1.12	1.396
		9.14	72.03	0.2631	0.7664	1.578	1.388	0.03191	0.1484	0.0777	0.124	1.856	1.016
			9.02		0.2629		1.606		0.03186		0.07763		1.856
	150	84.79	65.36	0.9594	1.005	2.335	1.205	0.321	0.3539	0.2144	0.2256	1.404	1.41
		9.14	19.64	0.3222	0.8561	1.6	1.512	0.0557	0.2325	0.1107	0.174	2.16	1.277
			9.00		0.3217		1.495		0.05552		0.1105		2.159
15	100	83.62	31.31	1.156	1.282	6.721	5.412	0.6657	0.7743	0.369	0.387	2.006	1.897
		9.16	52.51	0.3866	1.104	2.261	6.334	0.07665	0.5393	0.127	0.3128	2.065	1.779
			9.04		0.3862		2.327		0.0766		0.1271		2.068
	150	83.62	75.18	1.416	1.461	0.5184	2.125	0.6386	0.5803	0.289	0.2545	1.283	1.095
		9.16	8.66	0.4735	1.178	1.785	6.934	0.06902	0.7083	0.09341	0.3852	1.24	2.055
			9.01		0.4726		1.731		0.06907		0.09364		1.245
25	100	82.63	14.97	1.901	2.257	2.907	3.844	0.8045	0.9458	0.2712	0.2684	0.8962	0.7472
		9.13	67.86	0.6345	1.854	1.736	3.032	0.1053	0.7809	0.1063	0.2699	1.053	0.9145
			9.02		0.6339		1.728		0.1053		0.1065		1.055
	150	82.63	54.28	2.328	2.472	3.331	3.233	0.9163	0.8861	0.2522	0.2297	0.6806	0.5838
		9.13	28.57	0.777	2.138	1.784	5.543	0.1619	0.9443	0.1335	0.283	1.079	0.832
			9.00		0.776		1.768		0.1604		0.1324		1.072
40	100	82.05	47.79	3.019	3.235	2.566	1.78	0.7654	0.753	0.1624	0.1491	0.3381	0.2896
		9.10	34.48	1.007	2.808	1.201	2.908	0.3548	0.8042	0.2258	0.1835	1.409	0.4105
			8.97		1.006		1.204		0.354		0.2256		1.41
	150	82.05	45.83	3.697	3.975	1.015	0.8318	0.6916	0.5945	0.1198	0.09583	0.2037	0.1515
		9.10	36.44	1.233	3.452	8.338	1.179	0.7696	0.7589	0.3999	0.1409	2.038	0.2564
			8.97		1.232		8.36		0.7688		0.4		2.041

Table 5.62: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the L'Aquila 2009 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	49.82	0.1171	0.1276	1.072	0.5616	0.004936	0.004961	0.02712	0.02491	1.463	1.228
			50.18		0.1107		0.5346		0.005329		0.03088		1.755
	150	100.00	43.72	0.1434	0.1581	1.17	1.338	0.007046	0.007136	0.03152	0.02894	1.384	1.152
			56.28		0.1366		0.6969		0.006358		0.02971		1.363
2	100	94.72	21.85	0.1894	0.2202	0.1814	0.7217	0.009554	0.01311	0.0323	0.03814	1.071	1.088
		5.28	72.97	0.07236	0.184	0.2076	0.5999	0.001494	0.008639	0.01308	0.03003	1.127	1.025
			5.18		0.07232		0.1916		0.001491		0.01306		1.126
	150	94.72	29.04	0.232	0.2624	0.3584	0.793	0.01491	0.01816	0.04115	0.04436	1.114	1.063
		5.28	65.78	0.08862	0.2235	0.631	0.3537	0.00272	0.01362	0.01963	0.039	1.392	1.096
			5.18		0.08857		0.6174		0.002717		0.01962		1.392
3	100	91.41	34.13	0.2631	0.2921	0.8135	0.5705	0.01818	0.018	0.04429	0.03946	1.058	0.8487
		7.49	57.42	0.09389	0.2515	0.7905	0.6037	0.003258	0.01754	0.02209	0.04468	1.473	1.116
			7.37		0.09381		0.7516		0.003249		0.02205		1.471
	150	91.41	9.95	0.3222	0.4105	0.8592	1.503	0.02169	0.05064	0.0431	0.07906	0.8401	1.21
		7.49	81.58	0.115	0.317	0.8731	0.8632	0.005065	0.02056	0.02835	0.04154	1.559	0.8233
			7.38		0.1149		0.9056		0.005069		0.02839		1.562
5	100	87.95	59.33	0.4113	0.4359	1.466	1.164	0.05058	0.04958	0.07879	0.07287	1.204	1.05
		8.72	28.80	0.1409	0.3764	0.7687	1.086	0.007055	0.04464	0.03209	0.07595	1.432	1.267
			8.58		0.1407		0.7412		0.007056		0.03213		1.435
	150	87.95	57.71	0.5038	0.535	0.7369	0.8484	0.07069	0.07689	0.0899	0.09207	1.121	1.081
		8.72	30.42	0.1726	0.4628	0.6814	0.7732	0.007803	0.05844	0.02898	0.0809	1.056	1.098
			8.58		0.1724		0.6334		0.007798		0.02899		1.058

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$\mathrm{T}_{S,i}$ [s]		FAS($\mathrm{T}_{S,i}$) [m/s]		$\mathrm{S}_d(\mathrm{T}_{S,i})$ [m]		$\mathrm{S}_{pv}(\mathrm{T}_{S,i})$ [m/s]		$\mathrm{S}_{pa}(\mathrm{T}_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	54.88	0.7834	0.8327	1.064	0.8842	0.1022	0.1038	0.08358	0.07987	0.6705	0.6026
	100	9.14	30.12	0.2631	0.7206	0.8135	1.244	0.01818	0.1045	0.04429	0.09295	1.058	0.8105
			9.01		0.2627		0.8034		0.01817		0.04433		1.061
		84.79	19.95	0.9594	1.109	0.7917	1.063	0.1144	0.1197	0.07643	0.06914	0.5006	0.3918
	150	9.14	65.04	0.3222	0.9304	0.8592	0.4953	0.02169	0.1125	0.0431	0.07748	0.8401	0.5232
			9.02		0.3219		0.8635		0.0216		0.04297		0.8384
15		83.62	45.81	1.156	1.244	1.026	0.6557	0.1196	0.1172	0.06629	0.06036	0.3604	0.3047
	100	9.16	38.02	0.3866	1.081	2.598	1.024	0.04811	0.1193	0.07972	0.07074	1.296	0.4112
			9.03		0.3861		2.592		0.04797		0.07959		1.295
		83.62	41.59	1.416	1.535	0.5778	0.4909	0.1073	0.1066	0.04855	0.04449	0.2155	0.1821
	150	9.16	42.23	0.4735	1.333	1.679	0.7572	0.06206	0.1121	0.08397	0.0539	1.114	0.2541
			9.03		0.4729		1.621		0.06188		0.08383		1.114
25		82.63	30.62	1.901	2.11	0.29	0.3646	0.1239	0.1288	0.04177	0.03911	0.1381	0.1164
	100	9.13	52.22	0.6345	1.817	1.109	0.2427	0.09389	0.1209	0.09481	0.04262	0.939	0.1474
			9.01		0.6338		1.151		0.09377		0.09479		0.9398
		82.63	39.55	2.328	2.533	0.3647	0.3345	0.1305	0.1297	0.0359	0.03282	0.09689	0.08143
	150	9.13	43.29	0.777	2.198	1.247	0.3781	0.1029	0.1298	0.08482	0.03785	0.686	0.1082
			9.01		0.7761		1.274		0.103		0.085		0.6882
40		82.05	41.51	3.019	3.27	0.2761	0.2207	0.1231	0.1184	0.02612	0.0232	0.05437	0.04458
	100	9.10	40.77	1.007	2.84	0.9293	0.313	0.117	0.1261	0.07446	0.02845	0.4647	0.06295
			8.97		1.006		0.9279		0.117		0.07452		0.4656
		82.05	36.69	3.697	4.043	0.2013	0.1924	0.1119	0.1039	0.01939	0.01647	0.03295	0.02559
	150	9.10	45.58	1.233	3.504	0.7144	0.2042	0.1177	0.1166	0.06115	0.02133	0.3116	0.03825
			8.97		1.232		0.7218		0.1177		0.06125		0.3125

Table 5.63: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Chile 2010 earthquake (Angol station), with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	42.56	0.1171	0.1293	0.3008	0.4516	0.007033	0.009105	0.03833	0.04508	2.052	2.19
			57.44		0.1116		0.5228		0.005858		0.03351		1.882
	150	100.00	10.09	0.1434	0.1905	0.9657	5.243	0.01271	0.0315	0.05654	0.1059	2.47	3.494
			89.91		0.1415		1.966		0.01189		0.0537		2.38
2	100	94.72	48.50	0.1894	0.2053	4.391	5.055	0.03126	0.03319	0.1057	0.1036	3.506	3.175
		5.28	46.32	0.07236	0.1783	0.6809	3.847	0.002092	0.02822	0.0183	0.1014	1.576	3.574
			5.18		0.07231		0.4736		0.002086		0.01826		1.574
	150	94.72	30.91	0.232	0.2609	2.661	6.47	0.03668	0.048	0.1013	0.1179	2.745	2.839
		5.28	63.91	0.08862	0.2231	1.291	5.175	0.003453	0.03424	0.02497	0.0983	1.773	2.768
			5.18		0.08857		1.07		0.003452		0.02497		1.774
3	100	91.41	42.07	0.2631	0.2873	8.289	2.675	0.04803	0.04298	0.117	0.09595	2.796	2.101
		7.49	49.48	0.09389	0.249	1.523	1.555	0.003671	0.04154	0.02504	0.1068	1.68	2.695
			7.36		0.09381		0.5472		0.003667		0.02504		1.681
	150	91.41	40.82	0.3222	0.3527	4.169	3.217	0.03242	0.03389	0.06448	0.06155	1.258	1.096
		7.49	50.73	0.115	0.3055	1.206	2.795	0.006582	0.03493	0.03648	0.07334	1.986	1.511
			7.36		0.1149		1.219		0.00656		0.03639		1.983
5	100	87.95	30.89	0.4113	0.4588	1.46	0.648	0.0301	0.03077	0.04688	0.04297	0.7161	0.5885
		8.72	57.23	0.1409	0.3941	0.7892	0.4712	0.01164	0.03196	0.05282	0.052	2.352	0.8299
			8.59		0.1408		0.6155		0.01158		0.05261		2.346
	150	87.95	40.73	0.5038	0.5496	2.136	0.5838	0.04058	0.04326	0.05158	0.05044	0.6431	0.5767
		8.72	47.40	0.1726	0.4766	3.961	1.737	0.02686	0.03587	0.09969	0.04821	3.63	0.6355
			8.59		0.1724		4.04		0.02682		0.09964		3.632

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$\mathrm{T}_{S,i}$ [s]		FAS($\mathrm{T}_{S,i}$) [m/s]		$\mathrm{S}_d(\mathrm{T}_{S,i})$ [m]		$\mathrm{S}_{pv}(\mathrm{T}_{S,i})$ [m/s]		$\mathrm{S}_{pa}(\mathrm{T}_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	43.91	0.7834	0.8471	0.4892	1.038	0.05263	0.06123	0.04305	0.04631	0.3453	0.3435
	100	9.14	41.08	0.2631	0.7357	8.289	1.182	0.04803	0.06303	0.117	0.05491	2.796	0.4691
			9.01		0.2628		8.821		0.04803		0.1171		2.803
		84.79	39.41	0.9594	1.046	0.6696	0.1287	0.05192	0.04838	0.03467	0.02963	0.2271	0.178
	150	9.14	45.58	0.3222	0.9072	4.169	1.58	0.03242	0.06217	0.06448	0.04391	1.258	0.3042
			9.01		0.3218		3.748		0.03247		0.06467		1.263
15		83.62	36.11	1.156	1.267	0.1293	0.3786	0.04829	0.05326	0.02677	0.02693	0.1455	0.1335
	100	9.16	47.71	0.3866	1.097	0.9144	0.4806	0.03397	0.04294	0.05632	0.02507	0.9159	0.1436
			9.04		0.3862		0.8137		0.03403		0.05649		0.9197
		83.62	38.72	1.416	1.544	0.8986	0.6044	0.05956	0.05056	0.02696	0.02098	0.1197	0.08541
	150	9.16	45.10	0.4735	1.339	1.358	0.6533	0.03513	0.06029	0.04753	0.02885	0.6306	0.1354
			9.03		0.4729		0.9553		0.035		0.0474		0.6297
25		82.63	43.55	1.901	2.053	0.5929	0.5064	0.08635	0.08546	0.0291	0.02667	0.0962	0.08165
	100	9.13	39.29	0.6345	1.783	1.505	0.5698	0.04843	0.06969	0.04892	0.02504	0.4845	0.08826
			9.01		0.6337		1.453		0.04854		0.04909		0.4869
		82.63	29.64	2.328	2.591	0.226	0.3267	0.06119	0.07113	0.01684	0.01759	0.04544	0.04265
	150	9.13	53.19	0.777	2.229	1.002	0.3251	0.05304	0.06165	0.04374	0.01772	0.3537	0.04997
			9.01		0.7762		1.035		0.0531		0.04384		0.3549
40		82.05	20.49	3.019	3.469	0.1738	0.1758	0.0743	0.09171	0.01577	0.01694	0.03282	0.03068
	100	9.10	61.77	1.007	2.923	0.2448	0.252	0.05097	0.07592	0.03244	0.01664	0.2025	0.03577
			8.98		1.006		0.3007		0.05108		0.03254		0.2033
		82.05	13.74	3.697	4.43	0.1903	0.3289	0.1044	0.1332	0.01809	0.01926	0.03074	0.02732
	150	9.10	68.52	1.233	3.611	0.7513	0.2107	0.05514	0.09767	0.02865	0.01733	0.146	0.03015
			8.98		1.232		0.7799		0.05517		0.02869		0.1463

Table 5.64: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Chile 2010 earthquake (Concepcion San Pedro station), with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	26.43	0.1171	0.1353	1.404	1.23	0.004959	0.0067	0.02709	0.0317	1.453	1.473
			73.57		0.1135		0.5452		0.004628		0.02606		1.441
	150	100.00	45.71	0.1434	0.1575	2.016	1.398	0.007765	0.009183	0.0346	0.03739	1.513	1.494
			54.29		0.1362		0.4234		0.006779		0.03185		1.469
2		94.72	38.29	0.1894	0.2092	2.03	1.633	0.01948	0.01993	0.06586	0.06105	2.184	1.835
	100	5.28	56.53	0.07236	0.1806	0.2498	1.407	0.001209	0.01748	0.01062	0.062	0.9184	2.157
			5.18		0.07232		0.172		0.001207		0.01061		0.9177
		94.72	35.20	0.232	0.258	4.15	2.893	0.02437	0.02437	0.06727	0.06052	1.821	1.474
	150	5.28	59.62	0.08862	0.222	0.2863	2.617	0.002246	0.02043	0.01619	0.05889	1.146	1.665
			5.18		0.08857		0.2226		0.002242		0.01617		1.146
3		91.41	44.19	0.2631	0.2862	6.357	2.596	0.02463	0.02899	0.05999	0.06486	1.433	1.423
	100	7.49	47.36	0.09389	0.2483	0.6775	3.073	0.002748	0.02464	0.01862	0.06363	1.241	1.612
			7.36		0.09381		1.029		0.002739		0.01858		1.24
		91.41	39.87	0.3222	0.3533	0.43	2.372	0.03292	0.04222	0.06541	0.07659	1.275	1.363
	150	7.49	51.68	0.115	0.3059	0.8884	1.883	0.004764	0.03423	0.02648	0.07165	1.446	1.471
			7.36		0.1149		0.4051		0.004755		0.02645		1.446
5		87.95	55.62	0.4113	0.4381	2.857	1.565	0.05334	0.05911	0.08309	0.08643	1.269	1.239
	100	8.72	32.51	0.1409	0.3795	0.4941	5.222	0.007267	0.04878	0.033	0.08235	1.471	1.363
			8.58		0.1408		0.3461		0.007234		0.0329		1.468
		87.95	46.07	0.5038	0.5444	3.939	1.634	0.0784	0.07711	0.09973	0.09074	1.244	1.047
	150	8.72	42.06	0.1726	0.4728	0.7283	1.98	0.01429	0.0738	0.05292	0.1	1.923	1.329
			8.58		0.1724		0.4074		0.01421		0.05269		1.917

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	15.61	0.7834	0.9287	2.256	1.404	0.1004	0.1403	0.08209	0.09678	0.6584	0.6548
	100	9.14	69.37	0.2631	0.7638	6.357	4.874	0.02463	0.09253	0.05999	0.07761	1.433	0.6384
			9.02		0.2628		5.475		0.02461		0.06		1.435
		84.79	58.01	0.9594	1.015	1.701	2.733	0.1338	0.1231	0.08937	0.07768	0.5853	0.4807
	150	9.14	26.99	0.3222	0.876	0.43	5.329	0.03292	0.1789	0.06541	0.1309	1.275	0.9386
			9.00		0.3218		0.9677		0.03264		0.06494		1.267
15		83.62	26.58	1.156	1.3	1.8	2.257	0.1244	0.1504	0.06894	0.07418	0.3748	0.3586
	100	9.16	57.24	0.3866	1.111	1.736	1.644	0.04938	0.1062	0.08184	0.06121	1.331	0.3461
			9.04		0.3862		2.473		0.04935		0.08189		1.333
		83.62	44.42	1.416	1.528	2.579	0.6384	0.1442	0.1571	0.06527	0.06589	0.2897	0.271
	150	9.16	39.40	0.4735	1.327	1.703	0.896	0.07374	0.1476	0.0998	0.07128	1.325	0.3376
			9.03		0.4729		1.694		0.07379		0.09999		1.329
25		82.63	25.63	1.901	2.142	0.4937	0.8964	0.1499	0.1215	0.05053	0.03635	0.167	0.1066
	100	9.13	57.21	0.6345	1.829	2.07	0.7823	0.07339	0.1579	0.07411	0.05532	0.734	0.19
			9.01		0.6338		1.764		0.07327		0.07406		0.7342
		82.63	11.67	2.328	2.848	0.689	0.5603	0.1269	0.2009	0.03491	0.0452	0.09421	0.09971
	150	9.13	71.16	0.777	2.281	1.671	0.438	0.09769	0.1238	0.08054	0.03478	0.6512	0.09581
			9.02		0.7764		1.452		0.09743		0.0804		0.6506
40		82.05	67.28	3.019	3.146	0.4496	0.4798	0.1649	0.1289	0.03499	0.02625	0.07282	0.05243
	100	9.10	15.00	1.007	2.644	1.952	0.7652	0.1225	0.2153	0.07796	0.05216	0.4866	0.124
			8.96		1.005		1.8		0.1224		0.07801		0.4876
		82.05	46.04	3.697	3.974	0.8669	0.2684	0.1596	0.1794	0.02765	0.02893	0.04699	0.04575
	150	9.10	36.23	1.233	3.451	3.724	0.3557	0.1326	0.1349	0.06889	0.02504	0.3511	0.0456
			8.97		1.232		3.804		0.1325		0.06892		0.3517

Table 5.65: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the New Zealand 2010 earthquake, with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	31.13	0.1171	0.1331	0.1176	0.02404	0.004085	0.005444	0.02227	0.02616	1.192	1.234
			68.87		0.113		0.2134		0.003604		0.02035		1.129
	150	100.00	47.15	0.1434	0.1571	0.16	0.8558	0.006616	0.007488	0.02951	0.03057	1.292	1.225
			52.85		0.136		0.4536		0.005771		0.02713		1.252
2	100	94.72	36.02	0.1894	0.2103	0.5903	0.5677	0.008999	0.01519	0.03042	0.04627	1.009	1.383
		5.28	58.80	0.07236	0.1811	0.1247	0.5042	0.001272	0.007542	0.01115	0.02666	0.9621	0.9243
			5.18		0.07232		0.1007		0.001269		0.01113		0.9611
	150	94.72	29.60	0.232	0.2619	0.4309	0.5149	0.0161	0.01801	0.0445	0.044	1.207	1.054
		5.28	65.22	0.08862	0.2234	0.2133	0.5114	0.002205	0.0166	0.01592	0.04764	1.129	1.341
			5.18		0.08857		0.2014		0.002203		0.01592		1.129
3	100	91.41	58.09	0.2631	0.2802	0.4451	0.6044	0.01845	0.02144	0.04487	0.04902	1.07	1.099
		7.49	33.46	0.09389	0.2427	0.2263	0.7248	0.002364	0.01398	0.01613	0.03695	1.083	0.9579
			7.36		0.0938		0.3029		0.002362		0.01613		1.084
	150	91.41	21.73	0.3222	0.3726	0.3768	0.6539	0.0279	0.03419	0.05549	0.05878	1.083	0.9911
		7.49	69.82	0.115	0.3125	0.2071	0.9912	0.003839	0.02785	0.02129	0.0571	1.16	1.148
			7.37		0.1149		0.1922		0.003829		0.02125		1.159
5	100	87.95	8.72	0.4113	0.5303	0.9338	1.708	0.05217	0.09237	0.08124	0.1116	1.241	1.322
		8.72	79.39	0.1409	0.4051	0.4136	0.4571	0.006322	0.04825	0.02872	0.07627	1.28	1.182
			8.60		0.1408		0.4262		0.006311		0.02869		1.28
	150	87.95	47.72	0.5038	0.5429	2.805	0.8185	0.09326	0.0888	0.1186	0.1048	1.479	1.213
		8.72	40.41	0.1726	0.4716	0.3961	1.25	0.007799	0.07382	0.02899	0.1003	1.058	1.336
			8.58		0.1724		0.3801		0.00781		0.02907		1.061

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	14.01	0.7834	0.9403	0.8324	2.005	0.2135	0.2352	0.1746	0.1603	1.401	1.071
	100	9.14	70.97	0.2631	0.7654	0.4451	0.7891	0.01845	0.2023	0.04487	0.1693	1.07	1.39
			9.02		0.2629		0.4217		0.01836		0.0447		1.067
		84.79	37.22	0.9594	1.051	1.855	2.29	0.241	0.3013	0.1609	0.1837	1.054	1.099
	150	9.14	47.77	0.3222	0.9101	0.3768	3.17	0.0279	0.2395	0.05549	0.1686	1.083	1.164
			9.01		0.3218		0.4152		0.02792		0.0556		1.086
15		83.62	15.78	1.156	1.366	2.097	2.946	0.3468	0.3705	0.1923	0.1737	1.045	0.7989
	100	9.16	68.03	0.3866	1.126	0.412	2.129	0.03721	0.3383	0.06166	0.1925	1.002	1.074
			9.04		0.3862		0.3877		0.03715		0.06162		1.002
		83.62	53.65	1.416	1.506	2.354	2.746	0.3365	0.3062	0.1523	0.1303	0.6762	0.5436
	150	9.16	30.17	0.4735	1.303	1.322	4.999	0.07508	0.3819	0.1016	0.1877	1.348	0.9049
			9.03		0.4729		1.242		0.07467		0.1012		1.344
25		82.63	26.72	1.901	2.134	2.519	3.336	0.5109	0.7142	0.1722	0.2144	0.5691	0.6313
	100	9.13	56.12	0.6345	1.827	1.632	3.41	0.1185	0.4544	0.1196	0.1594	1.185	0.5483
			9.01		0.6338		1.629		0.1181		0.1194		1.183
		82.63	25.28	2.328	2.626	2.742	2.492	0.8362	0.8224	0.2301	0.2006	0.6211	0.48
	150	9.13	57.55	0.777	2.241	1.02	1.687	0.2092	0.7915	0.1724	0.2263	1.394	0.6344
			9.01		0.7762		1.044		0.2086		0.1722		1.394
40		82.05	25.18	3.019	3.404	1.742	1.405	0.5379	0.4768	0.1142	0.08974	0.2376	0.1656
	100	9.10	57.09	1.007	2.906	3.141	1.028	0.2717	0.6127	0.1729	0.1351	1.079	0.2921
			8.98		1.006		3.176		0.2709		0.1726		1.078
		82.05	11.05	3.697	4.551	1.16	2.05	0.5459	1.04	0.0946	0.1464	0.1608	0.202
	150	9.10	71.21	1.233	3.625	3.549	1.172	0.355	0.5011	0.1844	0.08857	0.9399	0.1535
			8.98		1.232		3.537		0.355		0.1846		0.9415

Table 5.66: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Tohoku 2011 earthquake (Tsukidate station), with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	6.54	0.1171	0.1734	1.801	10.61	0.01326	0.05283	0.07224	0.1953	3.865	7.086
		0.00	93.46	0	0.1161	0	2.104	0	0.01283	0	0.07044	0	3.799
	150	100.00	22.45	0.1434	0.1687	6.966	6.643	0.03522	0.05076	0.1569	0.1926	6.861	7.17
		0.00	77.55	0	0.1396	0	7.012	0	0.03168	0	0.1453	0	6.539
2	100	94.72	40.58	0.1894	0.2083	11.9	11.83	0.05673	0.1069	0.1919	0.3282	6.365	9.886
		5.28	54.24	0.07236	0.1801	0.6534	7.171	0.008903	0.05411	0.07843	0.1925	6.8	6.714
		0.00	5.18	0	0.07232	0	0.7055	0	0.008889	0	0.07836	0	6.798
	150	94.72	44.90	0.232	0.253	9.743	7.988	0.1741	0.1829	0.4806	0.4636	13.01	11.53
		5.28	49.92	0.08862	0.2194	2.487	14.18	0.009623	0.1438	0.06989	0.4197	4.988	12.01
		0.00	5.18	0	0.08857	0	1.865	0	0.009633	0	0.07002	0	5.002
3	100	91.41	59.69	0.2631	0.2796	6.91	6.616	0.1698	0.1447	0.414	0.3317	9.903	7.456
		7.49	31.86	0.09389	0.2419	1.409	15.47	0.009491	0.1859	0.06487	0.4926	4.36	12.8
		0.00	7.36	0	0.0938	0	0.6697	0	0.009487	0	0.06492	0	4.367
	150	91.41	44.75	0.3222	0.3502	4.607	3.549	0.1346	0.105	0.2678	0.1921	5.225	3.447
		7.49	46.80	0.115	0.3039	2.783	2.505	0.01235	0.1333	0.06844	0.2811	3.725	5.815
		0.00	7.36	0	0.1149	0	1.867	0	0.01231	0	0.06825	0	3.718
5	100	87.95	36.85	0.4113	0.4523	2.101	1.186	0.08863	0.09535	0.138	0.1351	2.108	1.877
		8.72	51.28	0.1409	0.3912	7.933	0.9569	0.03287	0.08158	0.1492	0.1336	6.649	2.147
		0.00	8.59	0	0.1408	0	7.617	0	0.03273	0	0.1488	0	6.637
	150	87.95	50.11	0.5038	0.5409	0.9457	1.073	0.08216	0.0855	0.1045	0.1013	1.304	1.176
		8.72	38.02	0.1726	0.4697	12.15	2.418	0.05269	0.09334	0.1956	0.1273	7.128	1.703
		0.00	8.58	0	0.1724	0	6.454	0	0.05265	0	0.1957	0	7.139

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
10		84.79	14.56	0.7834	0.9361	2.143	2.623	0.08092	0.1132	0.06618	0.07749	0.5308	0.5201
	100	9.14	70.42	0.2631	0.7648	6.91	1.208	0.1698	0.0738	0.414	0.06181	9.903	0.5077
		0.00	9.02	0	0.2628	0	8.048	0	0.1702	0	0.4154	0	9.944
		84.79	43.05	0.9594	1.039	1.579	1.453	0.1217	0.11	0.08126	0.06784	0.5322	0.4103
	150	9.14	41.94	0.3222	0.9022	4.607	1.599	0.1346	0.1138	0.2678	0.08084	5.225	0.563
		0.00	9.01	0	0.3218	0	6.042	0	0.1349	0	0.2686	0	5.246
15		83.62	24.46	1.156	1.309	1.553	2.339	0.112	0.1699	0.06211	0.08317	0.3377	0.3992
	100	9.16	59.36	0.3866	1.114	0.9273	1.736	0.0828	0.1067	0.1373	0.06136	2.232	0.3461
		0.00	9.04	0	0.3862	0	0.7653	0	0.08294	0	0.1377	0	2.241
		83.62	34.99	1.416	1.556	1.009	1.646	0.1309	0.1432	0.05924	0.05894	0.263	0.238
	150	9.16	48.84	0.4735	1.346	3.072	1.183	0.09204	0.1633	0.1246	0.07774	1.654	0.3629
		0.00	9.04	0	0.4729	0	2.07	0	0.09223	0	0.125	0	1.661
25		82.63	44.81	1.901	2.048	2.112	1.721	0.3118	0.2737	0.1051	0.08562	0.3473	0.2626
	100	9.13	38.03	0.6345	1.779	2.658	4.75	0.08233	0.2641	0.08318	0.09512	0.8242	0.336
		0.00	9.01	0	0.6337	0	2.377	0	0.08296	0	0.08391	0	0.8324
		82.63	20.17	2.328	2.681	0.196	1.096	0.2631	0.2371	0.07241	0.05667	0.1954	0.1328
	150	9.13	62.67	0.777	2.256	1.149	1.883	0.07887	0.2812	0.06502	0.07988	0.5257	0.2225
		0.00	9.01	0	0.7763	0	0.8451	0	0.07858	0	0.06485	0	0.5249
40		82.05	20.61	3.019	3.467	1.093	2.435	0.3183	0.4596	0.06754	0.08494	0.1406	0.1539
	100	9.10	61.65	1.007	2.923	0.9518	0.8021	0.1196	0.2906	0.07611	0.0637	0.475	0.1369
		0.00	8.98	0	1.006	0	0.7017	0	0.1198	0	0.07633	0	0.4769
		82.05	64.53	3.697	3.868	1.211	0.9878	0.43	0.3913	0.07452	0.06483	0.1266	0.1053
	150	9.10	17.75	1.233	3.282	2.242	1.676	0.1603	0.425	0.08329	0.08298	0.4244	0.1589
		0.00	8.96	0	1.231	0	2.568	0	0.16	0	0.08327	0	0.4249

Table 5.67: *Comparison of modal parameters for the main modes of vibration before and after the insertion of the optimum TMD, for all the considered primary structures subjected to the Tohoku 2011 earthquake (Sendai station), with μ = 0.02, ζ_S = 0.05.*

$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$T_{S,i}$ [s]		FAS($T_{S,i}$) [m/s]		$S_d(T_{S,i})$ [m]		$S_{pv}(T_{S,i})$ [m/s]		$S_{pa}(T_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
1	100	100.00	31.78	0.1171	0.1328	0.6939	1.298	0.006818	0.008484	0.03729	0.04085	2.003	1.931
			68.22		0.1129		0.2654		0.00649		0.0368		2.05
	150	100.00	36.49	0.1434	0.1606	0.6962	0.5341	0.01037	0.01141	0.04627	0.04552	2.027	1.782
			63.51		0.1376		1.136		0.00936		0.0435		1.985
2	100	94.72	37.86	0.1894	0.2094	1.252	1.425	0.01372	0.01658	0.04641	0.05072	1.54	1.522
		5.28	56.96	0.07236	0.1807	0.04522	1.227	0.002551	0.0129	0.0225	0.04574	1.953	1.591
			5.18		0.07232		0.185		0.002548		0.02249		1.953
	150	94.72	61.42	0.232	0.2469	1.761	0.813	0.01876	0.02044	0.0518	0.05305	1.403	1.35
		5.28	33.41	0.08862	0.2138	0.7576	1.48	0.003876	0.01699	0.02798	0.05092	1.983	1.497
			5.17		0.08856		1.045		0.003871		0.02796		1.983
3	100	91.41	20.42	0.2631	0.3061	1.262	2.963	0.02366	0.03565	0.05755	0.07461	1.373	1.532
		7.49	71.12	0.09389	0.2555	0.4326	0.7676	0.004478	0.02162	0.03043	0.05419	2.033	1.333
			7.37		0.09382		0.259		0.004469		0.03039		2.032
	150	91.41	15.34	0.3222	0.387	1.861	3.16	0.03653	0.06708	0.07265	0.1111	1.417	1.805
		7.49	76.20	0.115	0.3148	0.4312	1.972	0.006653	0.03625	0.03704	0.07379	2.027	1.473
			7.37		0.1149		0.3507		0.006647		0.03704		2.027
5	100	87.95	32.69	0.4113	0.4566	4.092	2.053	0.06691	0.09543	0.1042	0.1338	1.592	1.841
		8.72	55.43	0.1409	0.3933	0.4855	1.725	0.009947	0.06651	0.0452	0.1084	2.015	1.732
			8.59		0.1408		0.9642		0.009924		0.04514		2.014
	150	87.95	37.46	0.5038	0.5532	6.509	3.717	0.124	0.162	0.1577	0.1875	1.967	2.129
		8.72	50.66	0.1726	0.4787	0.7074	4.678	0.01214	0.1158	0.04505	0.155	1.641	2.034
			8.59		0.1724		0.301		0.01212		0.04503		1.642

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$n_{S,n}$	$m_{S,i}$ [t]	$M_{me,S,i}$ [%]		$\mathrm{T}_{S,i}$ [s]		FAS($\mathrm{T}_{S,i}$) [m/s]		$\mathrm{S}_d(\mathrm{T}_{S,i})$ [m]		$\mathrm{S}_{pv}(\mathrm{T}_{S,i})$ [m/s]		$\mathrm{S}_{pa}(\mathrm{T}_{S,i})$ [m/s^2]	
		No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD	No TMD	TMD
		84.79	44.98	0.7834	0.8455	2.624	1.603	0.2784	0.2494	0.2277	0.189	1.827	1.405
	100	9.14	40.01	0.2631	0.7344	1.262	1.18	0.02366	0.2728	0.05755	0.238	1.373	2.037
10			9.01		0.2628		1.897		0.02352		0.05728		1.368
		84.79	30.31	0.9594	1.068	5.797	2.439	0.3938	0.3902	0.263	0.234	1.722	1.377
	150	9.14	54.68	0.3222	0.9185	1.861	1.091	0.03653	0.3538	0.07265	0.2468	1.417	1.688
			9.01		0.3219		1.781		0.03651		0.07267		1.419
		83.62	74.62	1.156	1.194	1.902	3.304	0.3395	0.3387	0.1882	0.1818	1.023	0.9567
	100	9.16	9.22	0.3866	0.9678	2.543	9.288	0.06714	0.3995	0.1113	0.2644	1.81	1.717
15			9.02		0.3859		2.013		0.06723		0.1116		1.819
		83.62	26.03	1.416	1.594	2.425	1.862	0.3373	0.4225	0.1527	0.1698	0.6776	0.6691
	150	9.16	57.79	0.4735	1.362	4.632	3.083	0.1118	0.3481	0.1512	0.1638	2.007	0.7556
			9.04		0.473		4.723		0.1114		0.1509		2.004
		82.63	42.44	1.901	2.057	0.9958	0.7215	0.2978	0.2986	0.1004	0.09301	0.3318	0.2841
	100	9.13	40.40	0.6345	1.786	4.215	1.324	0.2516	0.312	0.254	0.1119	2.516	0.3937
25			9.01		0.6337		5.084		0.2508		0.2536		2.514
		82.63	27.56	2.328	2.607	1.267	1.915	0.3118	0.3282	0.08582	0.08066	0.2316	0.1944
	150	9.13	55.28	0.777	2.235	6.381	1.565	0.2793	0.3278	0.2303	0.094	1.862	0.2643
			9.01		0.7762		6.888		0.2794		0.2306		1.867
		82.05	51.99	3.019	3.214	1.129	0.348	0.3845	0.3474	0.08161	0.06926	0.1699	0.1354
	100	9.10	30.28	1.007	2.784	6.661	1.697	0.4085	0.3763	0.26	0.08661	1.622	0.1955
40			8.97		1.005		6.43		0.4087		0.2605		1.628
		82.05	49.52	3.697	3.952	0.8733	0.2634	0.2923	0.2668	0.05066	0.04326	0.08609	0.06879
	150	9.10	32.75	1.233	3.428	2.444	0.7907	0.3379	0.3208	0.1756	0.05997	0.8947	0.1099
			8.97		1.231		2.31		0.3381		0.1759		0.8975

Chapter 6

Conclusions

The subject of the present doctoral dissertation was the optimum tuning of Tuned Mass Dampers, within several different structural and dynamic frameworks, with a main slant to the civil engineering context, but providing also indications of general character and validity. The TMD devices, in their different forms, are certainly one of the most important means for the controlled reduction of structural vibrations. The versatility of TMDs allows for their application in many engineering fields, and the insertion in a generic system either at design stage or also in existing structures. Main achievements are outlined below.

The optimisation of the TMD device is first investigated for benchmark ideal excitations, namely harmonic and white noise excitations, acting either as force on the primary structure or as base acceleration, for the general case of damped SDOF primary structures.

The considered excitations represent the most representative cases contemplated in the literature and constitute suitable models for possible real loading situations. The main task of this study was the development of general tuning formulas, generated from a shared fitting model, for a range of values suitable for civil engineering applications. The optimum parameters have been obtained through a nonlinear optimisation algorithm and then fitted by means of nonlinear least squares.

Then, two fitting models have been proposed and calibrated: the first is based on a polynomial combination of terms and is completely originated from the surface fitting process of the obtained optimum parameters; the second is based on literature formulas for the special and simplified case of undamped primary structure and then enriched by proper additional terms taking into account the effect of damping. A global comparison pointed out the effectiveness of the newly achieved tuning formulas based on the proposed fitting model, which allows for the actual optimum tuning of TMDs, through very simple expressions, which could likely turn out quite useful in view of practical applications.

Within the context of ideal excitations, another important part has been dedicated to the analysis of the TMD performance in reducing the structural transient response, through the optimisation of the control device in the case of unit impulse excitation, for the general case of damped SDOF primary structures.

First, the passive TMD has been tuned through the same optimisation procedure described previously for the other excitations. At that stage, it was found that the control device was not significantly able to reduce as well the peak response occurring at the very beginning of the time history, while the overall response turned out to be remarkably decreased. Such outcome is likely due to the inertial nature of the passive TMD, which therefore does not react promptly when a sudden excitation occurs on the structural system.

Hence, the TMD has been successively upgraded to a hybrid form, with the addition of an active controller. Two specific control strategies have been analysed in details for the controller, the first basically following the mainstream literature and composed of the total primary structure kinematic response, then limited to velocity and displacement only, and the second newly-proposed based on the relative velocity between primary structure and TMD and on the displacement of the primary structure. This latter control law was found to be the best one, since it provided large response reductions and with a shorter settling time, which denotes a more stable behaviour from the structural point of view.

The last chapter concerned the investigation on the seismic performance improvement of simple structural systems (shear-type frames) due to the insertion of a passive TMD. Due to the intrinsic uncertainties related to the nature of the earthquake events, a wide group composed of 16 shear-type frame structures and 18 earthquakes, for a total of 288 cases, has been considered. The primary structures have been conceived so that their modal parameters could cover a wide range of the response spectrum values, while the assumed seismic signals exhibit quite different characteristics in terms of magnitude, duration, frequency content. A quite high structural damping has been adopted, so that to provide a challenging test for the control device. Indeed, a lower structural damping would certainly increase the TMD performance. The special character of this study concerns the seismic optimisation of the TMD by means of an innovative tuning method, which directly involves the earthquake event, by embedding the seismic input signal within the optimisation process, and therefore provides a TMD tuned for each specific considered case. This feature is very important, since it establishes a sort of objective for the TMDs, and therefore it becomes possible to explore the highest level of TMD effectiveness, at least for the response index assumed as objective function.

The so obtained optimum TMD parameters globally exhibit the trends obtainable from the tuning in the case of canonical excitations, with the exception of some cases where different values are recovered. The seismic response, evaluated for different kinematic and energy indicators, displayed a general significant reduction, especially with consideration of the indexes related to the overall response, while the peak response mainly showed smaller percentage reductions. A minor group of cases actually exhibited results, in terms of both TMD parameters and response reduction, not in line with the mainstream, which fact could be motivated by different reasons, among which the characteristics of the signal, the correlation between structural modal parameters and seismic signal characteristics and, mostly, the shape of the objective function involved in the optimisation process.

A further investigation focused on this direction has been developed, concerning a crossed comparison involving the response reduction and relevant indicators representing both input and output of the analysis, such as modal parameters, frequency content of the seismic signal and response spectra. Following this way, an inspection for possible motivations of the TMD effectiveness has been carried out. As a main outcome of this analysis, it appears that the TMD performance could be further supported when the effective modal mass of the mode on which the device is tuned turns out equally partitioned on the two achieved split modes and the attached values of the response spectra corresponding to the modal periods decrease, which denotes a general reduction of the structural seismic response. As a further indirect validation of this observation, the cases where the obtained TMD performance was less remarkable, more often corresponding to situations where the post-tuning modes displayed quite unbalanced modal masses and the main of these two modes exhibited just small reductions in terms of response spectra values.

In conclusions, this doctoral thesis has represented an effort in establishing the actual effectiveness of Tuned Mass Damper devices within different structural dynamic contexts, assessed through models of the structural system and of the dynamic excitation representative of real situations. Hence, outcomes and guidelines outlined in the present research study could likely be implemented in potential engineering applications of such control devices.

The overall results of this wide investigation clearly pointed out basically a significant positive effect related to the addition of the TMD to the primary structure. Indeed, a general remarkable reduction of dynamic response is achieved, which could take values from 20% to 50% of the total, depending on the structural and excitation parameters. This outcome should, in principle, encourage the further adoption of the Tuned Mass Dampers in practical engineering applications, including in the realm of civil engineering.

However, the passive TMD turns out quite able to reduce mostly the overall response, and especially the part related to the steady-state evolution, while the level of performance for the mitigation of the peak and the beginning of the transient response could be improved with the addition of an active controller, especially when the excitation exhibits characteristics of typical impulse loading, such as unit impulse and earthquake excitations, as analysed here. For this latter case, the uncertainties related to this typology of loading, which is characterised by a purely random nature, made necessary a wide study, where different situations have been investigated. Such research provided not only a wide scenario of the potential seismic performance of the TMD, but also started to point out the main connections between a given earthquake response abatement due to TMD addition and the corresponding structural context. These results should play an important role in explaining the actual behaviour of TMDs in earthquake engineering, and also could significantly contribute in improving the seismic tuning process, which could therefore be set in order to match favourable conditions from both the structure and the excitation points of view, so that to maximise the benefit coming from the control device.

With these remarks in mind, future studies could first focus on further and more definite analyses on the effectiveness of TMDs in reducing the seismic response, with the task of developing *ad hoc* tuning formulas, and set a comparison to those available for classical ideal excitations.

Also, the performance of the hybrid TMD should be deepened in different contexts. Such typology of TMD looks very interesting, since it provides a basic effective passive TMD and the active controller brings in the chance to further improve the performance, with specific reference to the inertial component of the passive device alone.

The indications provided within this research could provide important guidelines for further studies on the optimal design of TMDs in earthquake engineering. In particular, an improved tuning process could consider and take advantage from the salient positive features that motivate the remarkable level of effectiveness of the control device in decreasing the structural seismic response. Summary and final outcome of these investigations and studies could likely be the development of a special version of a TMD, designed specifically for civil engineering applications, which should be able to control and reduce efficiently the structural dynamic response.

Bibliography

[1] M. Abé, T. Igusa, "Semi-active dynamic vibration absorbers for controlling transient response", *Journal of Sound and Vibration*, Vol. 198, No. 5, p. 547–569, 1996.

[2] C. Adam, T. Fürtmuller, "Seismic Performance of Tuned Mass Dampers", in: H. Irschik, M. Krommer, K. Watanabe, T. Furukawa (Eds.), *Mechanics and Model-Based Control of Smart Materials and Structures*, Springer-Verlag, 1st edition, p. 11–18, 2010.

[3] A.S. Ahlawat, A. Ramaswamy, "Multi-objective optimal design of FLC driven hybrid mass damper for seismically excited structures", *Earthquake Engineering and Structural Dynamics*, 31(12):1459-1479, 2002.

[4] M. Al-Dawod, B. Samali, J. Li, "Experimental verification of an active mass driver system on a five-storey model using a fuzzy controller", *Structural Control and Health Monitoring*, 13(5):917-943, 2006.

[5] A.M. Aly, "Proposed robust tuned mass damper for response mitigation in buildings exposed to multidirectional wind", *The Structural Design of Tall and Special Buildings*, Early view, 28 pages, 2012.

[6] F. Amini, N. Khanmohammadi Hazaveh, A. Abdolah Rad, "Wavelet PSO-Based LQR Algorithm for Optimal Structural Control Using Active Tuned Mass Dampers", *Computer-Aided Civil and Infrastructure Engineering*, 28(7):542-557, 2013.

[7] T. Asami, O. Nishihara, A.M. Baz, "Analytical solutions to H_∞ and H_2 optimization of dynamic vibration absorber attached to damped linear systems", *Journal of Vibrations and Acoustics* (ASME), 124(2):284–295, 2002.

[8] T. Asami, O. Nishihara, "Closed-form Exact Solution to H_∞ Optimization of Dynamic Vibration Absorber (Application to Different Transfer Functions and Damping Systems)", *Journal of Vibrations and Acoustics* (ASME), 125(3):398–405, 2003.

[9] S.M. Avila, P.B. Gonçalves, "Hybrid control to mitigate excessive vibrations caused by dynamics loadings with random perturbations in tall buildings", *ABCM Symposium Series in Mechatronics*, 1:295–303, 2004.

[10] E.O. Ayorinde,G.B. Warburton, "Minimizing structural vibrations with absorbers", *Earthquake Engineering and Structural Dynamics*, 8(3):219–236, 1980.

[11] S.V. Bakre, R.S. Jangid, "Optimum parameters of tuned mass damper for damped main system", *Structural Control and Health Monitoring*, 14(3):448–470, 2006.

[12] V.A. Bapat, H.V. Kumaraswamy, "Effect of primary system damping on the optimum design of an untuned viscous damping vibration absorber", *Journal of Sound and Vibration*, 63(4):469–474, 1979.

[13] G. Bekdaş, S.M. Nigdeli, "Estimating optimum parameters of tuned mass dampers using harmony search", *Engineering Structures*, 33(9):2716–2723, 2011.

[14] P.Bisegna, G.Caruso, "Closed-form formulas for the optimal pole-based design of tuned mass dampers", *Journal of Sound and Vibration*, 331(10):2291–2314, 2012.

[15] J.E. Brock, "A Note on the Damped Vibration Absorber", *Journal of Applied Mechanics* (ASME), 13(4):A–284, 1946.

[16] B. Brown, T. Singh, "Minimax design of vibration absorbers for linear damped systems", *Journal of Sound and Vibration*, 330(11):2437–2448, 2011.

[17] H.Cao, Q.S.Li, "New control strategies for active tuned mass damper systems", *Computers and Structures*, 82(27):2341-2350, 2004.

[18] H. Cao, A.M. Reinhorn, T.T. Soong, "Design of an ative mass damper for a tall TV tower in Nanjing, China", *Engineering Structures*, 20(3):134-143, 1998.

[19] F. Casciati, F. Giuliano, "Performance of Multi-TMD in the Towers of Suspension Bridges", *Journal of Vibration and Control*, 15(6):821–847, 2009.

[20] A.K. Chopra, *Dynamics of Structures - Theory and Applications to Earthquake Engineering*, Prentice-Hall, 2nd ed., 2001.

[21] J.J. Connor, *Introduction to Structural Motion Control*, Prentice-Hall, 1st ed., 2002.

[22] M.C. Constantinou, T.T. Soong, G.F. Dargush, *Passive Energy Dissipation Systems for Structural Design and Retrofit*, MCEER Monography no. 1, 1998.

[23] S.H. Crandall, W.D. Mark, *Random Vibration in Mechanical Systems*, Academic Press, 1st ed., 1963.

[24] D.I. Cretu, I.R. Racanel, A. Ercusi, "Passive Control Devices Used to Improve the Dynamic Response of a Footbridge", Proc. of 5th European Conference on Structural Control (EACS 2012), Genoa, Italy, 18–20 June 2012, 8 pages, 2012

[25] T.K. Datta, "A state-of-the-art review on active control of structures", *ISET Journal of Earthquake Technology*, Paper No. 430, 40(1):1-17, 2003.

[26] J.P. Den Hartog, *Mechanical Vibrations*, McGraw-Hill, 4th ed., 1956.

[27] I.M. Diaz, E. Pereira, P. Reynolds, C.M. Casado, "Human-induced vibration cancelation using an active mass damper modfied by an inner loop", Proc. of 8th Int. Conf. on Structural Dynamics (EURODYN 2011), Leuven, Belgium,

4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.), 6 pages, 2011.

[28] A.Farshidianfar, S.Soheili, “Bee Colony Optimization of Tuned Mass Dampers for vibrations of high-rise buildings including Soil Structure Interaction”, Proceedings of 6th National Congress on Civil Engineering, Semnan, Iran, 26–27 April 2011, pp. 1–8, 2011.

[29] M.Q. Feng, A. Mita, “Vibration control of tall buildings using mega subconfiguration”, *Journal of Engineering Mechanics*, 121(10):1082–1088, 1995.

[30] N.R. Fisco, H. Adeli, “Smart structures: Part I - Active and semi-active control”, *Scientia Iranica*, transcription A: civil engineering, 18(3):275-284, 2011.

[31] N.R. Fisco, H. Adeli, “Smart structures: Part II Hybrid control systems and control strategies”, *Scientia Iranica*, transcription A: civil engineering, 18(3):285-295, 2011.

[32] H. Frahm, “Device for damping vibrations of bodies”, U.S. Patent No. 989958, 3576–3580, 1911.

[33] Y. Fujino, M. Abé, “Design formulas for tuned mass dampers based on a perturbation technique”, *Earthquake Engineering and Structural Dynamics*, 22(10):833–854, 1993.

[34] C.R. Fuller, S.J. Elliott, P.A. Nelson, *Active Control of Vibration*, Academic Press, 1st ed., 1996.

[35] A.Ghosh, B.Basu, “A closed-form optimal tuning criterion for TMD in damped structures”, *Structural Control and Health Monitoring*, 14(4):681–692, 2007.

[36] Y.P. Gupta, A.R. Chandrasekaran, “Absorber system for earthquake excitations”, Proceedings of 4th World Conference on Earthquake Engineering, Santiago (Chile), Vol. II, pp. 139–148, 1969.

[37] E. Hahnkamm, “Die Dämpfung von Fundamentschwingungen bei veränderlicher Erregerfrequenz”, *Ingenieur-Archiv*, 4:192–201, 1933 (in German).

[38] N. Hoang, P. Warnitchai, “Design of multiple tuned mass dampers by using a numerical optimizer”, *Earthquake Engineering and Structural Dynamics*, 34(2):125–144, 2005.

[39] N. Hoang, Y. Fujino, P. Warnitchai, “Optimal tuned mass damper for seismic applications and practical desing formulas”, *Engineering Structures*, 30(3):707–715, 2008.

[40] G.W. Housner, L.A. Bergman, T.K. Caughey, A.G. Chassiakos, R.O. Claus, S.F. Masri, R.E. Skelton, T.T. Soong, B.F. Spencer, J.T.P. Yao, “Structural Control: Past, Present and Future”, *Journal of Engineering Mechanics*, 123(9):897–971, 1997.

[41] T. Ioi, K. Ikeda, “On the dynamic vibration damped absorber of the vibration system”, *Bulletin of the Japanese Society of Mechanical Engineering*, 21(151):64–71, 1978.

[42] R.G. Jacquot, D.L. Hoppe, “Optimal Random Vibration Absorbers”, *Journal of the Engineering Mechanics Division* (ASCE), 99(3):612–616, 1973.

[43] R.S. Jangid, “Optimum Multiple Tuned Mass Dampers for base-excited undamped system”, *Earthquake Engineering and Structural Dynamics*, 28(9):1041–1049, 1999.

[44] K. Kanai, “An empirical formula for the spectrum of strong earthquake motions”, *Bulletin of the Earthquake Research Institute*, 39(1):85–95, 1961.

[45] A.M. Kaynia, D. Veneziano, J.M. Biggs, “Seismic effectiveness of Tuned Mass Dampers”, *Journal of the Structural Division* (ASCE), 107(8):1465–1484, 1981.

[46] L. Kela, P. Vähäoja, “Recent Studies of Adaptive Tuned Vibration Absorbers/Neutralizers”, *Applied Mechanics Reviews*, 62(6):060801-1–060801-9, 2009.

[47] S.G. Kelly, *Fundamentals of Mechanical Vibrations*, McGraw-Hill, 2nd ed., 2000.

[48] F. Kerber, S. Hurlebaus, B.M. Beadle, U. Stöbener, “Control concepts for an active vibration isolation system”, *Mechanical Systems and Signal Processing*, 21(8):3042-3059, 2007.

[49] S.Krenk, “Frequency Analysis of the Tuned Mass Damper”, *Journal of Applied Mechanics* (ASME), 72(6):936–942, 2005.

[50] S. Krenk, J. Høgsberg, “Tuned mass absorbers on damped structures under random load”, *Probabilistic Engineering Mechanics*, 23(4):408–415, 2008.

[51] K.C.S. Kwok, B. Samali, “Performance of Tuned Mass Dampers under wind loads”, *Engineering Structures*, 17(9):655–667, 1995.

[52] C.-L. Lee, Y.-T. Chen, L.-L. Chung, Y.-P. Wang, “Optimal design theories and applications of tuned mass dampers”, *Engineering Structures*, 28(1):43–53, 2006.

[53] A.Y.T. Leung, H. Zhang, “Particle Swarm Optimization of tuned mass dampers”, *Engineering Structures*, 31(3):715–728, 2009.

[54] A.Y.T. Leung, H. Zhang, C.C. Cheng, Y.Y. Lee, “Particle Swarm Optimization of TMD by non-stationary base excitation during earthquake”, *Earthquake Engineering and Structural Dynamics*, 37(9):1223–1246, 2008.

[55] C. Li, B. Zhu, “Estimating double tuned mass dampers for structures under ground acceleration using a novel optimum criterion”, *Journal of Sound and Vibration*, 298(1–2):280–297, 2006.

[56] C.-C. Lin, J.-M. Ueng, T.-C. Huang, “Seismic response reduction of irregular buildings using passive tuned mass dampers”, *Engineering Structures*, 22(5):513–524, 1999.

[57] K. Liu, J. Liu, “The damped dynamic vibration absorbers: revisited and new result”, *Journal of Sound and Vibration*, 284(3–5):1181–1189, 2005.

[58] C.H. Loh, C.H. Chao, “Effectiveness of active tuned mass damper and seismic isolation on vibration control of multi-storey building”, *Journal of Sound and Vibration*, 193(4):773–792, 1996.

[59] A. Lorenzana, J.M.W. Brownjohn, A.V. Poncela, C.E. Lavin, A. Vasallo, "Dynamic response of a RC chimney without and with TMD under wind action. Experimental and simulation results", Proc. of 8th Int. Conf. on Structural Dynamics (EURODYN 2011), Leuven, Belgium, 4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.), 7 pages, 2011.

[60] R.W. Luft, "Optimal Tuned Mass Dampers for Buildings", *Journal of the Structural Division* (ASCE), 105(12):2766–2772, 1979.

[61] L.E.Mackriell, K.C.S.Kwok, B.Samali, "Critical mode control of a wind-loaded tall building using an active tuned mass damper", *Engineering Structures*, 19(10):834–842, 1997.

[62] G.C. Marano, R. Greco, "Stochastic optimum design of linear tuned mass dampers for seismic protection of high towers", Proceedings of 14th World Conference on Earthquake Engineering (14WCEE), Beijing, China, 12–17 October 2008, p. 1–8, 2008.

[63] G.C. Marano, R. Greco, F. Trentadue, B. Chiaia, "Constrained reliability-based optimization of linear tuned mass dampers for seismic control", *International Journal of Solids and Structures*, 44(22-23):7370–7388, 2007.

[64] E. Matta, "Effectiveness of Tuned Mass Dampers against Ground Motion Pulses", *Journal of Structural Engineering*, 139(2):188–198, 2013.

[65] R.J. McNamara, "Tuned Mass Dampers for Buildings", *Journal of the Structural Division* (ASCE), 103(9):1785–1798, 1977.

[66] C. Meinhardt, "Tuned Mass Dampers for Offshore-Structures under Wave-Loading", Proc. of 8th Int. Conf. on Structural Dynamics (EURODYN 2011), Leuven, Belgium, 4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.), 7 pages, 2011.

[67] J.C. Miranda, "On tuned mass dampers for reducing the seismic response of structures", *Earthquake Engineering and Structural Dynamics*, 34(7):847–865, 2005.

[68] J.C. Miranda, "System intrinsic, damping maximized, tuned mass dampers for seismic applications", *Structural Control and Health Monitoring*, 19(9):405–416, 2012.

[69] M. Mohebbi, A. Joghataie, "Designing optimal tuned mass dampers for nonlinear frames by Distributed Genetic Algorithms", *The Structural Design of Tall and Special Buildings*, 21(1):57–76, 2012.

[70] S. Nagarajaiah, "Adaptive passive, semiactive, smart tuned mass dampers: identification and control using empirical mode decomposition, hilbert transform, and short-term fourier transform", *Structural Control and Health Monitoring*, 16(7–8):800–841, 2009.

[71] I. Nagashima, "Optimal displacement feedback control law for active tuned mass damper", *Earthquake Engineering and Structural Dynamics*, 30(8):1221–1242, 2001.

[72] O. Nishihara, T. Asami, "Closed-form solutions to the exact optimizations of dynamic vibration absorbers (minimizations of the maximum amplitude magnifications factors)", *Journal of Vibrations and Acoustics* (ASME), 124(4):576–582, 2002.

[73] I. Nishimura et al., "Active tuned mass damper", *Smart Materials and Structures*, 1(4):306–311, 1992.

[74] K. Ogata, *Modern Control Engineering*, Prentice Hall, 5th ed., 2009.

[75] M.R. Okhovat, M. Rahimian, A.K. Ghorbani-Tahna, "Tuned mass damper for seismic response reduction of Tehran Tower", Proceedings of 4th International Conference on Earthquake Engineering (4ICEE), Taipei, Taiwan, 12–13 October 2006, Paper No. 132, p. 1–9, 2006.

[76] J. Ormondroyd, J.P. Hartog, "The theory of the dynamic vibration absorber", *Journal of Applied Mechanics* (ASME), 50(7):9–22, 1928.

[77] M.B. Ozer, T.J. Royston, "Application of Sherman-Morrison matrix inversion formula to damped vibration absorbers attached to multi-degree of freedom systems", *Journal of Sound and Vibrations*, 283(3–5):1235–1249, 2005.

[78] M.B. Ozer, T.J. Royston, "Extending Den Hartog's Vibration Absorber Technique to Multi-Degree-of-Freedom Systems", *Journal of Vibrations and Acoustics*, 127(4):341–350, 2005.

[79] M.M. Paredes, R.C. Barros, A. Cunha, "A parametric study of TMD's for regular buildings under earthquakes", Proceedings of 2nd International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2009), Rhodes, Greece, 22–24 June 2009, M. Papadrakakis, N.D. Lagaros, M. Fragiadakis (Eds.), p. 1–15, 2009.

[80] E. Pennestrì, "An application of Chebyshev's Min-Max criterion to the optimal design of a damped dynamic vibration absorber", *Journal of Sound and Vibration*, 217(4):757–765, 1998.

[81] A.G. Piersol, T.L. Paez, *Harris' Shock and Vibration Handbook*, McGraw-Hill, 6th ed., 2010.

[82] T. Pinkaew, Y. Fujino, "Effectiveness of semi-active tuned mass dampers under harmonic excitation", *Engineering Structures*, 23(7):850–856, 2001.

[83] R. Rana, T.T. Soong, "Parametric study and simplified design of tuned mass dampers", *Engineering Structures*, 20(3):193–204, 1998.

[84] S.E. Randall, D.M. Halsted, D.L. Taylor, "Optimum vibration absorbers for linear damped systems", *Journal of Mechanical Design* (ASME), 103(4):908–913, 1981.

[85] S.S. Rao, *Mechanical Vibrations*, Pearson, 5th ed., 2011.

[86] S.K. Rasouli, M. Yahyai, "Control of response of structures with passive and active tuned mass dampers", *The Structural Design of Tall Buildings*, 11(1):1–14, 2002.

[87] E. Rizzi, D. Brescianini, M. Scotti, "On the optimal tuning of Tuned Mass Dampers in structural systems", Proc. of ECCOMAS Thematic Conference - 2nd Int. Conf. on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2009), Rhodes, Greece, 22–24 June 2009, M. Papadrakakis, N.D. Lagaros, M. Fragiadakis (Eds.), 24 pages, 2009.

[88] M. Saberi, C. von Scholten, "Design and installation of Tuned Mass Dampers into a very slender footbridge across Aagade in Copenaghen", Proc. of ECCOMAS Thematic Conference - 2nd Int. Conf. on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2009), Rhodes, Greece, 22–24 June 2009, M. Papadrakakis, N.D. Lagaros, M. Fragiadakis (Eds.), 17 pages, 2009.

[89] C. Sabirin, T. Roglin, D. Mayer, "Design and Control of Adaptive Vibration Absorbers Mounted on an Air Conditioning Compressor", Proc. of 8th Int. Conf. on Structural Dynamics (EURODYN 2011), Leuven, Belgium, 4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.), 7 pages, 2011.

[90] F. Sadek, B. Mohraz, A.W. Taylor, R.M. Chung, "A method of estimating the parameters of Tuned Mass Dampers for seismic applications", *Earthquake Engineering and Structural Dynamics*, 26(6):617–635, 1997.

[91] J. Salvi, E. Rizzi, "Minimax optimization of Tuned Mass Dampers under seismic excitation", Proc. of 8th Int. Conf. on Structural Dynamics (EURODYN 2011), Leuven, Belgium, 4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.), Book of Abstracts, ISBN: 978-90-760-1931-4, p. 68; CD-ROM Proceedings, p. 1892–1899, 8 pages, 2011.

[92] J.Salvi, E.Rizzi, "Numerical tuning of Tuned Mass Dampers under earthquake loading", Technical Report, Università di Bergamo (Dalmine), Italy, ISBN: 978-88-905817-4-8, 35 pages, 2011.

[93] J. Salvi, E. Rizzi, "A numerical approach towards best tuning of Tuned Mass Dampers", Proc. of 25th Int. Conf. on Noise and Vibration Engineering (ISMA 2012), Leuven, Belgium, 17–19 September 2012, P. Sas, D. Moens,

S. Jonckheere(Eds.), Book of Abstracts, ISBN: 978-90-738-0289-6, p. 141; CD-ROM Proceedings, p. 2419-2434, 16 pages, 2012.

[94] J. Salvi, E. Rizzi, "Minimax tuning of Tuned Mass Dampers for frame structures under earthquake loading", submitted for publication, 2013.

[95] J. Salvi, E. Rizzi, E. Rustighi, N.S. Ferguson, "Analysis and optimisation of Tuned Mass Dampers for impulsive excitation", Proc. of the 11th Int. Conf. on Recent Advances in Structural Dynamics (RASD 2013), Pisa, Italy, July 1–3, 2013, E. Rustighi (Ed.), Book of Abstracts, p. 64, CD-ROM Proceedings, ISBN: 9780854329649, Paper ID 1002, p. 1-15, 15 pages, 2013.

[96] J. Salvi, E. Rizzi, E. Rustighi, N.S. Ferguson, "On the optimisation of a hybrid Tuned Mass Damper for impulse loading", submitted for publication, 2013.

[97] D. Siepe, C. Meinhardt, "Passive Damping Devices for Earthquake Protection of Bridges", Proc. of 8th Int. Conf. on Structural Dynamics (EURODYN 2011), Leuven, Belgium, 4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.), 7 pages, 2011.

[98] J.R. Sladek, R.E. Klingner, "Effect of tuned mass dampers on seismic response", *Journal of the Structural Division* (ASCE), 109(8):2004–2009, 1983.

[99] M. Smrz, R. Bastaits, A. Preumont, "Active Damping of a Cherenkov Gamma Ray Telescope", Proc. of 8th Int. Conf. on Structural Dynamics (EURODYN 2011), Leuven, Belgium, 4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.), 8 pages, 2011.

[100] S. Soheili, M. Abachizadeh, A. Farshidianfar, "Tuned mass dampers for earthquake oscillations of high-rise structures using Ant Colony Optimization technique", Proceedings of 18th Annual International Conference on Mechanical Engineering (ISME 2010), Tehran, Iran, 11–13 May 2010, pp. 1–6, 2010.

[101] T.T.Soong, "Active structural control in civil engineering", *Engineering Structures*, 10(2):74–84, 1988.

[102] T.T. Soong, G.F. Dargush, *Passive Dissipation Systems in Structural Engineering*, Wiley, 1st ed., 1997.

[103] T.T. Soong, B.F. Spencer Jr., "Supplemental energy dissipation: state-of-the-art and state-of-the-practice", *Engineering Structures*, 24(3):243–259, 2002.

[104] R. Soto-Brito, S.E. Ruiz, "Influence of ground motion intensity on the effectiveness of tuned mass dampers", *Earthquake Engineering and Structural Dynamics*, 28(11):1255–1271, 1999.

[105] B.F. Spencer Jr., M.K. Sain, "Controlling Buildings: A New Frontier in Feedback", *Control Systems Magazine*, 17(6):19–35, 1997.

[106] R. Steinbuch, "Bionic Optimisation of the Earthquake Resistance of High Buindings by Tuned Mass Dampers", *Journal of Bionic Engineering*, 8(3):335–344, 2011.

[107] C.Sun, S.Nagarajaiah, "Study on semi-active tuned mass damper with variable damping and stiffness under seismic excitations", The Structural Design of Tall and Special Buildings, Early view, 2014.

[108] P. Tan, Y. Liu, Y. Zhang, F. Zhou, J. Teng, Y. Xiao, "Application of Active-passive Hybrid Tuned Mass Damper for Canton Tower", Proc. of 5th European Conference on Structural Control (EACS 2012), Genoa, Italy, 18–20 June 2012, 7 pages, 2012.

[109] The MathWorks Inc., *MATLAB User's Guide and Optimization Toolbox*, USA, 2011.

[110] A.G. Thompson, "Optimizing the untuned viscous dynamic vibration absorber with primary system damping: a frequency locus method", Letter to the editor, *Journal of Sound and Vibration*, 73(3):469–472, 1980.

[111] A.G. Thompson, "Optimum tuning and damping of a dynamic vibration absorber applied to a force excited and damped primary system", *Journal of Sound and Vibration*, 77(3):403–415, 1981.

[112] O.F. Tigli, "Optimum vibration absorber (tuned mass damper) design for linear damped systems subjected to random loads", *Journal of Sound and Vibration*, 331(13):3035–3049, 2012.

[113] N. Tondini, O.S. Bursi, A. Bonelli, S. Francescotti, "Dynamic and aeroelastic behaviour of a twin deck curved cable-stayed footbridge equipped with passive devices", Proc. of 8th Int. Conf. on Structural Dynamics (EURODYN 2011), Leuven, Belgium, 4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.), 7 pages, 2011.

[114] A. Tributsch, C. Adam, T. Fürtmuller, "Mitigation of earthquake induced vibrations by Tuned Mass Dampers", Proceedings of 8th International Conference on Structural Dynamics (EURODYN 2011), Leuven, Belgium, 4–7 July 2011, G. De Roeck, G. Degrande, G. Lombaert, G. Müller (Eds.); CD-ROM Proceedings, pp. 1742–1749, 2011.

[115] A. Tributsch, C. Adam, "Evaluation and analytical approximation of Tuned Mass Dampers performance in an earthquake environment", *Smart Structures and Systems*, 10(2):155–179, 2012.

[116] H.-C. Tsai, G.-C. Lin, "Optimum tuned-mass dampers for minimizing steady-state response of support-excited and damped systems", *Earthquake Engineering and Structural Dynamics*, 22(11):957–973, 1993.

[117] H.-C. Tsai, G.-C. Lin, "Explicit formulae for optimum absorber parameters for force-excited and viscously damped systems", *Journal of Sound and Vibration*, 176(5):585–596, 1994.

[118] A. Ussia, O.S. Bursi, A. Bonelli, "Optimal Semi Active MR Control Strategies of a Footbridge Endowed with TMDs", Proc. of 5th European Conference on Structural Control (EACS 2012), Genoa, Italy, 18–20 June 2012, 11 pages, 2012.

[119] R. Villaverde, "Reduction in seismic response with heavily-damped vibration absorbers", *Earthquake Engineering and Structural Dynamics*, 13(1):33–42, 1985.

[120] R. Villaverde, N.M. Newmark, "Seismic response of light attachments to buildings", SRS No. 469, University of Illinois, Urbana (IL), February 1980.

[121] R. Villaverde, L.A. Koyama, "Damped resonant appendages to increase inherent damping in buildings", *Earthquake Engineering and Structural Dynamics*, 22(6):491–507, 1993.

[122] A.-P. Wang, R.-F. Fung, S.-C. Huang, "Dynamic analysis of a tall building with a tuned-mass-damper device subjected to earthquake excitation", *Journal of Sound and Vibration*, 244(1):123–136, 2001.

[123] J.-F. Wang, C.-C. Lin, C.-H. Lian, "Two-stage optimum design of tuned mass dampers with consideration of stroke", *Structural Control and Health Monitoring*, 16(1):55–72, 2009.

[124] G.B. Warburton, E.O. Ayorinde, "Optimum absorber parameters for simple systems", *Earthquake Engineering and Structural Dynamics*, 8(3):197–217, 1980.

[125] G.B. Warburton, "Optimum absorber parameters for minimizing vibration response", *Earthquake Engineering and Structural Dynamics*, 9(3):251–262, 1981.

[126] G.B. Warburton, "Optimum absorber parameters for various combinations of response and excitation parameters", *Earthquake Engineering and Structural Dynamics*, 10(3):381–401, 1982.

[127] P.H. Wirshing, G.W. Campbell, "Minimal structural response under random excitation using the vibration absorber", *Earthquake Engineering and Structural Dynamics*, 2(4):303–312, 1974.

[128] K.K.F. Wong, "Seismic Energy Dissipation of Inelastic Structures with Tuned Mass Dampers", *Journal of Engineering Mechanics* (ASCE), 134(2):163–172, 2008.

[129] K.K.F. Wong, Y.L. Chee, "Energy dissipation of tuned mass dampers during earthquake excitations", *The Structural Design of Tall and Special Buildings*, 13(2):105–121, 2004.

[130] W.O. Wong, Y.L. Cheung, "Optimal design of a damped dynamic vibration absorber for vibration control of structure excited by ground motion", *Engineering Structures*, 30(1):282–286, 2008.

[131] J.N. Yang, T.T. Soong, "Recent advances in active control of civil engineering structures", *Probabilistic Engineering Mechanics*, 3(4):179–188, 1988.

[132] M. Zilletti, S.J. Elliott, E. Rustighi, "Optimisation of dynamic vibration absorbers to minimise kinetic energy and maximise internal power dissipation", *Journal of Sound and Vibration*, 331(18):4093–4100, 2012.

[133] K. Zhou, J.C. Doyle, K. Glover, *Robust and Optimal Control*, Prentice Hall, 1st ed., 1996.

[134] L. Zuo, S.A. Nayfeh, "Minimax optimization of multi-degree-of-freedom tuned-mass dampers", *Journal of Sound and Vibrations*, 272(3–5):893–908, 2004.

[135] L. Zuo, S.A. Nayfeh, "The Two-Degree-of-Freedom Tuned-Mass Damper for Suppression of Single-Mode Vibration Under Random and Harmonic Excitation", *Journal of Vibrations and Acoustics*, 128(1):56–65, 2006.

Appendix

In the following, a representative sample of the codes assembled within a MATLAB environment, for the solution of the tuning problems presented all along the thesis, is reported. In particular, the codes list the main scheme of the numerical implementations developed within this research work for the following topics:

- Optimisation of the passive TMD for ideal loading (Chapter 3);
- Tuning of the passive TMD for impulse loading (Chapter 4);
- Tuning of the hybrid TMD for impulse loading (Chapter 4);
- Seismic tuning of the passive TMD (Chapter 5).

```
%% OPTIMISATION OF THE TMD FOR IDEAL LOADING %%

%% Initialisations
close all; clear all; clc; format long;

%% Numerical optimisation
% Variables
mu=0.0025:0.0025:0.1;
Zs=[0:0.0025:0.05,0.055:0.005:0.1];
g=0:0.0002:2; % frequency range

% Options, bounds and constraints
lb=[1e-5;1e-5]; ub=[5;5];
options=optimset('Display','iter',...
        'TolX',1e-6,'TolFun',1e-6,'TolCon',1e-6,...
        'MaxFunEvals',300,'MaxIter',300);

% Optimisation loop
for i=1:length(mu);
    for n=1:length(Zs);
x0=[1./(1+mu(i));
    sqrt(3*mu(i)/(8*(1+mu(i))))]; % Den Hartog tuning
v=[mu(i) Zs(n)];
[x,fval,maxfval]=fminimax(@(x) HF_OF(x,v),x0,...
                 [],[],[],[],lb,ub,[],options);

As=x(1).^2-g.^2;
At=x(1).^2;
Ats=g.^2;
Bs=2.*x(2).*x(1).*g;
Bt=2.*x(2).*x(1).*g;
```

```
Bts=0;
C=g.^4-g.^2.*(1+x(1).^2.*(1+v(1))+4.*v(2).*x(2).*x(1))+x(1).^2;
D=-g.^3.*(2.*x(2).*x(1).*(1+v(1))+2.*v(2))+...
    g.*(2.*x(2).*x(1)+2.*v(2).*x(1).^2);
Xs=max(sqrt((As.^2+Bs.^2)./(C.^2+D.^2)));
Xt=max(sqrt((At.^2+Bt.^2)./(C.^2+D.^2)));
Xts=max(sqrt((Ats.^2+Bts.^2)./(C.^2+D.^2)));

RT(i,8*n-7:8*n)=[Zs(n) mu(i) x(1) x(2) Xs Xt Xts 0];
f(n,i)=x(1); Zt(n,i)=x(2);
vXs(n,i)=Xs; vXt(n,i)=Xt; vXts(n,i)=Xts;
    end
end

%% THE END %%
```

```
%% IMPULSE TUNING OF THE PASSIVE TMD %%

%% Initialisations
close all; clear all; clc; format long;

%% Optimisation parameters and options
% Time vector
t0=0; ta=10; fs=1e3; ni=fs*ta; dt=1/fs; t=t0:dt:ta;

% Structural parameters
m1=100; k1=10000; w1=sqrt(k1/m1);
mu=0.0025:0.0025:0.1; z1=[0,0.01,0.02,0.03,0.05];

% Optimisation options
lb=[0.001*w1;0.001]; ub=[5*w1;1];
options=optimset('Display','iter','Algorithm','sqp',...
        'TolX',1e-10,'TolFun',1e-10,'TolCon',1e-10,...
        'MaxIter',50,'MaxFunEvals',300);

% Result vectors
f=zeros(length(z1),length(mu)); z2g=f; dresp_H2=f;

for l=1:length(mu);
    for n=1:length(z1);
%% Structural parameters
% SDOF system
c1=2*z1(n)*sqrt(k1*m1); Hsn=1; Hsd=[m1,c1,k1];

% 2DOF system
m2=mu(l)*m1; w2=w1; k2=w2^2*m2; z2=10/100; c2=2*z2*sqrt(k2*m2);
```

```
% Impulse loading
Xg=0.01; % amplitude of the base displacement [m]
F1s=Xg*[c1 k1]; F2s=[0 0];

% Transfer function
N1=conv(F1s,Hsn); D=Hsd; [res1,p,]=residue(N1,D);

% Time response
xt1=zeros(length(p),length(t)); xdt1=xt1; xddt1=xt1;
for i=1:length(p);
    xt1(i,:)=res1(i)*exp(p(i)*t);
    xdt1(i,:)=p(i)*xt1(i,:); xddt1(i,:)=p(i)^2*xt1(i,:);
end
xt1=sum(xt1); xdt1=sum(xdt1); xddt1=sum(xddt1);

% Time response (analytical formulation, base displacement)
xt1=exp(-c1./(2*m1).*t)./(m1.*sqrt(4*k1*m1-c1^2)).*...
    ((2*k1*m1-c1^2).*sin(sqrt(4*k1*m1-c1^2)./(2*m1).*t)+...
    (c1.*sqrt(4*k1*m1-c1^2)).*cos(sqrt(4*k1*m1-c1^2)./(2*m1).*t));

% Peak response (analytical formulation)
tmax=2*m1/sqrt(4*m1*k1-c1^2)*atan((c1^2-m1*k1)*sqrt(4*m1*k1-c1^2)/...
     (c1^3-3*m1*c1*k1));
xmax=sqrt(k1/m1)*sqrt((c1^2-3*m1*k1)^2)/(3*m1*k1-c1^2)*...
     exp(-c1/sqrt(4*m1*k1-c1^2)*...
     atan((c1^2-m1*k1)*sqrt(4*m1*k1-c1^2)/(c1^3-3*m1*c1*k1)));

% Optimisation loop (w2, z2)
x0=[w2; z2];
[x]=fmincon(@(x) of_p(x,t,m1,c1,k1,m2,F1s,F2s),x0,[],[],[],[],...
    lb,ub,[],options);
```

```
% TMD parameters
w2=x(1); z2=x(2); k2=w2^2*m2; c2=2*z2*sqrt(k2*m2);

% Transfer function
Hsn11p=[m2,c2,k2]; Hsn12p=[0,c2,k2];
Hsn21p=[0,c2,k2]; Hsn22p=[m1,c1+c2,k1+k2];
Hsdp=[m1*m2,c1*m2+c2*(m1+m2),c1*c2+k1*m2+k2*(m1+m2),...
      c1*k2+c2*k1,k1*k2];

% Time response
[xt1p,xdt1p,xddt1p,xt2p,xdt2p,xddt2p]=...
iltr_2dof(t,F1s,F2s,Hsn11p,Hsn12p,Hsn21p,Hsn22p,Hsdp);

dr_H2=(norm(xt1)-norm(xt1p))/norm(xt1); frat=w2/w1;
f(n,l)=frat; z2g(n,l)=x(2); dresp_H2(n,l)=dr_H2;
    end
end

%% THE END %%
```

```
%% IMPULSE TUNING OF THE HYBRID TMD %%

%% Initialisations
close all; clear all; clc; format long;

%% Optimisation parameters and options
% Time vector
t0=0; ta=10; fs=1e3; ni=fs*ta; dt=1/fs; t=t0:dt:ta;

% Structural parameters
m1=100; k1=10000; w1=sqrt(k1/m1); mu=0.02; z1=0.05;

% Optimisation options
options=optimset('Display','iter','Algorithm','sqp','TolX',1e-10,...
        'TolFun',1e-10,'TolCon',1e-10,'MaxIter',50,'MaxFunEvals',300);

%% Passive TMD
% Structural parameters
c1=2*z1*sqrt(k1*m1); Hsn=1; Hsd=[m1,c1,k1];
m2=mu*m1; w2=w1; z2=10/100;

% Impulse loading
Xg=0.01; % amplitude of the base displacement [m]
F1s=Xg*[c1 k1]; F2s=[0 0]; % base displacement

% Transfer function
N1=conv(F1s,Hsn); D=Hsd; [res1,p,]=residue(N1,D);

% Time response
xt1=zeros(length(p),length(t)); xdt1=xt1; xddt1=xt1;
for i=1:length(p);
```

```
    xt1(i,:)=res1(i)*exp(p(i)*t);
    xdt1(i,:)=p(i)*xt1(i,:); xddt1(i,:)=p(i)^2*xt1(i,:);
end
xt1=sum(xt1); xdt1=sum(xdt1); xddt1=sum(xddt1);

% Optimisation loop (w2, z2)
x0=[w2; z2]; lb=[0.001*w1;0.001]; ub=[5*w1;1];
[x]=fmincon(@(x) of_p(x,t,m1,c1,k1,m2,F1s,F2s),x0,[],[],[],[],...
    lb,ub,[],options);

% TMD parameters
w2=x(1); z2=x(2); k2=w2^2*m2; c2=2*z2*sqrt(k2*m2); f=w2/w1;

% System matrices
M=[m1 0;0 m2]; C=[c1+c2 -c2;-c2 c2]; K=[k1+k2 -k2;-k2 k2];

% Transfer function
Hsn11p=[m2,c2,k2]; Hsn12p=[0,c2,k2];
Hsn21p=[0,c2,k2]; Hsn22p=[m1,c1+c2,k1+k2];
Hsdp=[m1*m2,c1*m2+c2*(m1+m2),c1*c2+k1*m2+k2*(m1+m2),...
      c1*k2+c2*k1,k1*k2];

[xt1p,xdt1p,xddt1p,xt2p,xdt2p,xddt2p]=...
iltr_2dof(t,F1s,F2s,Hsn11p,Hsn12p,Hsn21p,Hsn22p,Hsdp);

%% Hybrid TMD
% Selection of the control law
fcl=1;

% Gains optimisation
if fcl==1;
```

```
gv=20; gd=12000; lb=[0;10000]; ub=[50;25000]; x0h=[gv; gd];
[xh]=fmincon(@(x) of_h(x,t,m1,c1,k1,m2,c2,k2,F1s,F2s,fcl),x0h,...
             [],[],[],[],lb,ub,[],options);
gv=xh(1); gd=xh(2); sprintf('gv=%g, gd=%g',gv,gd)
Mc=M; Cc=C+[gv 0; -gv 0]; Kc=K+[gd 0; -gd 0];

% Closed loop poles
cl_poles=polyeig(Kc,Cc,Mc); % closed loop poles

% Transfer function
Hsn11h=[m2,c2,k2]; Hsn12h=[0,c2+gv,k2+gd];
Hsn21h=[0,c2+gv,k2]; Hsn22h=[m1,c1+c2+gv,k1+k2+gd];
Hsdh=[m1*m2,c1*m2+c2*(m1+m2)+gv*m2,...
      c1*c2+k1*m2+k2*(m1+m2)+gd*m2,c1*k2+c2*k1,k1*k2];

% Time response
[xt1h,xdt1h,xddt1h,xt2h,xdt2h,xddt2h]=...
iltr_2dof(t,F1s,F2s,Hsn11h,Hsn12h,Hsn21h,Hsn22h,Hsdh);

% Control force
fct=gv*xdt1h+gd*xt1h;

elseif fcl==2;
ga=-90; gv=30; lb=[-95;20]; ub=[-85;40]; x0h=[ga; gv];
% ga=-90; gv=30; lb=[-90;20]; ub=[-75;45]; x0h=[ga; gv];
[xh]=fmincon(@(x) of_h(x,t,m1,c1,k1,m2,c2,k2,F1s,F2s,fcl),x0h,...
     [],[],[],[],lb,ub,[],options);
ga=xh(1); gv=xh(2); sprintf('ga=%g, gv=%g',ga,gv)
Mc=M+[ga 0; -ga 0]; Cc=C+[gv -gv; -gv gv]; Kc=K;

% Closed loop poles
```

```
cl_poles=polyeig(Kc,Cc,Mc); % closed loop poles

% Transfer function
Hsn11h=[m2,c2+gv,k2]; Hsn12h=[ga,c2+gv,k2];
Hsn21h=[0,c2+gv,k2]; Hsn22h=[m1+ga,c1+c2+gv,k1+k2];
Hsdh=[m1*m2+ga*m2,c1*m2+c2*(m1+m2)+gv*(m1+m2),...
      c1*c2+k1*m2+k2*(m1+m2)+gv*c1,c1*k2+c2*k1+gv*k1,k1*k2];

% Time response
[xt1h,xdt1h,xddt1h,xt2h,xdt2h,xddt2h]=...
iltr_2dof(t,F1s,F2s,Hsn11h,Hsn12h,Hsn21h,Hsn22h,Hsdh);

% Control force
fct=ga*xddt1h+gv*(xdt1h-xdt2h);

end

%% THE END %%
```

```
%% SEISMIC TUNING OF THE TMD %%

%% Initialisations
close all; clear all; clc; format long;

%% Seismic analysis of the primary structure
% Constants and primary structure parameters
g=9.8066; run Structures/sp_01_100;

% Seismic input
run Strong_motions/Accel_LBE1933_LBE_NS;
acc=reshape(acc_LBE1933_LBE_NS',[],1);
dt=dt_acc_LBE1933_LBE_NS; ta=0:dt:dt*length(acc)-dt;

% Number of degrees of freedom and initial conditions
u0=zeros(ndof,1); ud0=zeros(ndof,1);

% Evaluation and assignment of seismic forces
F=zeros(ndof,length(accp));
for i=1:length(accp);
  F(:,i)=-M*ones(ndof,1)*accp(i);
end

%% Dynamic response (time domain)
[u,ud,udd]=newmark(M,C,K,u0,ud0,F,dt,ta);
[u_tst_Hinf,u_tst_H2,ud_tst_Hinf,ud_tst_H2,...
udd_tst_Hinf,udd_tst_H2,Ee,Te,De]=seisres(u,ud,udd,M,K,C,accp);

% PSa - periods
PSa_T=lint_PSaT(T,PSa,Trs);
```

```
%% Optimization and seismic analysis of the structural system
% Variable parameters
mu=0.02; % mass ratio

% Options and constraints
x0=[1;0.1]; lb=[0.7;0.001]; ub=[1.1;0.1];
options=optimset('Display','iter','Algorithm','interior-point',...
        'TolX',1e-10,'TolFun',1e-10,'MaxFunEvals',500,'MaxIter',500);

for r=1:length(mu);
    for n=1:length(Z);
v=[mu(r) Z(n)];
x0=[1/(1+mu(r));sqrt(3*mu(r)/(8*(1+mu(r))))];

% Optimization algorithm
[x]=fmincon(@(x) objfun(x,v,ndof,M,K,C,w,Mm,accp,dt,ta),...
    x0,[],[],[],[],lb,ub,[],options);

% TMD and structural system parameters
[M_tmd,K_tmd,C_tmd,ndof_ss,M_ss,K_ss,C_ss]=...
                  tmdpar(x,v,ndof,M,K,C,w,Mm);

% Boundary conditions
u0_ss=zeros(ndof_ss,1); ud0_ss=zeros(ndof_ss,1);

% Evaluation and assignment of seismic forces
F_ss=zeros(ndof_ss,length(accp));
for i=1:length(accp);
    F_ss(:,i)=-M_ss*ones(ndof_ss,1)*accp(i);
end
```

```
% Dynamic response
[u_ss,ud_ss,udd_ss]=newmark(M_ss,C_ss,K_ss,u0_ss,ud0_ss,F_ss,dt,ta);
[u_ss_tst_Hinf,u_ss_tst_H2,ud_ss_tst_Hinf,ud_ss_tst_H2,...
udd_ss_tst_Hinf,udd_ss_tst_H2,u_tmd_Hinf,u_tmd_H2,...
ud_tmd_Hinf,ud_tmd_H2,udd_tmd_Hinf,udd_tmd_H2,Ee_pt,Te_pt,De_pt]=...
seisres_ss(u_ss,ud_ss,udd_ss,M,K,C,accp);

% Table of results
RT(n*length(mu)-(length(mu)-r),1:28)=[Z(n) mu(r) x(1) x(2)...
u_ss_tst_Hinf 100*(u_tst_Hinf-u_ss_tst_Hinf)/u_tst_Hinf...
ud_ss_tst_Hinf 100*(ud_tst_Hinf-ud_ss_tst_Hinf)/ud_tst_Hinf...
udd_ss_tst_Hinf 100*(udd_tst_Hinf-udd_ss_tst_Hinf)/udd_tst_Hinf...
u_ss_tst_H2 100*(u_tst_H2-u_ss_tst_H2)/u_tst_H2...
ud_ss_tst_H2 100*(ud_tst_H2-ud_ss_tst_H2)/ud_tst_H2...
udd_ss_tst_H2 100*(udd_tst_H2-udd_ss_tst_H2)/udd_tst_H2...
max(Ee_pt) 100*(max(Ee)-max(Ee_pt))/max(Ee)...
max(Te_pt) 100*(max(Te)-max(Te_pt))/max(Te)...
max(De_pt) 100*(max(De)-max(De_pt))/max(De)...
norm(Ee_pt) 100*(norm(Ee)-norm(Ee_pt))/norm(Ee)...
norm(Te_pt) 100*(norm(Te)-norm(Te_pt))/norm(Te)...
norm(De_pt) 100*(norm(De)-norm(De_pt))/norm(De)];

% Modal analysis
[eval_ss,evec_ss,w_ss,f_ss,T_ss,Mm_ss,Rho_ss,Mmeff_ss]=...
                                        modan(M_ss,K_ss);

% PSa - periods
PSa_T_ss=lint_PSaT(T_ss,PSa,Trs);
    end
end
```

%% THE END %%