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On the bright side of correlation information transmission*

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Abstract

We consider a situation in which a decision maker solicits information from two partially informed experts with uncertain biases. Experts' private information about an underlying state might be conditionally correlated across them. We show that although correlation tightens the conditions on preferences for a truth-telling equilibrium, in the presence of uncertainty, a high level of correlation may discipline the strongly biased expert and may foster the informational content of the experts' advice under high uncertainty about correlation. In contrast to what may be expected, it may be optimal for the decision maker to consult two experts with correlated information than consulting two experts with independent information. This result suggests that getting independent opinions may not always be helpful for decision-making.

JEL Classification: C72, D82, D83.

Keywords: Cheap Talk, Uncertain Preferences, Correlated Information.

*All remaining errors are ours.

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1 Introduction

Decision makers often solicit experts' advice to make sound decisions. However, experts may be biased towards specific directions, usually unknown to the decision maker. This issue is important, especially when the communication is cheap and unverifiable, since biased experts may strategically alter their advice, pushing the decision makers in a specific direction. For instance, consider e-commerce platforms such as Taobao, eBay, and Amazon. Such platforms use a feedback system that provides information regarding the seller or the product to help potential consumers make the right purchasing decisions. However, due to their unverifiability, online reviews create a strong incentive to distort them, resulting in biased reviews. More recently, it has been documented that the Trump administration consulted two different groups of advisors before taking an effective measure to combat the COVID-19 pandemic in early 2020: the White House Coronavirus Task Force and the Physician Advisory Group. However, besides their common educational background, both groups of scientific advisers ended up giving conflicting recommendations.[1] These examples suggest that bias uncertainty may prevent effective decision-making since it may limit the decision maker's ability to draw inferences after receiving several opinions.

Many existing models explain how, and under which conditions, experts with uncertain preferences might transmit information to a decision maker (See, for instance, Li and Madarasz, 2008, and Karakoç, 2022, among many others). However, most of these models assume that the experts' private information comes from independent sources. Still, little is known about the communication when experts' private information comes from correlated sources, limiting the decision maker's ability to react to multiple experts' opinions. Under which conditions are experts incentivized to share their correlated private information with the decision maker? Can correlated information sources ease information transmission with uncertain biases? And, as an outsider, is it better for the decision maker to avoid correlated private information or uncertainty on experts' preferences when seeking information?

In this paper, we analyze a cheap talk model adapted from Austen-Smith (1993), in which an uninformed decision maker solicits advice from two partially informed experts whose private information is (conditionally) correlated across players. Each expert receives a private signal about the state of the world and then provides information to the decision maker through simultaneous cheap talk.[2] The experts' may differ in terms of how biased they are relative to the decision maker. Since an expert's bias measures how distant his preferences are relative to the decision maker, we introduce two types of experts: a moderate expert and a radical expert. The difference between the two types is that, in absolute terms, the moderate expert is assumed to be less biased than a radical expert. Crucially, we assume the experts' private signals might be (positively) correlated, which is a natural assumption in many real-life situations. One could think that people with similar preferences often generate strongly correlated information. For instance, in online market places, a seller may write a

[1] See, e.g., https://www.science.org/content/article/inside-story-how-trumps-covid-19-coordinator-undermined-cdc

[2] For ease of discussion, hereafter, I refer to the decision maker as "she," and each expert "he."

review on both his and his competitor's product to manipulate reviews.[3]

Building on this insight, we focus on two informative equilibria in which the decision-maker may extract some information from the experts. As a benchmark, we consider a conflict-revealing equilibrium in which experts of either type truthfully reveal their privately observed signals about the state of the world and the decision-maker believes them. We then turn our attention to a conflict-hiding equilibrium in which the moderate expert is willing to communicate truthfully to the decision-maker about his privately observed signal, while the radical expert reports the same message irrespective of his private information.

We first analyze the experts' truth-telling incentives. We show that in a conflict-revealing equilibrium, the interval that supports truth-telling as an equilibrium depends only on the degree of correlation and not on the uncertain biases. Specifically, the higher the correlation, the more likely the experts' private information comes from the same sources, hence the lower the impact of receiving identical messages from the experts. In this case, the correlation limits the gains from consulting multiple sources, weakening the experts' incentives to reveal truthful information to the decision maker. It is important to notice that, in a conflict-revealing equilibrium, correlation matters only if the decision maker receives identical messages. The reason is that when the decision maker receives conflicted messages, she can infer with certainty that the messages come from independent sources. Hence, experts' correlated information has no consequence on her action. By contrast, in a conflict-hiding equilibrium, the decision maker is skeptical about the informational content of the experts' messages, negatively affecting the experts' incentives to disclose truthful information. In fact, uncertain biases shrink the interval that supports truth-telling as equilibrium compared to the one in a conflict-revealing equilibrium. This happens because, in a conflict-hiding equilibrium, bias uncertainty erodes the credibility of an expert's message. Consequently, the experts' messages have a lower impact on the decision maker's action. This, of course, provides an incentive to lie, making the truth-telling condition tighter. Finally, in a conflict-hiding equilibrium, correlation matters not only upon receiving identical messages but also mixed messages because of the bias uncertainty.

We show that in both conflict-revealing and conflict-hiding equilibria, information transmission is more difficult under correlated information. When the experts' private information is correlated, each expert knows that his report is less relevant in affecting the decision maker's final action. Hence, considering the experts' interaction between bias and information, the transmission of truthful information is mainly shaped by the so-called *overshooting effect* (Morgan and Stocken, 2008): the decision maker's reaction to undetectable lies from the experts. The stronger is the correlation, the weaker is the overshooting effect, and the lower the reaction of the decision maker through optimal actions. Instead, in a conflict-hiding equilibrium, there is also a novel effect shaping the expert's incentives to communicate truthfully with the decision-maker, i.e., the *uncertain biases effect.* Specifically, the uncertain biases effect reflects an expert's relative gain from the decision maker's skepticism. The higher the degree of bias uncertainty, the higher the magnitude of the uncertain biases effect; hence, the weaker is the moderate expert's incentive to communicate truthfully. Which of the two effect

[3]See, e.g., Amazon's customer product reviews policies at https://sellercentral.amazon.co.uk/.

dominate depends on the relative likelihood of receiving distorted messages by the radical type and the degree of correlation.

We then develop a welfare analysis. As a welfare measure, we use the ex-ante expected utility of the decision maker to measure how much information she may extract from the experts after simultaneous communication within and across equilibria. We first make a welfare comparison within equilibria to pin down the effect of correlation on information transmission under bias uncertainty. Interestingly, we show that a conflict-hiding equilibrium with correlated information is informationally superior to a conflict-hiding equilibrium with independent information. The trade-off that the decision maker faces is the following. First, correlation has a direct negative effect on information transmission since it weakens the benefits of consulting distinct experts. Second, it has an indirect positive effect because, in the presence of bias uncertainty, the decision maker can use correlation as a tool to discipline the experts with uncertain preferences to communicate truthfully. Surprisingly, when both the degree of bias uncertainty and the degree of correlation is sufficiently large, the indirect effect dominates, and correlated information transmission helps the decision maker extract superior information from potentially biased experts.

We then make a welfare comparison across equilibria to understand, from the decision maker's perspective, whether it is better to avoid correlation among experts' private information or uncertainty about experts' biases. To answer this question, we compare a conflict-revealing equilibrium with correlated information (so that bias uncertainty is absent) with a conflict-hiding equilibrium with independent information (so correlated information is absent). Interestingly, when both bias uncertainty and the degree of correlation are large, the decision maker may prefer to avoid correlated information transmission. The reason is that, although noisy information erodes the credibility of the experts' messages, the decision maker's marginal gain from talking to independent experts fully internalizes her marginal loss due to receiving correlated opinions. Instead, when the bias uncertainty is large, while the degree of correlation is low, the decision maker may prefer to avoid receiving noisy information. In that case, the decision maker faces two opposing forces. On the one hand, a low degree of correlation strengthens the overshooting effect, pushing the moderate expert's incentive to communicate truthfully. On the other hand, a large bias uncertainty makes stronger the uncertain biases effect, weakening the moderate expert's incentive to communicate truthfully. On balance, under a low degree of correlation, the overshooting effect dominates the uncertain biases effect, enabling the decision maker to better deal with the bias uncertainty. Hence, our results show that experts' correlated private information may have a bright side in information transmission.

The rest of the paper is organized as follows. After discussing the related literature, Section 2 describes the model. While Section 3 describes the equilibrium. Section 3.1 provides some preliminary insights, Section 3.2 characterizes the conditions under which a conflict-revealing equilibrium exists and Section 3.2 characterizes the conditions under which a conflict-hiding equilibrium exists. Welfare analysis is illustrated in Section 4. The last section concludes. All proofs are in the Appendix.

Related Literature.

Our paper is built on and contributes to two strands of literature on cheap talk. On the one hand, it is related to the contributions of multiple experts and uncertain individual preferences. On the other hand, it is related to a small subset of the literature dealing with the correlation of signals.

In the first stream of the literature, the analysis in the present paper is mainly related to that in Austen-Smith (1993), who considers a uniform state space and assumes that the experts are partially informed about the underlying state, as this paper does. However, we allow the decision maker to be uncertain about the experts' biases as well as correlated information between the experts. Few papers focus on the informativeness of costless communication with uncertain biases. Morgan and Stocken (2003), Li (2004), Dimitrakas and Sarafidis (2005) and Li and Madarász (2008), among many others, show that the revelation of the expert's bias weakens the communication when the magnitude of the bias is uncertain. These results are partially in line with our insights, *about the role of uncertainty*, suggesting that the transparency of biases improves information transmission and increases experts' incentives to communicate truthfully due to the risk of overshooting the decision maker's ideal action. Furthermore, let us quote a growing literature on the experts' reputational concerns - e.g. Sobel (1985), Bénabou and Laroque (1992), Morris (2001), Gentzkow and Shapiro (2006), Ottaviani and Sørensen (2006)- even if this paper does not address this issue.

The second niche of the literature is related to the tension between information extraction and information aggregation. For instance, Battaglini (2004) shows that a decision maker may reduce noise by aggregating a larger number of signals, but at the cost of receiving less truthful signals from each expert. Gerardi et al. (2009) develop a mechanism by which the decision-maker achieves almost full information extraction when experts' signals are accurate and the number of experts is high. In our paper, we take a slightly different perspective, which is closer to the approach of Currarini et al. (2020). Specifically, they consider a situation in which a decision maker gathers information from imperfectly informed experts, receiving coarse signals about a uniform state of the world, and their private information is correlated. In this framework, they show when the number of senders is not too small, an increase of correlation may be beneficial to the receiver via its effect on the likelihood of informative communication. We add some new insights to this result. By introducing the uncertainty on the information bias, we show that, even with a small number of experts, under some conditions, the correlation of information across players enhances the informative value of the messages.

2 The model

Players and Decision Sequence. Consider a (female) decision maker**,** D who solicits information from two experts $E_i, i = 1, 2$. The decision maker takes a payoff relevant action $y \in \mathbb{R}$, which depends on an underlying state of the world θ, a random variable uniformly distributed on the unit interval $[0, 1]$, with $f(\theta) = 1$. The decision maker has no further information about θ, and each expert privately observes an information signal about the realization of $s_i \in \{0, 1\}$ about the realization of θ. Clearly, due to uniform distribution, the marginals are $\Pr[s_i] = \frac{1}{2}, \forall s_i \in \{0, 1\}$.

Following Austen-Smith (1993), we assume that signals are equally informative for the experts, and hence, each signal has the following conditional probability

$$\Pr[s_i|\theta] = \theta^{s_i}(1-\theta)^{1-s_i}, \quad s_i \in \{0,1\}.$$

Although each expert is privately informed about the state of the world, we allow their signals to be positively correlated. Specifically, conditional on the state of the world, the signals are drawn from the following joint distribution

- $\Pr[s_i = 0, s_j = 0|\theta] = (1-\theta)k + (1-\theta)^2(1-k)$,
- $\Pr[s_i = 0, s_j = 1|\theta] = \Pr[s_i = 1, s_j = 0|\theta] = \theta(1-\theta)(1-k)$,
- $\Pr[s_i = 1, s_j = 1|\theta] = \theta k + \theta^2(1-k)$.

The parameter $k \geqslant 0$ captures the degree of correlation between the signals and is assumed to be state-independent. Therefore, the experts information may come from the same source, which happens with probability k, or from independent sources with a complementary probability. Nevertheless, we assume that the experts do not know ex-post whether they drew identical signals.

We focus on the case of simultaneous communication in which, based on the realized signal, each expert then simultaneously reports a binary message $m_i \in \{0,1\}$ to the decision maker. After receiving the messages, D takes her payoff relevant action denoted by $y(m_i, m_j)$.

As standard in the cheap talk literature, we consider quadratic loss utility functions[4] Specifically, D's utility is

$$\mathcal{U}_D(y,\theta) = -(y-\theta)^2,$$

and E_i's utility is

$$\mathcal{U}_i(y,\theta,b_i) = -(y-\theta-b_i)^2, \quad i = 1,2,$$

where the constant b_i denotes the expert i's bias.

The quadratic loss specification has two important implications. First, it guarantees the concavity of D's objective function and uniqueness of the optimal action. Second, it allows us to obtain tractable closed form solutions. Hence, given the quadratic loss specification, it is straightforward to see that experts' preferences are not perfectly aligned with those of the decision maker: in state θ, the decision maker's preferred action is θ, the expert i's preferred action is $\theta + b_i$.

The parameter $b_i \in \{b_M, b_R\}$, instead, represents E_i's individual preferences and it measures how distant E_i's preferences are to those of the decision maker, which we normalized to zero. We consider two types of experts; moderate or radical. Specifically, when $b_i = b_M$, then expert E_i is moderately biased vis-à-vis the decision maker. In that case, E_i is called a moderate expert. Instead, when $b_i = b_R$, then E_i is extremely biased relative to the decision maker, and in that case, E_i is called a radical expert. Moreover, the moderate type is assumed to be less biased than the radical type — i.e., such that $|b_M| < b_R$. We assume that

[4]See, e.g., Crawford and Sobel, 1982; Austen-Smith, 1993: Morgan and Stocken, 2008; for a similar approach.

the radical expert is rightward biased so that he wants as high action as possible relative to the decision maker.[5]

Crucially, we assume that the experts' biases are not common knowledge in the sense that each expert is privately informed about his own bias, which is drawn from the following distribution:

$$b_i \equiv \begin{cases} b_M & \text{with probability } \nu \\ b_R & \text{with probability } 1-\nu \end{cases} \qquad i = 1, 2.$$

Hence, while E_i knows his own bias, D and E_j hold only prior about that. Both experts and the decision maker are expected utility maximizers.

Timing. The timing of the game is as follows:

- Nature randomly chooses θ according to the uniform distribution on the unit interval.
- Nature privately informs each expert about his type.
- Each expert privately observes signals.
- Experts simultaneously send messages to the decision maker.
- The decision maker receives the messages and then takes an action $y \in \mathbb{R}$.

Equilibrium. The solution concept we adopt here is Perfect Bayesian Equilibrium (PBE). Multiple equilibria may exist, which is a common feature of cheap talk games.[6] However, this paper is interested in information transmission from the informed parties to the uninformed one. In the analysis that follows, we are going to focus on two classes of informative equilibria. First, we characterize the conflict-revealing equilibrium (or truth-telling equilibrium) as a benchmark. In this equilibrium, experts of either type reveal their privately observed signals truthfully to the decision maker and the decision maker believes them. Secondly, and interestingly, we characterize the conflict-hiding equilibrium in which the moderate type truthfully reports his private signal to the decision maker while the radical expert hide his type, and accordingly, sends the same message taking into account his biased preferences.

3 Equilibrium Analysis

3.1 Conflict-Revealing Benchmark

As a benchmark, first consider an equilibrium in which experts of either type truthfully reveal their privately observed signals about the state of the world — i.e., such that $m_i = s_i, i = 1, 2$ — and D believes them. In that case, uncertainty about the expert's types will have no consequence on the optimal actions as D believes that experts truthfully report their private signals regardless of their type. In fact, D's utility maximizing action as a response to such strategy is given by

[5] Since the experts are ex-ante symmetric and the message space is binary, this assumption is without loss of generality.

[6] In particular, a pooling equilibrium always exists, in which all messages from the experts are uninformative. In that case, no information is being transmitted from the experts to the decision maker, and hence, this equilibrium is outside the scope of this paper.

$$\begin{aligned} y^T_{s_i,s_j} &\equiv \arg\max_{y\in\mathbb{R}} \int_\theta -(y-\theta)^2 f(\theta|s_1,s_2)\,d\theta, \\ &= \mathbb{E}[\theta|s_i,s_j], \end{aligned} \tag{1}$$

where, abusing slightly notation $y^T(s_i, s_j)$ denotes the optimal action taken by the decision maker after being truthfully informed about the signals. Consequently, D's optimal action coincides with her conditional expectation of θ. Simple algebra yields the decision maker's optimal actions in a conflict-revealing equilibrium (See the appendix)

$$y^T_{0,0} = \frac{1+k}{2(2+k)}, \qquad y^T_{0,1} = y^T_{1,0} = \frac{1}{2}, \qquad y^T_{1,1} = \frac{3+k}{2(2+k)}.$$

Hence, in a conflict-revealing equilibrium, the expert's types have no impact on the optimal actions since D believes that experts truthfully report their private signals regardless of their type. However, D's optimal actions depend on the parameter k due to the possible correlation between the experts' information sources. Notice that correlation matters only when D receives identical messages from the experts, reflected by the probability k, which reflects the probability that the experts' information may come from identical sources. The higher is k, the more likely that the experts' private information is coming from the same source, the lower is the impact of the identical messages from the experts. In other words, when D receives the same signals, due to correlation, she has a less precise idea regarding the state, shifting the optimal action leftward when she receives $(s_i, s_j) = (0, 0)$, and rightward when she receives $(s_i, s_j) = (1, 1)$ from the experts, — i.e.,

$$\frac{dy^T_{0,0}}{dk} > 0, \quad \text{and} \quad \frac{dy^T_{1,1}}{dk} < 0,$$

By contrast, in the case of mixed signals, experts' correlated information has no consequence on the optimal actions since D can infer with certainty that the messages come from independent sources. Consequently, she takes an action based on her prior beliefs about the state.

In a conflict-revealing equilibrium, the fact that experts are ex ante identical, there exists a conflict-revealing equilibrium in which E_i has an incentive to reveal his information rather than reporting false information if and only if

$$\sum_{s_j\in\{0,1\}} \mathbb{E}_\theta\left[\mathcal{U}\left(y^T_{s_i,s_j},\theta,b_i\right)|s_i\right] \geqslant \sum_{s_j\in\{0,1\}} \mathbb{E}_\theta\left[\mathcal{U}\left(y^T_{1-s_i,s_j},\theta,b_i\right)|s_i\right], \quad \forall s_i \in \{0,1\}, \tag{2}$$

given that E_j communicates truthfully with the decision maker— i.e., in equilibrium $m_j = s_j$.

Hence, E_i's incentives for truthful communication is shaped by how an undetectable lie from E_i may displace D's action as compared to the case of truth telling— i.e., the overshooting effect (highlighted in Morgan and Stocken, 2008). Let $\Delta y^T(s_i, s_j)$ be the displacement in D's optimal actions after receiving falsified message from E_i such that

$$\Delta y^T(s_i, s_j) \equiv \underbrace{y^T_{1-s_i,s_j} - y^T_{s_i,s_j}}_{\text{Overshooting Effect}}. \tag{3}$$

From the expression in (3), the higher is the magnitude of the overshooting effect, that is the higher is $\left|\Delta y^T(s_i, s_j)\right|$, the higher is the impact of E_i's deviation from a truthful message on D's action too far from his preferred action, and hence, the lower is E_i's incentive to falsify information.[7]

Having established that experts' incentives for truthful communication depend on the overshooting effect, the following proposition characterizes a conflict-revealing equilibrium with two experts whose information coming from the same source (i.e., correlated information source).

Proposition 1 *A conflict-revealing equilibrium exists if and only if*

$$|b_i| \leqslant \frac{1}{4(2+k)}, \qquad b_i \in \{b_M, b_R\}, i = 1, 2.$$

The key observation about Proposition 1 is that the truth-telling threshold shrinks as correlation increases. The reason is that a higher k implies that when one expert's signal, for example, is zero (resp. one), the other expert's signal is more likely to be zero (resp. one) too. Hence, as the correlation increases, this magnifies the experts' information to distort information since D's ability to react E_i's undetectable lie decreases. As a result, due to correlated information sources, in a conflict-revealing equilibrium, experts' preferences should be even closer to those of the decision maker to reveal their private information as compared to the case where the experts' private information coming from independent information sources.

3.2 Conflict-Hiding Equilibrium

Now consider an equilibrium in which the moderate expert reports truthfully while the rightward biased radical expert distorts information and sends a message equal to 1 irrespective of his privately observed signal. In that case, the bias uncertainty matters for D as messages may not reflect the experts' private information. In that case, letting $y^H_{m_1,m_2}$ be the decision maker's optimal action, in a conflict-hiding equilibrium D solves the following

$$\begin{aligned} y^H_{m_1,m_2} &\equiv \arg\max_{y\in\mathbb{R}} \int_\theta -(y-\theta)^2 f(\theta|m_1,m_2)\,d\theta, \\ &= \underbrace{\sum_{(s_1,s_2)\in\{0,1\}\times\{0,1\}} \Pr[s_1,s_2|m_1,m_2]\,\mathbb{E}[\theta|s_1,s_2]}_{\equiv\mathbb{E}_\nu[\theta|m_1,m_2]}. \end{aligned}$$

[7]. To understand this point, suppose that a leftward biased expert observes a signal $s_i = 0$. In that case, such expert has no incentive to falsify his private information because this would shift D's optimal action rightward, which is undesirable for him, that is $\Delta y^T(s_i = 0, s_j) > 0$. By the same token, when a rightward observes $s_i = 1$, he has no incentive to lie because, in this case, reporting a false signal to the decision maker cannot be incentive compatible.

Hence, the optimal actions in a conflict-hiding equilibrium depend on the parameter ν and hence, reflects D's skepticism about the experts' biases. In that case, the optimal actions depend on her posterior beliefs according to the Bayes rule.

The decision maker's optimal actions with and without uncertainty about experts' individual preferences are reported below (See the Appendix for the details).

$$y^H_{0,0} = \frac{1+k}{2\left(2+k\right)}, \qquad y^H_{0,1} = y^H_{1,0} = \frac{2-\nu-k\nu}{2\left(3-2\nu-k\nu\right)}, \qquad y^H_{1,1} = \frac{\nu^2\left(1+k\right)-4\nu+6}{2\nu^2\left(2+k\right)-12\nu+12}, \tag{4}$$

A couple of remarks are in order. First, in contrast to the case of conflict-revealing, D's optimal actions in a conflict-hiding equilibrium depend on correlation upon receiving identical as well as mixed messages from the experts. The reason is that, in a conflict-hiding equilibrium, bias uncertainty erodes the credibility of an expert's message equal to one in equilibrium. Second, different from the case of conflict revealing equilibrium, when D receives a mixed messages, a higher degree of correlation shifts the optimal actions leftward when $(m_1 = 1, m_2 = 1)$. Additionally, this is because the higher is the correlation, the lower is the impact of the messages from the two experts. Finally, the higher is ν, the lower are the chances of receiving biased information from the radical expert, and higher is the optimal actions when the decision maker receives at least one message equal to 1.

Now, since the moderately biased expert can separate himself from the extremely biased expert and communicate truthfully in equilibrium, the moderate expert E_i's incentive compatibility condition writes as

$$\begin{aligned} &\sum_{b_j\in\{b_M,b_E\}} \Pr\left[b_j\right] \sum_{s_j\in\{0,1\}} \mathbb{E}_\theta\left[\mathcal{U}\left(y^H_{m_i,m_j},\theta,b_i=b_M\right)|m_i=s_i\right] \\ &\geqslant \sum_{b_j\in\{b_M,b_E\}} \Pr\left[b_j\right] \sum_{s_j\in\{0,1\}} \mathbb{E}_\theta\left[\mathcal{U}\left(y^H_{1-m_i,m_j},\theta,b_i=b_M\right)|m_i=s_i\right], \end{aligned}$$

which rearranging terms, simplifies to

$$\begin{aligned} &b_M \sum_{b_j\in\mathcal{B}} \Pr\left[b_j\right] \sum_{s_j\in\{0,1\}} \Pr\left(s_j|s_i\right)\Delta y^H\left(m_i,m_j\right) \leqslant \\ &\leqslant \sum_{b_j\in\mathcal{B}} \Pr\left[b_j\right] \sum_{s_j\in\{0,1\}} \Pr\left(s_j|s_i\right)\Delta y^H\left(m_i,m_j\right)\left[\frac{\Delta y^H(m_i,m_j)}{2} + \underbrace{\left(\mathrm{E}_\nu\left[\theta|m_i,m_j\right]-\mathrm{E}\left[\theta|s_i,s_j\right]\right)}_{\text{Uncertain Biases Effect}}\right]. \end{aligned} \tag{5}$$

The expression in (5) reflects that there are two forces that shape the moderate expert's incentives to report truthfully. First, as in the case of conflict-revealing equilibrium, there is an overshooting effect. However, in a conflict-hiding equilibrium, the overshooting effect has a different magnitude depending on the message sent by the other expert E_j. In particular, the overshooting effect is stronger when E_j reports $m_j = 1$ than when he reports $m_j = 0$ —

i.e.,

$$\left|\Delta y^H\left(m_i, m_j=0\right)\right|<\left|\Delta y^H\left(m_i, m_j=1\right)\right|, \qquad m_i \in\{0,1\}.$$

The reason is that when the other expert reports a message $m_j = 1$, due to bias uncertainty, the decision maker puts more weight in E_i's message. The overshooting effect now depends on uncertainty and the correlation between experts' private information.

Second, in a conflict-hiding equilibrium, there is a novel effect shaping the expert's incentives to communicate truthfully with the decision-maker, i.e., *uncertain biases effect.* Specifically, the uncertain biases effect reflects E_i's relative gain from D's skepticism due to uncertainty about experts' biases. The higher is the uncertainty — i.e., as ν goes to 0 — the higher is the magnitude of the uncertain biases effect, and hence, the weaker is the moderate expert's incentive to communicate truthfully.

Finally, suppose that E_i is a radical expert. To complete the characterization of the conflict-hiding equilibrium, radical E_i should have no incentive to report truthfully when his private signal is $s_i = 0$. This condition is

$$\begin{aligned}
&\sum_{b_j \in\{b_M, b_E\}} \operatorname{Pr}\left[b_j\right] \sum_{s_j \in\{0,1\}} \mathbb{E}_\theta\left[\mathcal{U}\left(y_{m_i, m_j}^H, \theta, b_i=b_E\right)\right] \\
&\leqslant \sum_{b_j \in\{b_M, b_E\}} \operatorname{Pr}\left[b_j\right] \sum_{s_j \in\{0,1\}} \mathbb{E}_\theta\left[\mathcal{U}\left(y_{1, m_j}^H, \theta, b_i=b_E\right)\right].
\end{aligned}$$

Clearly, when $s_i = 1$; a rightward biased radical expert has an incentive to report 1. The following proposition characterizes a conflict–hiding equilibrium with correlated signals.

Proposition 2 *There exists two thresholds $\alpha(k,\nu)$ and $\beta(k,\nu)$ such that a conflict-hiding equilibrium exists if and only if*

$$-\beta(k, \nu) \leqslant\left|b_M\right| \leqslant \alpha(k, \nu), \qquad \text{and} \qquad b_R \geqslant \alpha(k, \nu).$$

Hence, uncertainty about the experts' biases has two effects on information transmission. First, the interval that supports truth-telling as an equilibrium shrinks as the probability of being moderate tends to zero. This is because D updates her beliefs, discounting the possibility of receiving uninformative messages. Second, when the probability of being moderate increases (as ν goes to 1), the truth-telling interval in a conflict-hiding equilibrium enlarges, and eventually, it coincides with the truth-telling interval in a conflict-revealing equilibrium. Instead, the effect of the correlated signals on the truth-telling interval is the following: a higher correlation among experts' private information weakens the overshooting effect since the decision maker may fail to discipline the moderate expert due to weaker uncertain biases effect. In that case, the correlation may actually reinforce E_i's incentives to misreport. Instead, as ν increases, the overshooting effect becomes stronger.

4 Welfare Analysis

With the inherent multiplicity of equilibria, it is natural to attempt to pin down the most informative equilibrium that is particularly appealing to the decision maker according to some criteria. To this aim, we first consider the ex-ante expected utility of the decision maker as a measure of welfare to determine how much she may learn from the experts after simultaneous communication with two experts.[8] In order to study the welfare effects of the transmission of correlated information under uncertain preferences in a unified framework, we consider two distinct scenarios regarding the source of the experts' private information. First, the experts' private information may come from independent sources, and second, the experts' private information is likely to come from the same sources; hence, the experts' information is likely to be correlated.

Throughout the section, we say that one equilibrium is informationally superior to another when the former allows the decision maker to obtain higher welfare than the latter. Considering this definition, we first compare D's ex-ante expected utility within conflict-hiding equilibria with and without correlation.

We have the following result that highlights the bright side of correlated information sources.

Proposition 3 *There exist two thresholds ν^* and $k^*(\nu)$ such that a conflict-hiding with correlated information is superior to a conflict-hiding equilibrium with independent information only if $\nu < \nu^*$ and $k > k^*(\nu)$.*

In principle, consulting two experts with correlated private information has two contrasting effects on the decision maker's ex-ante expected utility. First, correlation limits the benefits of consulting two different experts. The reason is that, higher k increases the likelihood of receiving the identical messages from both experts, and hence, diminishes the marginal gain of seeking advice from multiple sources. Keeping everything else fixed, correlation among experts' information sources weakens D's ability to take action as a combination of two messages and lowers her chance of getting *at least* one truthful information. Second, in the presence of bias uncertainty, correlation can be used as a strategic tool by D so as to discipline them via uncertain biases effect. The reason is that, as k increases, the uncertain biases effect gets weaker, so as the experts' marginal gain from information distortion. This is because, under high correlation, each experts' message have a lower impact on D's action. On balance, which of the two effects dominates depends on the relative degrees of uncertainty and correlation among experts' signals.

Surprisingly, when the degree of bias uncertainty is large (i.e., ν is small) and, and the same time, the degree of correlation is sufficiently high (i.e., k is large), the decision maker prefers to receive correlated information from the experts. To see why, first notice the region in which the two equilibria obtain — i.e., a conflict hiding equilibrium with and without correlation (See Figure 1). It is straightforward to see that these two equilibria coexist when the radical expert is sufficiently biased vis-à-vis D — i.e., such that $\lim_{k\to 0} \alpha(\nu, k) \leqslant b_R$ —

[8] We then turn our attention to the ex interim expected payoff of the expert refers to her expected payoff after the value of her bias is realized, but before that of the state does.

so he reports the same message regardless of correlation. At the same time, the moderate type has preferences close enough to the decision maker (who assumed to be unbiased) and communicates truthfully — i.e., . $b_M \leqslant \alpha(\nu, k)$.

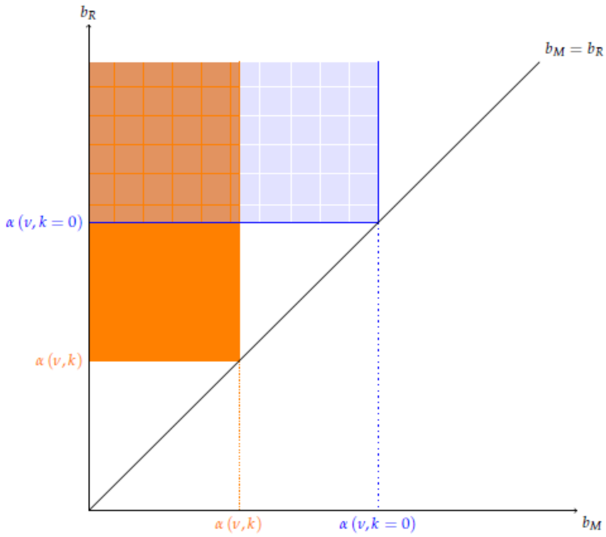

Figure 1: D's ex ante welfare in a conflict hiding equilibrium with and without correlation.

In the revelant region of parameters, when ν is small, the truth-telling interval in a conflict-hiding equilibrium is small, too, ceteris paribus. The reason is that since the bias uncertainty is a noise that erodes the credibility of the messages in equilibrium, the lower ν is, the higher is the relative likelihood of receiving false information from the experts, and the lower the ex-ante expected utility of the decision maker. However, the experts' truth-telling incentives depend also on the degree of correlation among experts' private signals. In fact, when k is large, the radical experts' incentives to lie get weaker via uncertain biases effect since a high correlation makes it more likely for D to react to an undetectable lie from the radical expert and react it through a stronger overshooting effect. On balance, the positive indirect effect dominates, and D's ex-ante welfare is higher with correlated messages since correlation disciplines the radical expert via overshooting effect and outweighs the marginal loss of receiving identical messages.

We then make a welfare comparison across equilibria to understand, from the decision maker's perspective, whether it is better to avoid correlation among experts' private information or uncertainty about experts' biases. To answer this question, we compare a conflict-revealing equilibrium with correlated information (so that bias uncertainty is absent) with a conflict-hiding equilibrium with independent information (so correlated information is absent). To answer this question, in what follows, we compare D's ex ante welfare across conflict hiding equilibrium without correlation and conflict revealing equilibrium with correlation. However, to make this comparison meaningful, we need to impose the following assumption to guarantee the coexistence of the aforementioned equilibria.

Assumption 1. The correlation between experts' private signals should not be too high —

i.e., $k < \overline{k}(\nu)$.[9]

In the light of Assumption 1, Figure 2 plots the decision maker's welfare maximizing equilibrium within each interval defined in Propositions 2 and 1. For the sake of simplicity and to ease discussion, we focus on the region where both types of experts are biased in the same direction — i.e., $0 < b_M < b_R$. We have the following result.

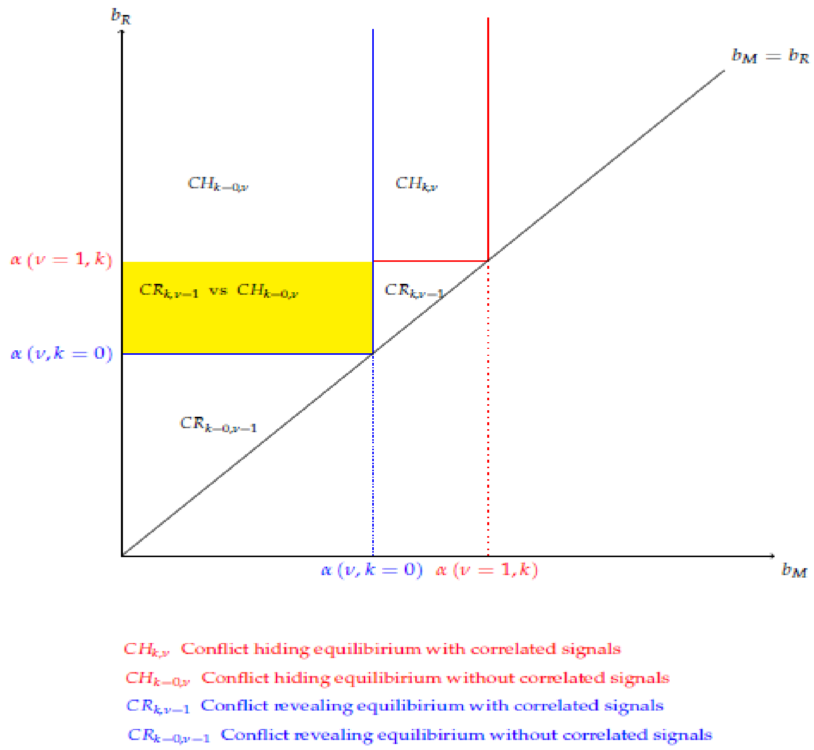

Figure 2: Decision maker's welfare maximizing equilibria under low correlation.

Proposition 4 *There exist two thresholds $\overline{\nu}$ and $\underline{k}(\nu)$ such that a conflict-revealing equilibrium with correlated signals is informationally superior to the conflict-hiding equilibrium with independent signals when when (i) $\nu \leqslant \overline{\nu}$ or (ii) $\nu > \overline{\nu}$ and $k \leqslant \underline{k}(\nu)$.*

Comparing the informativeness of the two equilibria just mentioned aims to quantify how much the model's driving forces - correlated signals and uncertain biases- weaken the information transmitted to D, separately. First, as mentioned above, correlated signals hinder the information transmitted to the decision maker because it diminishes the marginal benefit of soliciting advice from two independent experts. The higher degree of correlation between the signals makes it more likely that the decision maker receives a unanimous message from the experts, thereby lowering the impact of consulting another expert due to overshooting effect. Second, keeping everything else fixed, uncertain expert preferences makes D highly skeptical about the informational content of the two messages due to the uncertain biases effect. On the one hand, bias uncertainty is detrimental for D because it may drastically lower her ex-ante expected utility when the chances to talk to radical experts is sufficiently high. On the

[9] In the Appendix, we derive a closed-form solution for the thresholds $\overline{k}(\nu)$ as a function of the bias uncertainty parameter ν, which allows delivering additional comparative statics.

other hand, the degree of correlation between experts' private signals is also detrimental to her since higher correlation increases the likelihood of receiving a single unanimous message from the experts.

Interestingly, the decision maker may prefer to receive correlated information under two cases. First, when ν is low, so the likelihood of receiving distorted information is high, disciplining the radical expert outweighs the potential loss in the information transmission due consulting experts with correlated information. In that case, receiving correlated but undistorted information becomes more critical for the decision maker. Second, D prefers to receive correlated information when ν is sufficiently high, and the k is sufficiently low. The intuition is the following. If ν is high, the chances of talking to moderate experts are high too. However, since the degree of correlation is sufficiently low, talking to two experts with known biases fully offsets the loss in information transmission by receiving independent but potentially distorted information. Hence, even when the bias uncertainty is low, the overshooting effect dominates the uncertain biases effect, enabling the decision maker to better deal with the bias uncertainty. Hence, our result shows that experts' correlated private information may have a bright side in information transmission and can be used as a tool to prevent opportunistic behavior by the experts. In sum, seeking advice from independent sources to elicit information about the true state is not always ex-ante efficient, especially when the experts' preferences are unknown to the decision maker.

5 Conclusions

In the present paper, we analyze the experts' truth-telling incentives under simultaneous communication in the presence of bias uncertainty and correlated information. Focussing on the conflict-hiding equilibrium, we show that uncertainty over experts' biases negatively affects the informative power of the messages. At the same time, the correlation of the information mitigates this negative effect. From a welfare point of view, this implies that for a bias uncertainty sufficiently small, correlation in the information source discipline the experts' behaviors, such that receiving correlated messages may help the decision maker to extract more information. Hence, in the presence of bias uncertainty, the decision maker may prefer to consult two experts with correlated information rather than experts with independent private information. Our result suggests that even though the decision maker is uncertain about the experts' biases, consulting experts with correlated information can be used as a tool to prevent opportunistic behavior by the experts. Hence, in the presence of uncertain biases, talking to independent experts to elicit information from them about the true state is not always ex-ante efficient.

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6 Appendix

The Decision Maker's Optimal Actions

(i) D's *Optimal Actions in a conflict-revealing equilibrium with correlated signals.*

Suppose that D consults two experts who truthfully reports his signal but their information may come from correlated sources. From (1), we know that D's optimal action after receiving $m_i = s_i$ and $m_j = s_j$ is given by

$$y^T_{s_i,s_j} = \underbrace{\int_\theta \theta f(\theta|s_i, s_j)\, d\theta}_{\triangleq \mathrm{E}[\theta|s_i,s_j]}, \tag{6}$$

where the conditional density of θ given the signals s_i and s_j is

$$f(\theta|s_i, s_j) = \frac{\Pr[s_i, s_j|\theta]\, f(\theta)}{\int_\theta \Pr[s_i, s_j|\theta]\, f(\theta)\, d\theta},$$

Using the conditional probability distribution of the correlated signals defined in Section 2, we have

$$f(\theta|s_i = 0, s_j = 0) = \frac{6(1-\theta)(1-\theta+k\theta)}{k+2}, \qquad f(\theta|s_i = 1, s_j = 1) = 6\theta\frac{k+\theta(1-k)}{k+2}, \tag{7}$$

$$f(\theta|s_i = 0, s_j = 1) = f(\theta|s_i = 1, s_j = 0) = 6\theta(1-\theta). \tag{8}$$

Substituting (10) and (11) into (6) yields the decision maker's optimal actions

$$y^T_{0,0} = \frac{1+k}{2(2+k)}, \qquad y^T_{0,1} = y^T_{1,0} = \frac{1}{2}, \qquad y^T_{1,1} = \frac{3+k}{2(2+k)}, \tag{9}$$

as claimed.

(ii) D's *Optimal Actions in a conflict-hiding equilibrium with correlated signals*

Suppose that D consults two experts with unknown individual preferences and their information may come from correlated sources. From (1), we know that D's optimal action after receiving m_i and m_j is given by

$$\begin{aligned} y^H_{m_1,m_2} &\equiv \arg\max_{y\in\mathbb{R}} \int_\theta -(y-\theta)^2 f(\theta|m_1, m_2)\, d\theta, \\ &= \underbrace{\sum_{(s_1,s_2)\in\{0,1\}\times\{0,1\}} \Pr[s_1, s_2|m_1, m_2]\, \mathbb{E}[\theta|s_1, s_2]}_{\equiv \mathbb{E}_\nu[\theta|m_1,m_2]}. \end{aligned}$$

where the conditional density of θ given the signals s_i and s_j is

$$f(\theta|s_i, s_j) = \frac{\Pr[s_i, s_j|\theta]\, f(\theta)}{\int_\theta \Pr[s_i, s_j|\theta]\, f(\theta)\, d\theta},$$

Using the conditional probability distribution of the correlated signals defined in Section

2, we have

$$f(\theta|s_i=0,s_j=0)=\frac{6(1-\theta)(1-\theta+k\theta)}{k+2},\qquad f(\theta|s_i=1,s_j=1)=6\theta\frac{k+\theta(1-k)}{k+2},\tag{10}$$

$$f(\theta|s_i=0,s_j=1)=f(\theta|s_i=1,s_j=0)=6\theta(1-\theta).\tag{11}$$

Substituting (10) and (11) into (6) yields the decision maker's optimal actions

$$y^H_{0,0}=\frac{1+k}{2(2+k)},\qquad y^H_{0,1}=y^H_{1,0}=\frac{2-\nu-k\nu}{2(3-2\nu-k\nu)},\qquad y^H_{1,1}=\frac{\nu^2(1+k)-4\nu+6}{2\nu^2(2+k)-12\nu+12},\tag{12}$$

Proof of Proposition 1. In a conflict-revealing equilibrium experts of either type report truthfully. Hence, without loss of generality, consider E_i's incentives to report truthfully. Specifically, E_i's expected utility from reporting $m_i=s_i$ is higher than his expected utility from reporting $m_i=1-s_i$ if and only if

$$\sum_{s_j\in\mathcal{S}}\int_\theta-\left(y^T_{s_i,s_j}-\theta-b_i\right)^2f(s_j,\theta|s_i)\,d\theta\geqslant\sum_{s_j\in\mathcal{S}}\int_\theta-\left(y^T_{1-s_i,s_j}-\theta-b_i\right)^2f(s_j,\theta|s_i)\,d\theta,\tag{13}$$

which, substituting $f(s_j,\theta|s_i)=f(\theta|s_i,s_j)\Pr[s_j|s_i]$ by Bayes' rule and following the same steps as I did above, simplifies to

$$b_i\leqslant\sum_{s_j\in\mathcal{S}}\Pr[s_j|s_i]\frac{y^T_{1-s_i,s_j}-y^T_{s_i,s_j}}{2}.\tag{14}$$

In order to compute $\Pr[s_j|s_i]$, notice first that conditional probability distribution of the signals can be written as follows

$$\Pr(s_i,s_j|\theta)=\frac{f(s_i,s_j,\theta)}{f(\theta)}.\tag{15}$$

Then, using (15) together with the fact that $f(\theta)=1$, I obtain

$$\Pr[s_j|s_i]=\int_\theta f(s_j,\theta|s_i)\,d\theta=\int_\theta\frac{f(s_i,s_j,\theta)}{\Pr(s_i)}d\theta=\Pr[s_i]\int_\theta\Pr(s_i,s_j|\theta)\,d\theta.\tag{16}$$

Using the joint distributions defined in Section 2 together with $\Pr[s_i]=\frac{1}{2}$, $s_i\in\mathcal{S}$, it can be easily verified that

$$\Pr[s_j=0|s_i=0]=\Pr[s_j=1|s_i=1]=\frac{k+2}{3},\tag{17}$$

$$\Pr[s_j=1|s_i=0]=\Pr[s_j=0|s_i=1]=\frac{1-k}{3}.\tag{18}$$

Finally, substituting (17), (18) into (14) and using D's optimal actions defined in Section

3.1, when $s_i = 0$, truth-telling by E_i requires

$$b_i \leqslant \frac{1}{4(k+2)},$$

while, when $s_i = 1$, truth-telling by E_i requires

$$b_i \geqslant -\frac{1}{4(k+2)}.$$

where $b_i \in \{b_M . b_R\}, i = 1, 2$. The result follows immediately. ■

Proposition 2 Without loss of generality, we focus on E_i's incentives to disclose his private information, since experts have symmetric payoffs. Consider first that E_i is a moderate such that $b_i = b_M$. Given that E_j's type is his private information, E_i's incentive compatibility constraints are written as

$$\sum_{b_j \in \{b_M, b_E\}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \mathbb{E}_\theta \left[\mathcal{U}\left(y^H_{m_i,m_j}, \theta, b_i = b_M \right) | m_i = s_i \right]$$
$$\geqslant \sum_{b_j \in \{b_M, b_E\}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \mathbb{E}_\theta \left[\mathcal{U}\left(y^H_{1-m_i,m_j}, \theta, b_i = b_M \right) | m_i = s_i \right].$$

Simplifying terms and rearranging the above constraint can be rewritten as follows

$$b_M \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|s_i) \Delta y^H(m_i, m_j) \leqslant$$
$$\leqslant \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|s_i) \left\{ \frac{\Delta y^H(m_i, m_j)^2}{2} + \Delta y^H(m_i, m_j) (\mathrm{E}_\nu[\theta|m_i, m_j] - \mathrm{E}[\theta|s_i, s_j]) \right\}. \quad (19)$$

Using the optimal actions from Table 1 and $\Pr[s_j|s_i]$ from equations (17) and (18) , when $s_i = 0$, truth-telling by the moderate expert requires

$$b_M \leqslant \alpha(k, \nu)$$

where

$$\alpha(k,\nu) \triangleq \frac{(7k+5)(1-k)(k+2)\nu^4 + \left(8k^3 + 24k^2 - 72k - 68\right)\nu^3}{((3\nu^2 - 4\nu)k + (3\nu^2 - 8\nu + 6))4(k+2)(6(1-\nu) + (2+k)\nu^2)(3-(2-k)\nu)} + \quad (20)$$
$$+ \frac{\left(168k - 12k^2 + 168\right)\nu^2 - 36(4k+5)\nu + 36(k+2)}{((3\nu^2 - 4\nu)k + (3\nu^2 - 8\nu + 6))4(k+2)(6(1-\nu) + (2+k)\nu^2)(3-(2-k)\nu)}.$$

:

Since the denominator is positive, the sign of (20) depends on the sign of the numerator

$$\mu(k,\nu) \triangleq (7k+5)(1-k)(k+2)\nu^4 + \left(8k^3 + 24k^2 - 72k - 68\right)\nu^3 +$$
$$+ \left(168k - 12k^2 + 168\right)\nu^2 - 36(4k+5)\nu + 36(k+2),$$

with

$$\frac{\partial \mu(k,\nu)}{\partial k} = 3\nu^3(8-7\nu)k^2 - 24\nu^2(1-\nu)^2 k + 3\left(3\nu^2-6\nu+2\right)\left(6-6\nu+\nu^2\right). \tag{21}$$

Setting (21) and solving for k, we find the critical points

$$k^{\text{Crit.1}} = \frac{\left(\sqrt{-96\nu^3+484\nu^4-848\nu^5+680\nu^6-256\nu^7+37\nu^8}+4\nu^2-8\nu^3+4\nu^4\right)}{\nu^3(8-7\nu)} > 0 \text{ iff } \nu \geqslant 0.42,$$

and

$$k^{\text{Crit.2}} = -\frac{\left(\sqrt{-96\nu^3+484\nu^4-848\nu^5+680\nu^6-256\nu^7+37\nu^8}-4\nu^2+8\nu^3-4\nu^4\right)}{\nu^3(8-7\nu)} < 0.$$

First, notice that since $k^{\text{Crit.1}} < 0$ when $\nu < 0.42$ and $k^{\text{Crit.2}} < 0$, these critical points are outside the interval of interest. In these region of parameters since

$$\mu(k=0,\nu) = 2(2-\nu)\left(18-36\nu+24\nu^2-5\nu^3\right) > 0,$$
$$\mu(k=1,\nu) = 108(1-\nu)^3 > 0,$$

it is immediate to see that $\mu(k,\nu)$ is strictly positive, and hence, the truth-telling threshold $\alpha(k,\nu) > 0$.

Second, suppose that $\nu \geqslant 0.42$ so that $k^{\text{Crit.1}} > 0$. Notice that, in the relevant region of parameters, we have

$$\lim_{k\to k^{\text{Crit.1}}} \frac{\partial^2 \mu(k,\nu)}{\partial k^2} = 6\sqrt{37\nu^8-256\nu^7+680\nu^6-848\nu^5+484\nu^4-96\nu^3} > 0,$$

so $k^{\text{Crit.1}}$ is a relative minimum. However, since

$$k^{\text{Crit.1}} - 1 = \frac{\sqrt{-96\nu^3+484\nu^4-848\nu^5+680\nu^6-256\nu^7+37\nu^8}+4\nu^2-16\nu^3+11\nu^4}{\nu^3(8-7\nu)} > 0$$

implying that $k^{\text{Crit.1}} > 1$, so that also in this case, the critical point lies outside the interval of interest. As above since

$$\mu(k=0,\nu) > 0 \text{ and } \mu(k=1,\nu) > 0,$$

when $\nu \geqslant 0.42$, the numerator $\mu(k,\nu) > 0$ implying that the truth-telling threshold $\alpha(k,\nu) > 0$.

By the same token, suppose that $s_i = 1$. In that case, truth-telling by the moderate expert requires

$$b_M \geqslant -\beta(k,\nu).$$

where it can be shown that $\beta(k,\nu) > 0$. Notice that we are able to find the closed form solution for $\beta(k,\nu)$ and simulations are available upon request. To complete the proof, I need to check that radical expert has no incentive to report $m_i = 0$ when his signal is $s_i = 0$.

Adopting the same logic used above, E_i's expected utility from reporting $m_R = 1$ is higher than his expected utility when reporting truthfully $m_i = 0$ if

$$(b_M - b_D) \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|0)\, \Delta y^H(0, m_j) >$$
$$> \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|0) \frac{\Delta y^H(0, m_j)^2}{2} + \Delta y^H(0, m_j)\left(\mathrm{E}_\nu[\theta|0, m_j] - \mathrm{E}[\theta|0, s_j]\right).$$

Using the optimal actions from Sections 3.1 and 3.2 when $s_1 = 1$, babbling by the rightward biased radical type requires

$$b_R - b_D > \alpha(k, \nu),$$

while when $s_1 = 1$, the radical expert has an incentive to report truthfully his signal. ■

Proof of Proposition 3. Let $\mathrm{E}U(k, \nu)$ be the decision maker's ex-ante expected utility. In that case, D's expected utility in a conflict revealing equilibrium — i.e., from consulting two experts with known biases — is given by

$$\mathrm{E}U(k, \nu = 1) \triangleq \int_\theta \sum_{(s_i, s_j) \in \mathcal{S}^2} -\left(y^T_{s_i, s_j} - \theta - b_D\right)^2 \Pr[s_i, s_j|\theta]\, f(\theta)\, d\theta. \tag{22}$$

Using the optimal actions from Sections 3.1 and 3.2, it yields

$$\mathrm{E}U(k, \nu = 1) = -\frac{1 + k}{12(2 + k)}. \tag{23}$$

Instead, D's expected utility in a conflict hiding equilibrium with correlated signals— is given by

$$\mathrm{E}U(k, \nu) \triangleq \int_\theta \sum_{(m_i, m_j) \in \mathcal{M}^2} -\left(y^H_{m_i, m_j} - \theta - b_D\right) \Pr[m_i, m_j|\theta]\, f(\theta)\, d\theta,$$

where

$$\Pr[m_i, m_j|\theta] = \sum_{(s_i, s_j) \in \mathcal{S}^2} \Pr[m_i, m_j|s_i, s_j] \Pr[s_i, s_j|\theta]. \tag{24}$$

Substituting the conditional probability distribution of (s_i, s_j) and the corresponding prior probabilities into (24), we have

$$\Pr[m_i = 1, m_j = 1|\theta] = \theta(1 - \theta)\nu^2 k + (1 - \nu + \theta\nu)^2,$$
$$\Pr[m_i = 0, m_j = 0|\theta] = \nu^2(1 - \theta)(1 - \theta + k\theta)$$
$$\Pr[m_i = 0, m_j = 1|\theta] = -\nu(1 - \theta)(1 - \nu + (1 - k)\theta\nu).$$

Then using the optimal actions from Table 1, it is immediate to have

$$\mathrm{E}U(k, \nu) = -\frac{-\nu^3 k^3 - \left(5\nu^3 - 9\nu^2 + 6\nu\right)k^2 + \left(18 - 17\nu^3 + 48\nu^2 - 48\nu\right)k + \left(51\nu^2 - 13\nu^3 - 72\nu + 36\right)}{12(2 + k)\left(6(1 - \nu) + \nu^2(2 + k)\right)\left(3 - \nu(2 + k)\right)}. \tag{25}$$

We now compare D's ex ante expected utility in a conflict-hiding equilibrium with correlated signals and without correlation among experts' private signals. These two equilibria always coexist as, in a conflict hiding equilibrium, truth-telling threshold with correlated signals converges to the truth-telling threshold with independent signals.

Then, using D's ex-ante expected utility in a conflict-hiding equilibrium with correlation (25) and without correlation, we have

$$\mathrm{E}U(k,\nu)-\mathrm{E}U(k=0,\nu)=\frac{k\nu^2\delta(k,\nu)}{48(3-2\nu)(3-3\nu+\nu^2)(k+2)(6(1-\nu)+2\nu^2+k\nu^2)(3-2\nu-k\nu)}, \tag{26}$$

where

$$\begin{aligned}\delta(k,\nu)=&\left(5\nu^4-15\nu^3+12\nu^2\right)k^2+\left(38\nu^4-171\nu^3+297\nu^2-234\nu+72\right)k+\\&+\left(20\nu^4-84\nu^3+114\nu^2-36\nu-18\right).\end{aligned} \tag{27}$$

The sign of (26) depends on the sign of $\delta(k,\nu)$ with

$$\frac{\partial\delta(k,\nu)}{\partial k}=\left(10\nu^4-30\nu^3+24\nu^2\right)k+\left(38\nu^4-171\nu^3+297\nu^2-234\nu+72\right)>0.$$

Moreover,

$$\delta(k=0,\nu)=20\nu^4-84\nu^3+114\nu^2-36\nu-18<0,$$

and

$$\delta(k=1,\nu)=9(1-\nu)\left(6-7\nu^3-24\nu+23\nu^2\right)<0 \text{ if and only if } \nu\geqslant 0.36.$$

First, let $\nu\geqslant 0.36$ so that $\delta(k=1,\nu)<0$. In that case, since

$$\delta(k=0,\nu)<0 \text{ and } \delta(k=1,\nu)<0,$$

and

$$\frac{\partial\delta(k,\nu)}{\partial k}>0,$$

it is immediate to see that $\delta(k,\nu)<0$, for this region of parameters. Hence, when $\nu\geqslant 0.36$, D's ex-ante utility in a conflict hiding equilibrium with independent signals is higher than her ex-ante expected utility in a conflict revealing equilibrium with correlated signals.

Now let $\nu<0.36$ so that $\delta(k=1,\nu)>0$. In that case, setting $\delta(k,\nu)$ equal to zero and solving for k yields the (positive) critical point

$$\begin{aligned}k^*(\nu)=&\frac{234\nu-297\nu^2+171\nu^3-38\nu^4-72}{10\nu^4-30\nu^3+24\nu^2}+\\&+\frac{3\sqrt{116\nu^8-1124\nu^7+4837\nu^6-11\,974\nu^5+18\,493\nu^4-18\,108\nu^3+10\,932\nu^2-3744\nu+576}}{10\nu^4-30\nu^3+24\nu^2},\end{aligned}$$

where $k^*(\nu)\in(0,1)$. Hence, when the degree of uncertainty is sufficiently severe — i.e., $\nu<\nu^*=0.36$ — the decision maker's ex-ante expected utility from conflict-hiding equilibrium with correlated signals is higher than her ex-ante expected utility from conflict-hiding equilibrium with independent signals.

Proof of Proposition 4 We compare D's ex ante expected utility in a conflict-revealing equilibrium with correlated signals and her ex-ante expected utility in a conflict-hiding equilibrium with independent signals. To make this comparison meaningful, we need to focus on the region where these two equilibria coexist. This requires the following is satisfied

$$\lim_{\nu \to 1} \alpha(\nu, k) > \lim_{k \to 0} (\nu, k) \text{ if and only if } k < \overline{k}(\nu),$$

where

$$\overline{k}(\nu) = \frac{12\nu^5 - 101\nu^4 + 360\nu^3 - 654\nu^2 + 594\nu - 216}{10\nu^4 - 68\nu^3 + 168\nu^2 - 180\nu + 72} + \\ + \frac{\sqrt{144\nu^{10} - 2664\nu^9 + 21\,993\nu^8 - 106\,584\nu^7 + 336\,340\nu^6 - 723\,684\nu^5 + 1077\,300\nu^4 - 1097\,352\nu^3 + 732\,996\nu^2 - 290\,304\nu + 51\,840}}{10\nu^4 - 68\nu^3 + 168\nu^2 - 180\nu + 72},$$

with $0 < \overline{k}(\nu) < 1$ for every $\nu \in (0, 1)$.

Focusing on the region in which $k < \overline{k}(\nu)$, we compare D's ex-ante expected utilities from (23) and (25), we have

$$\mathrm{E}U(k = 0, \nu) - \mathrm{E}U(k, \nu = 1) = \frac{\left(5\nu^3 - 15\nu^2 + 12\nu\right) k + \left(18\nu^3 - 66\nu^2 + 84\nu - 36\right)}{48(3 - 2\nu)(3 - 3\nu + \nu^2)(k + 2)}. \quad (28)$$

Since the denominator is strictly positive, the sign of (28) depends on the sign of the numerator

$$\varphi(k, \nu) = \left(5\nu^3 - 15\nu^2 + 12\nu\right) k + \left(18\nu^3 - 66\nu^2 + 84\nu - 36\right),$$

with

$$\frac{\partial \varphi(k, \nu)}{\partial k} = 5\nu^3 - 15\nu^2 + 12\nu > 0.$$

Notice that

$$\varphi(k = 0, \nu) = -6(1 - \nu)\left(+6 - 8\nu + 3\nu^2\right) < 0,$$

and

$$\varphi(k = 1, \nu) = 23\nu^3 - 81\nu^2 + 96\nu - 36 < 0 \text{ if and only if } \nu \triangleq \overline{\nu} \leqslant 0.73.$$

First, let $\nu \leqslant \overline{\nu}$ so that $\varphi(k = 1, \nu) < 0$. In that case, since

$$\frac{\partial \varphi(k, \nu)}{\partial k} > 0,$$

and $\varphi(k = 0, \nu) < 0$, it is immediate to see that the numerator $\varphi(k, \nu)$ is increasing in k and it is always negative for any $k \in (0, 1)$. Hence, when $\nu \leqslant 0.73$, the decision maker's ex-ante expected utility is higher in a conflict-revealing equilibrium with correlated signals.

Now, let $\nu > \overline{\nu}$ so that $\varphi(k = 1, \nu) > 0$. In that case, setting $\varphi(k, \nu)$ equal to zero and solving for k yields the critical point

$$\underline{k}(\nu) = \frac{66\nu^2 - 18\nu^3 - 84\nu + 36}{5\nu^3 - 15\nu^2 + 12\nu},$$

where, $0 < \underline{k}(\nu) < \overline{k}(\nu)$.

Moreover, in this region of parameters since $\frac{\partial \varphi(k,\nu)}{\partial k} > 0$, and

$$\varphi\left(k=0,\nu\right)<0 \text{ and } \varphi\left(k=1,\nu\right)>0,$$

by the intermediate value theorem, it implies that when $k > \underline{k}(\nu)$ and $\nu < \overline{\nu}$ the decision maker's ex-ante expected utility is higher in a conflict-hiding equilibrium with independent signals. Otherwise, D's ex-ante expected utility is higher in a conflict-revealing equilibrium with correlated signals. ■