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Market timing and short-term portfolio selection based on state price density

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Abstract

Market timing models aim to anticipate short-term market movements according to a given source of information. Such information could be extracted from an analysis of history or a forecast of the future. In fact, the financial markets are driven mainly by the expectations of market investors and by exogenous sources. An explicit way for market investors to make clear their expectations about a certain asset is to define the implied volatility of the options that are written on that asset. Moreover, the literature proposed tools that generate the state price density of the underlying by observing the implied volatility of the options. The combination of the implied volatilities and the state price density can give deep insight into investors' expectations about the short-term movements of the underlying and, thus, can represent a reliable source of information to perform a market timing strategy or to select a portfolio for a risk-neutral investor with a short-term horizon. To avoid adjusting the procedure for dividend-paying assets, we develop our approach considering market price indexes. This approach constitutes a completely new technique to establish both a market timing strategy and a ranking among the considered indexes. In the empirical analysis, we considered both the market timing problem for a single index and the portfolio selection problem when multiple indexes are available.

Keywords Market timing · Portfolio selection · Index options · Implied volatility · State price density

JEL Classification: G11

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1 Introduction

The most famous and widely used technique to price options is the model of Black and Scholes (Black & Scholes, 1973) with contributions by Merton (Merton, 1973) (henceforth referred to as the BSM model). However, this model suffers from some drawbacks, such as the assumption that the log-returns of the underlying asset follow a Gaussian distribution and the assumption that their variance is constant and deterministic. Because of these disadvantages, the model's empirical observations of the option market prices differ from the prices obtained with the BSM model. Thus, traders typically invert the BSM formula to obtain the so-called implied volatility (IV), then quote the IVs rather than the option prices. Therefore, the information hidden in the IV somehow reflects market agents' ideas about the extent to which the future distribution of the returns of the underlying asset differs from the Gaussian distribution. At any time, for a single underlying asset, several call and put options with different strike prices and maturities are quoted. Thus, at any time, we can compute a whole set of IVs. Instead of the strike price, in this paper, we adopt the concept of *moneyness* (i.e., the ratio of the strike price to the current underlying asset price). Moneyness simplifies the comparison among options with different maturities. When, for a given maturity, we plot IVs against moneyness, we obtain a curve similar to a smile (especially for forex options) or a smirk (a nonsymmetric smile common for equities).

IV curves have been deeply analyzed in the last three decades. Corrado and Su (1997) focused on the relationship between the skewness and kurtosis of the underlying asset and of the IV. Skiadopoulos et al. (2000) applied principal component analysis to identify the components that better explain the IV smiles. Tompkins (2001) proposed an extensive work on IV surfaces, considering several markets and trying to capture similarities among the smile patterns. Hafner and Wallmeier (2007) extracted from IV curves suggestions regarding whether to invest or not in volatility-based financial instruments. Ielpo and Guillaume (2010) investigated the mean-reverting behavior of the IV through time. Silvia (2010) tried to exploit the information inside the IV to forecast the volatility for the underlying asset and demonstrated the quality of the BSM model. García-Machado and Rybczyński (2017) focused on the interpretation of the IV curves according to their particular shape in periods of market turmoil. Jang and Lee (2019) used neural network models to predict the IVs of the options. Thus, the importance of studying IV curves and of being able to smooth the IV function across the quoted values to capture its shape along the entire moneyness span is clear. For a review of parametric and semiparametric models to smooth the volatility surface please refer to Fengler (2012) and Homescu (2011). Unfortunately, there is no continuum of data with respect to moneyness, so we need some kind of interpolation and smoothing. In this paper, we adopted the IV smoothing proposed in Benko et al. (2007), which has been previously used in Kim and Lee (2013); Kopa et al. (2017), and Vitali et al. (2023). Benko et al. (2007) combine IV smoothing with a model to estimate the state price density (SPD). SPD is able to replace the BSM hypothesis of Gaussianity, giving a representation of the hidden distribution of the underlying forecast by market agents. Other non-arbitrage IV and SPD estimation procedures have been presented in, for example, Fengler (2012); Glaser and Heider (2012), Fengler and Hin (2015), and Ludwig (2015). The estimation procedures differ if the underlying of the options is a dividend-paying asset or not. To avoid excessive complications we limit the analysis in this work to non-dividend-paying asset, namely we use stock market price indexes.

The SPD (also known as Q-distribution) is, by construction, a risk-neutral distribution. To obtain the real-world probability density function (known as P-distribution) it is necessary to adjust the SPD, considering multiple elements as highlighted by Figlewski (2018). Moreover, Kostakis et al. (2011) show that it is important to try to perform such a transformation (from the Q-distribution to the P-distribution) to correctly quantify the risk when the calculation of an optimal portfolio is carried out. The same need is also noted by Kosolapova et al. (2023) for portfolio and risk management. However, they all agree that such a transformation is particularly challenging and that several elements are difficult to estimate and quantify. For these purposes, since we did not want to quantify, reduce, or control the risk, we replicated the behavior of a risk-neutral investor and, thus, as remarked in Kosolapova et al. (2023), we could rely on the information contained in the SPD. Indeed, this work aims to analyze whether some hidden information in the SPD distribution can be used to define a good trading strategy. If so, the advantage of using the SPD would be evident, since it can be easily estimated from option prices, while the P-distribution is difficult to obtain in practice.

To summarize, the aim of this paper is twofold:

- To propose a market timing strategy for a risk-neutral investor when the investment decision focuses on a single index;
- To propose a short-term portfolio selection strategy for a risk-neutral investor when the investment universe is composed of a set of indexes.

The idea of using the information from the IV to improve portfolio selection models has already been explored: DeMiguel et al. (2013) investigated the implied volatility and implied skewness of options within a portfolio optimization; Kempf et al. (2015) performed a minimum variance optimization based on implied estimators extracted from options. However, to the best of our knowledge, the idea of using the SPD to make investment decisions is new. To show the effectiveness of our approach, we considered a huge dataset of options for some of the main stock indexes that represent the worldwide financial markets, and we showed that it is possible to perform a good market timing for a single index and to rank the indexes to select the best ones to build a portfolio. The results show that the proposed market timing strategy nearly always performs better than the simple buy-and-hold strategy and, in multiple cases, our criteria suggest portfolios that beat the $1/N$ portfolio that we take as a benchmark.

This paper is structured as follows. Section 2 reviews the theory of IV and SPD. Section 2.1 describes the market timing and portfolio strategies that we propose. Section 4 presents the empirical analysis. Section 5 concludes the paper.

2 Implied volatility and state price density estimation

Implied volatility can be computed using the BSM-type model and market price data of European call (or put) options. Let us assume that the index value dynamic evolves as a geometric Brownian motion with constant drift and volatility. In this case, the value C_t of a call option on a non-dividend-paying underlying is expressed as (Black & Scholes, 1973):

$$C_t(S_t, K, \tau, r, \sigma) = S_t \Phi(d_1) - K e^{-r\tau} \Phi(d_2), \quad t < T, \tag{1}$$

$$d_1 = \frac{\log(S_t/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}, \tag{3}$$

where S_t is the spot value of the underlying, K represents the strike, T is the maturity of the option, $\tau = T - t$ is the time-to-maturity, r is the risk-free interest rate, σ is the volatility of the log returns of the underlying (assumed to be an unknown and constant parameter), and $\Phi(\cdot)$ represents the cumulative distribution function of the standard normal distribution. The implied volatility of the underlying log returns $\tilde{\sigma}$ is defined as the solution of the system (1)–(3) in which the price of call option C_t is substituted by the observed market price $\tilde{C}_t$.

It is well known that the implied volatility is not constant over time and not equal for all quoted strikes; see Brockhaus et al. (2000) and Fengler (2006). Indeed, for a given maturity, the implied volatility is a u-shaped function of the strike. This function is called the *implied volatility smile*.

Thus, we can assume the following model structure:

$$\tilde{\sigma}(K_i, \tau_i) = \sigma(K_i, \tau_i) + \varepsilon_i, \quad i = 1, \ldots, n, \tag{4}$$

where $\tilde{\sigma}_i = \tilde{\sigma}(K_i, \tau_i)$ is the implied volatility observed for K_i and τ_i, $\sigma_i = \sigma(K_i, \tau_i)$ is its theoretical counterpart, ε_i represents the noise of the model, and n corresponds to the number of observations. Following Benko et al. (2007), we do not introduce any explicit model expression for σ. Instead, we apply the local nonparametric smoothing technique presented by Fan and Gijbels (1996); Hardle (1990), and Fan and Yao (2003). Local polynomial estimators have been successfully applied to calibrate a volatility surface model (4) and are currently preferred to other approaches, such as a spline interpolation or higher-order polynomials; see Shimko (1993); Fengler et al. (2003), and Cont et al. (2002). Moreover, Benko et al. (2007) further improved this approach by imposing non-arbitrage constraints.

These constraints are actually imposed on the SPD that is a density function defined as:

$$q_{t,S_T}(x, \tau) := e^{r\tau} \left. \frac{\partial^2 C_t(K, \tau)}{\partial K^2} \right|_{K=x}. \tag{5}$$

A comparison of (1)–(3) and (5) makes clear the relation between the SPD and the implied volatility surface, which can be written as:

$$\begin{aligned} q_{t,S_T}(x, \tau) = e^{r\tau} S_t \sqrt{\tau} \varphi(d_1(x, \tau)) \Bigg\{ & \frac{1}{x^2 \sigma(x, \tau) \tau} + \frac{2 d_1(x, \tau)}{x \sigma(x, \tau) \sqrt{\tau}} \left. \frac{\partial \sigma(K, \tau)}{\partial K} \right|_{K=x} \\ & + \frac{d_1(x, \tau) d_2(x, \tau)}{\sigma(x, \tau)} \left(\left. \frac{\partial \sigma}{\partial K} \right|_{K=x} \right)^2 + \left. \frac{\partial^2 \sigma}{\partial K^2} \right|_{K=x} \Bigg\}, \end{aligned} \tag{6}$$

$$d_1(x, \tau) = \frac{\log(S_t/x) + (r + \sigma^2(x, \tau)/2)\tau}{\sigma(x, \tau)\sqrt{\tau}}, \tag{7}$$

$$d_2(x, \tau) = d_1(x, \tau) - \sigma(x, \tau)\sqrt{\tau}, \tag{8}$$

where $\varphi(\cdot)$ denotes the probability density function of the standard normal distribution; see Brunner and Hafner (2003). Therefore, the SPD is expressed as a function of the implied volatility surface and its derivatives. For a review, see Jackwerth (2004).

Benko et al. (2007) proves that a negative value of the SPD function highlights an arbitrage opportunity, while the nonnegativity of the SPD guarantees the opposite. Combining the local smoothing of (4) and the SPD, we can finally obtain a non-arbitrage estimation of both quantities jointly. To achieve this result, the implied volatility should be standardized with respect to the underlying value to make subsequent estimations comparable. Thus, instead of considering the strike K, we consider the *futures moneyness* $\kappa_t := K/F_t$ with $F_t = S_t e^{r\tau}$, meaning the relative position of the future value of the underlying with respect to the strike. The strikes K that we consider are the ones quoted in the market, therefore we do not utilize any artificial strike. In the following section, we focus on the estimation of the IVs for fixed maturity, and we apply this technique to a large set of data.

2.1 Implied volatility for fixed maturities

To estimate the implied volatility function, we consider a set of option market prices with a fixed time-to-maturity τ. In this case, the general setting of the model (4) can be further simplified considering the fixed τ and the futures moneyness κ_i:

$$\tilde{\sigma}(\kappa_i) = \sigma(\kappa_i) + \varepsilon_i, \quad i = 1, \ldots, n_\tau, \tag{9}$$

where $\tilde{\sigma}_i := \tilde{\sigma}(\kappa_i)$ and $\sigma_i := \sigma(\kappa_i)$ are assumed to be functions of κ_i only, ε_i is the error term, and n_τ is the number of observed implied volatilities with fixed time-to-maturity τ. If $\tau = T - t$ is given, the definition of the moneyness becomes $K_i = \kappa_i F_t$, where, observing daily data, the term $F_t = S_t e^{r\tau}$ is supposed to be constant, meaning $S_t := S$, $F_t := F$, and $K_i = \kappa_i F$.

Model (9) can be calibrated using a local polynomial fitting. In particular, Benko et al. (2007) suggests a local quadratic estimator. The estimator $\hat{\sigma}(\kappa)$ of the function $\sigma(\kappa)$ in the grid point κ is then computed as the solution of the following minimization problem:

$$\min_{\alpha_0,\alpha_1,\alpha_2} \sum_{i=1}^{n_\tau} \left[\tilde{\sigma}_i - \alpha_0 - \alpha_1(\kappa_i - \kappa) - \alpha_2(\kappa_i - \kappa)^2\right]^2 \mathcal{K}_h\left(\kappa - \kappa_i\right), \tag{10}$$

where the weighting term $\mathcal{K}_h(\kappa - \kappa_i)$ is defined as $\frac{1}{h}\mathcal{K}\left(\frac{\kappa-\kappa_i}{h}\right)$, in which $\mathcal{K}$ denotes a kernel function. The kernel function is a weighting function generally used in nonparametric estimation techniques. In particular, $\mathcal{K}$ is a nonnegative real-valued integrable function satisfying (i) $\int_{-\infty}^{\infty} \mathcal{K}(x)dx = 1$ and (ii) $\mathcal{K}(x) = \mathcal{K}(-x)$, $\forall x \in \Re$. For a review of the literature about kernel functions, see Fan and Gijbels (1996). The parameter $h > 0$ is the so-called bandwidth, and it controls the size of the local neighborhood considered for the estimation. As analyzed in Kopa et al. (2017), the choice of the bandwidth h has a huge impact in the local polynomial fitting procedure. On the one hand, a high bandwidth induces over-smoothing (i.e., modeling bias). On the other hand, a small bandwidth produces under-smoothing (i.e., noisy estimates). If $h = \infty$, the local polynomial fitting becomes a global quadratic fitting; see Fan and Yao (2003). Moreover, using the Taylor's expansion of σ in the problem (10), we can obtain the following: $\alpha_0 = \hat{\sigma}(\kappa)$, $\alpha_1 = \hat{\sigma}'(\kappa)$, and $\alpha_2 = \hat{\sigma}''(\kappa)/2$.

To impose the non-arbitrage condition, the minimization (10) must be subjected to $q_{t,S_T}(\kappa,\tau) \geq 0$. Moreover, because τ is fixed, we can further simplify the notation, as follows: $q_{t,S_T}(\kappa,\tau) =: q(\kappa)$. Therefore, equation (6) and the following formulas can be rewritten as

$$\begin{aligned} q(\kappa) = \sqrt{\tau}\varphi(d_1(\kappa)) &\left\{ \frac{1}{\kappa^2\sigma(\kappa)\tau} + \frac{2d_1(\kappa)}{\kappa\sigma(\kappa)\sqrt{\tau}}\frac{\partial\sigma(\kappa)}{\partial\kappa} + \frac{d_1(\kappa)d_2(\kappa)}{\sigma(\kappa)}\left(\frac{\partial\sigma(\kappa)}{\partial\kappa}\right)^2 \right. \\ &\left. + \frac{\partial^2\sigma(\kappa)}{\partial\kappa^2} \right\}, \end{aligned} \tag{11}$$

$$d_1(\kappa) = \frac{\sigma^2(\kappa)\tau/2 - \log(\kappa)}{\sigma(\kappa)\sqrt{\tau}}, \tag{12}$$

$$d_2(\kappa) = d_1(\kappa) - \sigma(\kappa)\sqrt{\tau}, \tag{13}$$

considering the following derived relations: $K = F\kappa$, $\frac{\partial K}{\partial \kappa} = F$, $\frac{\partial\sigma}{\partial K} = \frac{\partial\sigma}{\partial\kappa}\frac{1}{F}$, $\frac{\partial^2\sigma}{\partial K^2} = \frac{\partial^2\sigma}{\partial K^2}\frac{1}{F^2}$. With some analytical computations, the SPD can be standardized to fulfill $\int_{-\infty}^{\infty} q(\kappa)d\kappa = 1$.

It is clear that the true implied volatility function $\sigma(\kappa)$ and its derivatives are not generally known and, thus, that they must be substituted by suitable estimates. We can use α_0, α_1, and

α_2 as approximations, according to their characterization stated above. Then, the optimization model (9), constrained with the no-arbitrage condition, becomes the following:

$$\min_{\alpha_0,\alpha_1,\alpha_2} \sum_{i=1}^{n_\tau} \left[\tilde{\sigma}_i - \alpha_0 - \alpha_1(\kappa_i - \kappa) - \alpha_2(\kappa_i - \kappa)^2\right]^2 \mathcal{K}_h\left(\kappa - \kappa_i\right), \tag{14}$$

$$\text{s.t.} \quad \hat{q}(\kappa) = \sqrt{\tau}\varphi(\hat{d}_1(\kappa)) \left\{ \frac{1}{\kappa^2\alpha_0\tau} + \frac{2\hat{d}_1(\kappa)}{\kappa\alpha_0\sqrt{\tau}}\alpha_1 + \frac{\hat{d}_1(\kappa)\hat{d}_2(\kappa)}{\alpha_0}\alpha_1^2 + 2\alpha_2 \right\} \geq 0, \tag{15}$$

$$\hat{d}_1(\kappa) = \frac{\alpha_0^2\tau/2 - \log(\kappa)}{\alpha_0\sqrt{\tau}}, \tag{16}$$

$$\hat{d}_2(\kappa) = \hat{d}_1(\kappa) - \alpha_0\sqrt{\tau}. \tag{17}$$

If IV is estimated without no-arbitrage constraints (15)–(17), then the estimated SPD may be negative in some points.

To estimate the bias and variance of our results, we refer to the results reported by Kopa et al. (2017), whose work drew on the ideas of Fan and Gijbels (1996) and Fan and Yao (2003).

The previous results can be extended to cases in which the option is written on dividend-paying assets. In such a case, the options are priced as suggested in Merton (1973), and the semiparametric estimation proposed by Benko et al. (2007) must be adjusted accordingly, as shown by Kopa et al. (2017) and Vitali et al. (2017). For the purpose of this paper, we do not need to consider such adjustments, because we consider options written on price indexes that do not pay dividends and, thus, are considered as non-dividend-paying assets.

3 Investment strategies

In the following sections, we analyze four different strategies based on the shape of the SPD: two market timing strategies and two portfolio selection strategies. As mentioned in the introduction, we assume that the investor is risk neutral. Then, according to the information contained in the SPD, she simply wants to identify the best moment to invest (or to disinvest) in a given index, or which are the indexes (in a given universe) for which she expects a potential higher growth. In particular, in each trading day, the market timing strategies indicate either to buy or to sell the underlying index of the options. Meanwhile, in each trading day, the portfolio selection strategies aim to create a ranking among the available indexes, and to identify and invest in the best of them.

Strategy 1: market timing considering SPD areas

The first strategy, *Strat1*, is based on the observation of the area below the SPD. As discussed in the previous section, it is possible to interpret the SPD as the distribution of the future value of the underlying embedded in the quotation of the IV.Thus, if the area below the SPD for moneyness greater than one ($A_{\kappa>1}$) is bigger than the area for moneyness less than one ($A_{\kappa<1}$), then we can argue that the market believes that an increase of the value of the underlying is more probable than a loss. In this case, a risk-neutral investor will decide to invest (to buy or to hold) in the underlying because there is an higher probability of a potential increase. In Fig. 1, $A_{\kappa>1}$ is highlighted in blue, while $A_{\kappa<1}$ is in red. If $A_{\kappa>1} > A_{\kappa<1}$, then the strategy suggests investing in the underlying; otherwise, it suggests not investing in the

Fig. 1 Strategy 1: a comparison between area above and below monenyness 1

underlying. Such a strategy can be implemented as long-only (Strat1-L), meaning that either we invest or do not invest in the index, or as long-short (Strat1-LS), meaning that we could short sell the underlying if $A_{\kappa>1} < A_{\kappa<1}$.

Strategy 2: market timing considering SPD peak

The second strategy, *Strat2*, considers only the modal value of the SPD (the moneyness that corresponds to the SPD maximum), which represents the state that matters the most in the pricing of options, i.e. the state where an additional unit of payoff has the highest price impact. We denote such value as κ_{max}. If $\kappa_{max} > 1$, the state that carries the highest price impact for an additional unit of payoff is greater than the current state and, thus, we assume that the underlying will increase, and vice versa. Figure 2 presents an example of κ_{max}. If $\kappa_{max} > 1$, then the strategy suggests investing in the underlying; otherwise, it suggests not investing in the underlying. In addition, this strategy can be implemented as long-only (Strat2-L), meaning that either we invest or do not invest in the asset, or as long-short (Strat2-LS), meaning that we could short sell the underlying if $\kappa_{max} < 1$.

Strategy 3: Portfolio selection considering SPD areas

The third strategy, *Strat3*, is based on the area of the SPD above and below moneyness equal to 1. Let us assume there are I indexes in the investment universe. To generate a ranking among them, we compute for each index i, $i = 1, ..., I$; the areas $A_{i,\kappa>1}$ and $A_{i,\kappa<1}$; and the ratio $R_i = \frac{A_{i,\kappa>1}}{A_{i,\kappa<1}}$ (i.e., the ratio between the blue and red areas in Fig. 1). We decide the number N of indexes that we want to select among the I indexes available. Then, if we implement a long-only strategy (Strat3-L), we rank the ratios R_i in descending order, and we invest in the N indexes with higher R_i if their $R_i > 1$; otherwise, we invest only in the top one. It can happen that we invest in fewer than N scenarios. If we implement the long-short strategy (Strat3-LS), we rank the absolute value of the ratios $|R_i - 1|$ in descending order, and we invest in the top N indexes in the following way: If $R_i > 1$, we buy, and otherwise,

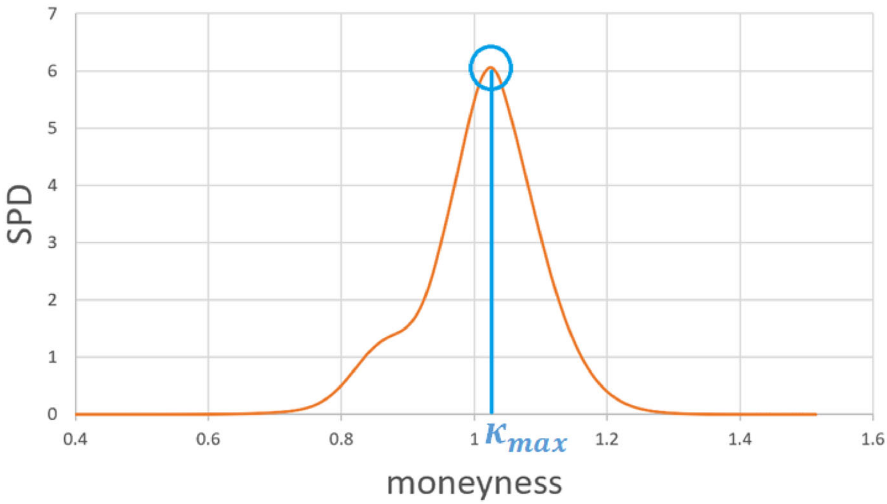

Fig. 2 Strategy 2: observation of the moneyness corresponding to the maximum value of the SPD

we short sell. Therefore, in the -LS case, we always invest in exactly N indexes, either long or short.

Strategy 4: Portfolio selection considering SPD peak

The fourth strategy, *Strat4*, is again based on the κ_{max}, meaning the value of moneyness in correspondence with the peak of the SPD (see Fig. 2). Similarly to Strategy 3, we decide the number N of indexes, and we generate a ranking to establish the best indexes. If we implement the long-only strategy (Strat4-L), we compute for each index $i = 1, ..., I$ its $\kappa_{i,max}$. Then we rank them, and we buy the N indexes with higher $\kappa_{i,max}$, if the $\kappa_{i,max} > 1$. Therefore, we could invest in fewer than N indexes if there are fewer than N indexes with $\kappa_{i,max} > 1$. If we implement a long-short strategy (Strat4-LS), we rank $|\kappa_{i,max} - 1|$ in descending order, and we invest in the top N indexes in the following way: If $\kappa_{i,max} > 1$ we buy, and otherwise, we short sell. Therefore, in this case, we always invest in exactly N indexes, either long or short.

Strategy 3 and strategy 4 with a weighted allocation

It would be difficult for a portfolio manager to implement Strategy 3 and Strategy 4, because they require sudden changes in a huge exposition of the market from one rebalancing day to the next. In other words, the two strategies do not consider any turnover constraints, exposing the portfolio to high rebalancing costs. Indeed, a portfolio manager typically has an open position in each of the available indexes and the day-by-day calibrating decision involves which indexes to overweight and which ones to underweight with respect to a reference allocation. From this perspective, we also define two weighted strategies in which the long/short decision is replaced with an overweight/underweight decision. In this case, the weights are assigned according to the number of indexes that the basic LS strategy suggests to buy/sell, and it follows the data presented in Table 1. We construct the table considering $N = 7$; for a different N, the table should be recalibrated. Clearly, case 0 (negative view

Table 1 Weights assigned to over-weighted and under-weighted indexes according to the number of indexes to overweight and underweight

Case	Indexes to over-(under-)weight	Overweight	Underweight
1	0 (7)	0	14.28%
2	1 (6)	40%	10%
3	2 (5)	25%	10%
4	3 (4)	20%	10%
5	4 (3)	17.5%	10%
6	5 (2)	16%	10%
7	6 (1)	15%	10%
8	7 (0)	14.28%	0%

on all indexes) and case 8 (positive view on all indexes) degenerate to the $1/N$ allocation. In other cases, we consider that with respect to the 14.28% weight of the $1/N$ portfolio, the under-weighted indexes have 10% allocation each, and the remaining allocation is equally split among the other indexes. To review, we again follow the idea of Strat3-LS and Strat4-LS, but instead of buying or selling, we overweight or underweight. Thus, we call these strategies Strat3-W and Strat4-W, respectively.

4 Empirical results

In this study, to perform of the strategies described in Sect. 3, we constructed a universe with seven indexes: S&P500, HangSeng, EuroStoxx50, FTSE100, DAX, CAC40, and Nikkei225. This set was built to include representative markets from around the world and considers those indexes for which, on each day, we can observe a sufficient number of IVs to perform the computation of the SPD. For each index, we downloaded from Datastream (Refinitiv Workspace) the implied volatilities of all the European options quoted on these indexes from January 2018 to December 2023 (data before 2018 are not available in DataStream). The amount of data used is reported in Table 2. In total, we analyzed 6,540,879 implied volatilities to perform our study. Moreover, for each index, we considered the series of the daily closing price over the six years of the study period.

For the market timing strategies, the computation of the returns of the optimal investment (i.e., the trading strategy to buy and to sell the underlying index) consists of the following steps:

1. We choose the index i on which we construct the strategy;
2. Each day d we collect the closing data related to all the quoted options with the nearest maturity having as underlying the chosen index, and the value of the index itself;
3. We run optimization (14)–(17) to identify the IVs and the SPDs of the selected options;
4. We apply the chosen strategy to decide to invest or not to invest in the index;
5. If the strategy suggests buying the index, then the portfolio daily return $r^p_{d,d+1}$ corresponds to the daily return of the index in the period $r^i_{d,d+1}$; if the strategy suggests to short sell the index, then the portfolio daily return $r^p_{d,d+1}$ corresponds to the opposite of the daily return of the index, meaning $-r^i_{d,d+1}$; and
6. The next trading day, we reiterate steps 2–5.

Table 2 Number of implied volatilities downloaded for each index and for each year, and (in brackets) the average number of options quoted on each day for each maturity

	2018	2019	2020	2021	2022	2023
SP500	252,187 (84)	282,764 (94)	366,456 (122)	418,098 (139)	356,811 (119)	385,473 (127)
HangSeng	108,622 (36)	105,700 (35)	114,530 (38)	114,065 (38)	129,931 (43)	179,188 (59)
Nikkei225	103,301 (34)	102,970 (34)	111,949 (37)	122,486 (41)	132,418 (44)	95,889 (31)
EuroStoxx50	151,229 (50)	138,808 (46)	172,436 (57)	188,197 (63)	218,259 (73)	269,178 (89)
FTSE100	148,292 (49)	135,391 (45)	186,101 (62)	154,664 (52)	141,270 (47)	117,201 (39)
DAX	117,120 (39)	115,378 (38)	148,601 (49)	134,135 (45)	117,865 (39)	114,191 (38)
CAC40	38,407 (13)	40,690 (14)	56,705 (19)	50,208 (16)	51,862 (17)	51,853 (17)

In Sect. 4.1, the performance of the optimal strategy is compared with the evolution of the chosen index. For concision, the figure and table related to the S&P500 are in Sect. 4.1, while the results regarding other indexes are reported in Appendix A.

For the portfolio strategies, the computation of the returns of the optimal portfolio (i.e., the portfolio obtained by applying the chosen strategy) consists of the following steps:

1. We choose the indexes $i = 1, ..., N$ that comprise the investment universe;
2. Each day d we collect the closing data related to all the quoted options with the nearest maturity having as underlying the chosen indexes, and the values of the indexes;
3. We run optimization (14)–(17) to find the IVs and the SPDs of the selected options;
4. We apply the chosen strategy to decide which indexes to buy or to short;
5. The portfolio daily return is computed as the average of the daily returns of the indexes in which the portfolio invests, meaning $r^p_{d,d+1} = \sum_{i=1}^{N} \frac{1}{N} r^i_{d,d+1}$; if the strategy suggests to short sell a certain index, we consider its opposite return; for the weighted strategies (Strat3-W and Strat4-W), the weights of the returns are not $1/N$ but follow Table 1; and
6. The next trading day, we reiterate steps 2–5.

In Sect. 4.2, the performance of the optimal portfolio is compared with the evolution of the equally weighted portfolio composed of all the indexes of the investment universe.

Here are further remarks on the two procedures:

- The amount of options used changes every day according to the quoted strikes for the nearest maturity; for instance, for the third of September 2018, we consider all the quoted options having maturity the 21st of September 2018 (typically the options expire on the third Friday of the month), and we will consider the same options (with their new quoted IV) also on the fourth of September, and so on until the 21st of September, when such options expire, and we start to consider the IVs of options expiring in October; in Table 2, we report in brackets the number of options available on average on each day: for example, for SP500 in each day of 2018 we have on average 84 quoted IVs to provide the IV smoothing and the SPD estimation, while for CAC40 in each day of 2018 we have on average only 13 quoted IVs;
- The procedure is repeated daily, meaning that, every day, the investor can update the investment position according to the updated information extracted from the SPD; indeed, even if the considered options are the same until they expire, and the investor starts to study the ones expiring the next month, their IVs change daily to reflect the sentiment of the market operators;
- The smoothing technique that we adopt utilizes only the quoted strikes, and we do not make any estimations for strikes that are not quoted;
- We collect the closing values for the IVs and the closing values for the underlying; we are aware that, in reality, the procedure cannot be perfectly simultaneous, and there must be some time between the observation of the IVs, the SPD computation, and the market operations; still, since (1) near to the closing time the markets are not particularly volatile, and (2) the IV smoothing and the SPD estimation for a single day requires less than one second to be performed (using MATLAB R2025a on a computer equipped with processor Snapdragon® X 12-core X1E80100 3.40 GHz and 16 GB of RAM), our approach gives realistic results;
- We assume to directly buy/sell the underlying index, while, in reality, one cannot directly buy/sell an index, but rather some futures or other replicating financial instruments;
- For all IV smoothing and SPD estimation, we always fix the bandwidth $h = 0.12$, which has been proved in Kopa et al. (2017) and Vitali et al. (2017) to provide a good balance in terms of smoothing.

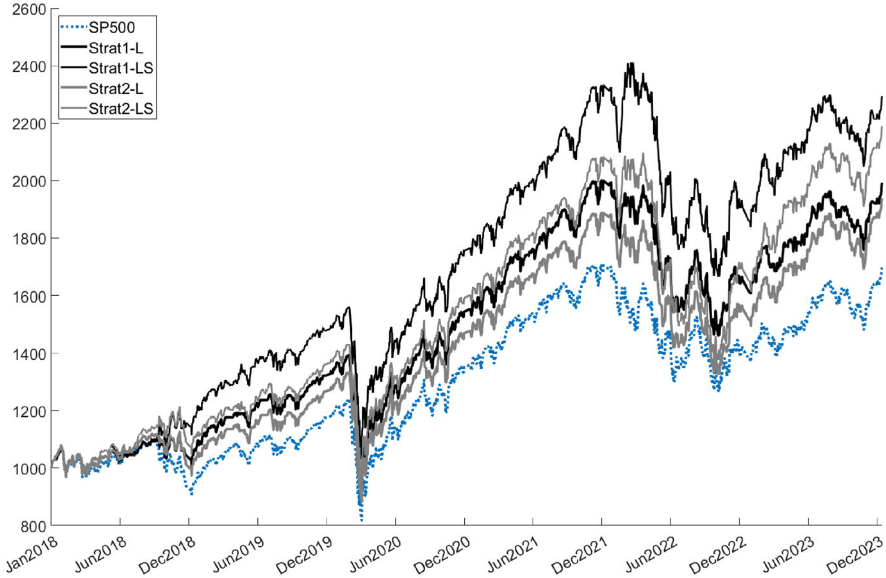

Fig. 3 Strategy 1 and Strategy 2: Results with S&P500. The dotted blue line represents the S&P500, i.e. the benchmark. The black lines represent Strat1 (thick line is Strat1-L and thin line is Strat1-LS). The gray lines represent Strat2 (thick line is Strat2-L and thin line is Strat2-LS)

4.1 Market timing strategies

S&P500 index

Figure 3 represents the evolution of the S&P500 index and of the optimal wealth evolution obtained by implementing Strat1-L, Strat 1-LS, Strat2-L, and Strat2-LS. Table 3 reports the annualized statistics of the daily returns of each process in each year. We note that the strategies that allow short selling perform generally better than the strategies that do not allow short selling and that Strat1 generates better returns than Strat2. In almost all the considered years, the strategies outperform the index both in terms of return and in terms of risk. In particular, the standard deviation of the returns generated with the strategies is lower than the index, while the tail risk is similar. In 2018, the strategies greatly beat the index; in the period from 2019–2021, the strategies are slightly better than the index; in 2022, the strategies suffer slightly more than the index during the financial crisis; and in 2023, the four strategies outperform the index remarkably. In general, we observe that the options written on S&P500 show optimism, whereas the strategies often suggest maintaining the long position. The few times that the market agents have a bad view of the next feature, and thus the strategies either exit the index or short it, the index actually suffers some turbulence that the strategies can either avoid or exploit. In particular, Table 4 reports, for each year, the number of trading days for which we have quoted IVs, the number of days in which each strategy suggests to stay long/short, and the number of times that the investment is changed from a long position to a short position or vice versa. From these data, we can confirm that the strategies mainly suggest having a long position. In particular, in 2020, during the COVID-19 pandemic, the market operators showed a strong confidence that the index would not suffer too much, and Strat1 suggests to go short eight days, while Strat2 suggests shorting only one day in the

Table 3 Statistics on the daily return of the S&P500 Index and of the four market-timing strategies

		SP500	Strat 1-L	Strat 1-LS	Strat 2-L	Strat 2-LS
2018	Annual. Average	−6.7%	3.8%	14.3%	0.8%	8.2%
	Annual. Std Dev	15.8%	14.5%	15.8%	14.9%	15.8%
	Skewness	−0.89	−1.05	−0.76	−0.99	−0.77
	Kurtosis	5.29	6.54	5.58	5.98	5.49
	V@R at 5%	−2.0%	−1.8%	−2.0%	−1.9%	−2.0%
	CV@R at 5%	−2.7%	−2.6%	−2.6%	−2.6%	−2.6%
2019	Annual. Average	25.2%	26.0%	26.9%	25.0%	24.8%
	Annual. Std Dev	12.4%	11.6%	12.4%	11.9%	12.4%
	Skewness	−0.57	−0.49	−0.33	−0.49	−0.36
	Kurtosis	6.26	7.42	6.16	6.81	6.15
	V@R at 5%	−1.2%	−0.9%	−1.1%	−1.2%	−1.2%
	CV@R at 5%	−2.0%	−1.8%	−1.8%	−1.9%	−1.9%
2020	Annual. Average	20.1%	21.6%	23.2%	20.9%	21.7%
	Annual. Std Dev	34.4%	34.4%	34.4%	34.4%	34.4%
	Skewness	−0.55	−0.55	−0.56	−0.55	−0.55
	Kurtosis	10.79	10.82	10.81	10.81	10.80
	V@R at 5%	−3.4%	−3.4%	−3.4%	−3.4%	−3.4%
	CV@R at 5%	−5.5%	−5.5%	−5.5%	−5.5%	−5.5%
2021	Annual. Average	23.2%	25.8%	28.4%	25.0%	26.8%
	Annual. Std Dev	12.8%	12.6%	12.7%	12.7%	12.7%
	Skewness	−0.38	−0.40	−0.42	−0.40	−0.42
	Kurtosis	3.86	4.02	3.94	3.95	3.92
	V@R at 5%	−1.3%	−1.3%	−1.3%	−1.3%	−1.3%
	CV@R at 5%	−1.9%	−1.9%	−1.9%	−1.9%	−1.9%
2022	Annual. Average	−18.1%	−18.9%	−19.7%	−19.2%	−20.3%
	Annual. Std Dev	24.0%	22.3%	24.0%	22.6%	24.0%
	Skewness	0.04	−0.04	−0.05	0.00	0.00
	Kurtosis	3.44	4.17	3.42	4.02	3.43
	V@R at 5%	−2.8%	−2.8%	−2.8%	−2.8%	−2.8%
	CV@R at 5%	−3.4%	−3.4%	−3.4%	−3.4%	−3.4%
2023	Annual. Average	21.6%	21.9%	22.1%	25.5%	29.4%
	Annual. Std Dev	12.9%	11.5%	12.9%	12.5%	12.9%
	Skewness	0.03	0.17	0.08	0.03	−0.02
	Kurtosis	2.84	3.60	2.82	3.11	2.86
	V@R at 5%	−1.4%	−1.2%	−1.3%	−1.3%	−1.3%
	CV@R at 5%	−1.6%	−1.5%	−1.6%	−1.6%	−1.6%

Table 4 Number of total trading days for SP500, days in which the strategy suggests to stay long, days in which the strategy suggests to stay short, number of changes from a long position to a short position or vice-versa

	Total	Strat1			Strat2		
		Long	Short	Changes	Long	Short	Changes
2018	253	204	49	38	225	28	32
2019	252	211	41	22	221	31	14
2020	251	243	8	12	250	1	2
2021	250	238	12	16	241	9	10
2022	249	217	32	44	222	27	45
2023	248	200	48	56	230	18	29

whole year. Moreover, we notice that in 2018 and 2019 the number of position changes is typically less than the number of days with short position, meaning that the short position is maintained for more days consecutively, while in 2022 and 2023 the position changes are more than the days with short position, meaning that these are not very consecutive. The information about the days in which the position changes also gives us an idea of the transaction costs that an investor would pay to implement such strategies.

Clearly, the days in which the SPD suggests having a short position are the most interesting ones. For this reason, as an example, we show in Fig. 4 the IV curves of four consecutive trading days: 13th, 16th, 17th, and 18th of July 2018; and in Fig. 5, the corresponding SPD curves. On the 13th, Strat1 and Strat2 give a long signal, while, on the next three days, the signal changes to short for both strategies. On the 13th, the IV curve touches the minimum at a moneyness of 1.0102 and the SPD curve touches the maximum at a moneyness of 1.0013 (both above 1); on the 16th, the 17th, and the 18th, the IV curves touch the minimum at moneyness 0.972, 0.989, and 0.989, respectively, and the SPD curves touch the maximum at moneyness 0.996, 0.998, 0.998, respectively (all below 1). Moreover, the three days that give a short signal show a right part of the IV curve more steep and the SPD curves more and more leptokurtic. A similar behavior with the IV minimum point that moves up and to the left and a consecutive change of the SPD occurs for other cases in which the SPD give short signals. For sake of conciseness, we do not propose other examples in this work but we refer to Vitali et al. (2023) for more details about the peculiar shape of smoothed IV curves during periods of turmoil.

HangSeng index

Figure 8 and Table 7 represent the results for the strategies that consider the HangSeng index. In this case, we observe that the IVs and the SPDs present long periods of pessimism about the next evolution of the index, in particular at the end of 2018, the beginning of 2019, and the first half of 2021 (as well as many other shorter periods). Indeed, Table 8 shows that this market is the one that gives the highest number of short and persistent signals. The strategies are able to catch this signal and stay out of the market (or short it) efficiently, thus accumulating a large, positive gap with respect to the index. Strat1 generates better returns than Strat 2 in 2018, 2019, and 2020. In the case of HangSeng, the strategies not only beat the index but significantly reduce the tail risk.

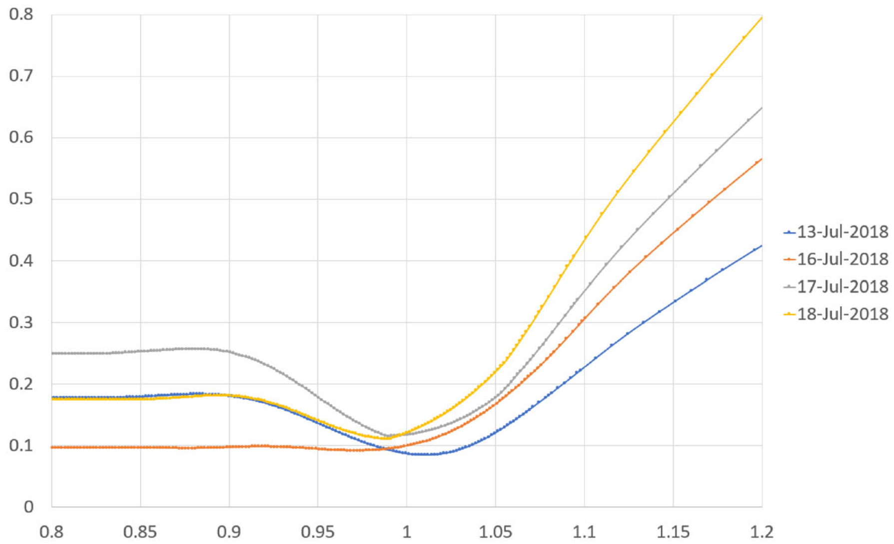

Fig. 4 IV curves for four consecutive trading days, the first with a long position, the next three suggesting a short position. For visualization purposes, the moneyness is restricted between 0.8 and 1.2

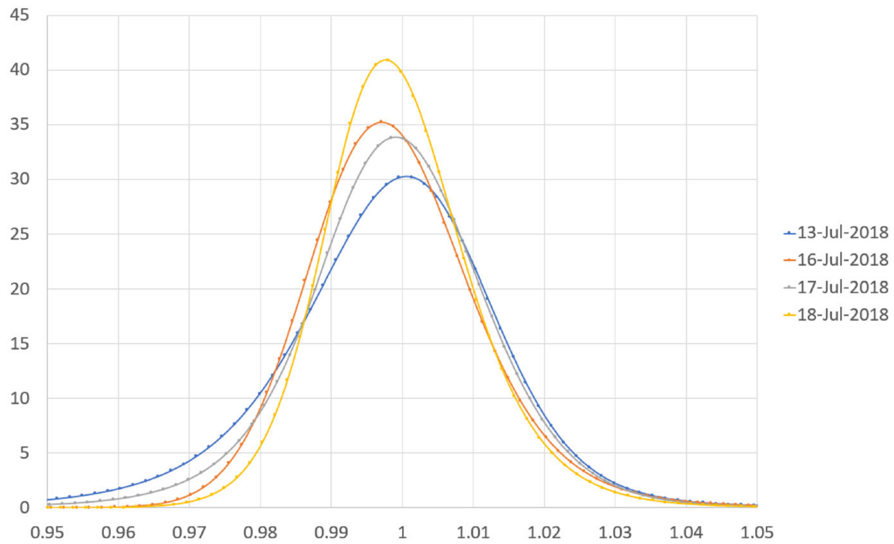

Fig. 5 SPD curves for four consecutive trading days, the first with a long position, the next three suggesting a short position. For visualization purposes, the moneyness is restricted between 0.95 and 1.05

Nikkei225 index

Figure 9 and Table 9 depict the results of the strategies considering the Nikkei225 index. Clearly, in this case, the information collected by the IVs and the SPDs are less informative than in the previous cases. Only in 2018 are the strategies able to beat the index (both in terms of return and of risk), while in subsequent years, the market agents are very positive, and the strategies almost identically follow the underlying index. Finally, in 2023, the market turmoil induces agents to be excessively pessimistic, and the strategies receive inaccurate signals from the SPDs (suggesting that the strategies exit the index or short sell it), losing whatever small recovery the index is able to achieve, even if the average return of this year is also negative for the index. However, the tail risk of the returns generated by the four strategies is always equal to or better than the index tail risk. In Table 10 we observe the same evidence: In 2018, there are a few (correct) short signals, while in 2019–2022, there are almost no short signals, and in the first half of 2023, the strategies suggest shorting the index.

EuroStoxx50 index

Figure 10 and Table 11 show the results of the strategies that consider the EuroStoxx50. During spring 2018, the SPDs show some pessimism for a few weeks in a row, while the index rebounds and, thus, the strategies generate a negative gap. This gap is reduced during the pandemic period, when the strategies are able to detect the negative market sentiment, at least for few days, and Strat1 overperforms the benchmark, while Strat2 remains very close to it. Indeed, in 2020, the average return of EuroStoxx50 is slightly negative, while the strategies generate an average return between 4.6 and 13.3%. The performance of the strategies improves during the initial months of 2021. Strat1 accumulates a more positive gap, and Strat2 slightly overperforms the benchmark as well. Table 12 shows that the signals are mainly positive for both strategies.

Figure 11 and Table 13 depict the performance of the strategies that consider the FTSE100. All strategies avoid the huge loss of the index at the beginning of 2018 and accumulate a notable positive gap. However, some uncertainty does not enable the entire rebound of mid-2018 to be caught, and the gap quickly reduces. Then, the strategies again diverge from the index at the beginning of 2019, and this margin increases until the initial period of the pandemic. At the beginning of 2021, the market agents forecast some turmoil, suggesting to stay out of the market; thus, the strategies lose some returns that the index generates in the first half of 2021 and at the end of 2022. The statistics confirm this observation, showing a large positive gap for the strategies during 2021, but a relatively bad performance during 2021 and 2022 nullifies this margin. Still, the volatility and the tail risk measures of the strategies are always better than or equal to those of the index. Table 14 highlights mainly long signals but also some relevant number of short signals.

DAX index

Figure 12 and Table 15 present the results of the strategies built on the DAX index. The strategies are not able to catch the positive return of the index in the first half of 2018, but all of them compensate for this gap in the second half of 2019, when the SPDs highlight some turbulence. The strategies are aligned with the index until the end of 2020, when they fall with the index but are not able to rebound because of pessimism in the market. Again, the negative margin is compensated after the beginning of the Russian invasion of Ukraine, when the strategies avoid some of the losses suffered by the index. The statistics show that the strategies perform especially well in 2019 and 2022, when both the volatility and tail risk measures are better than the DAX statistics. Table 16 reports that the market's operators are mainly positive, and the few negative signals are not very persistent, as shown by the high number of position changes.

CAC40 index

Figure 13 and Table 17 show the results of the strategies considering the CAC40 index. At the very beginning of the analyzed period, none of the strategies catch the first relevant positive returns of the index. This negative gap remains almost constant during the whole period. Strat1 is somewhat able to recover and reduce this gap, while Strat2 enlarges it until the very last year, when it also recovers. At the horizon, the final results are almost equal between the strategies and the index. The statistics confirm that the performance of Strat2 is very bad in 2019 but recovers greatly in 2022, while Strat1 is much more aligned with the index. However, the risk measures of the strategies are always equal to or better than those of the benchmark. Table 18 reports that the number of short days are relatively low and not very persistent.

4.2 Portfolio strategies

Strategy 3 and Strategy 4

Fig. 6 and Table 5 present the performance of the $1/N$ portfolio, which considers an equally weighted allocation among the five indexes considered in the previous section and the wealth generated by implementing Strat3 and Strat4. We note that in 2018, only Strat3-L generates a worse result than the benchmark, while the other three strategies are able to mitigate some of the losses of the markets and create a positive margin with respect to the $1/N$ portfolio. This margin is constantly enlarged during the following years, especially by Strat4. This result is not surprising because the portfolio strategies apply and highlight the evidence obtained in the previous section for the single indexes. Still, it is noteworthy that even among the indexes for which the SPDs indicates a positive return, the proposed approach identifies the best indexes, and this capability significantly improves the overall performance. A side effect of these positive gains is that in some years, the volatility and, especially, the tail risk measures show that the strategies embed a slightly higher risk than the $1/N$ portfolio. However, as shown in Fig. 6, it is clear that the strategies generate outperformance with respect to the benchmark portfolio. To better compare these results, we also perform a second-order stochastic dominance (SSD) test between the return distributions generated by the strategies and that of the $1/N$ portfolio. Interestingly, Strat3-L, Strat3-LS, and Strat4-LS dominate the $1/N$ portfolio according to SSD. As remarked previously, this comparison is not fully realistic because it assumes that the portfolio manager is able to completely rebalance the portfolio allocation every day. Moreover, none of the statistics presented in this work consider any transaction costs. However, the number of times the strategies actually suggest changing the allocation is relatively low, because most days, the strategies mimic the movement of the benchmark. For a fair comparison, in the next analysis, we limit the rebalancing effect of the portfolio.

Strategy 3 and strategy 4 with underweight and overweight

Fig. 7 and Table 6 present and summarize the results of the $1/N$ portfolio and of the strategies that rank the available indexes, but that overweight only best indexes while underweighting the worst indexes—but not excluding them completely from the allocation. This experiment is more aligned with the market operation of a portfolio manager, because a 100% turnover cannot occur every day during a daily rebalancing. Despite their limited capability to recal-

Table 5 Statistics on the daily return of the 1/N Portfolio and of the four portfolio strategies that allow a full rebalancing of the allocation

		1/N Port	Strat 3-L	Strat 3-LS	Strat 4-L	Strat 4-LS
2018	Annual. Average	−8.2%	−9.6%	−5.7%	−2.2%	−6.3%
	Annual. Std Dev	11.4%	11.7%	11.2%	11.6%	11.2%
	Skewness	−0.74	−0.55	−0.59	−0.44	−0.35
	Kurtosis	4.05	3.33	3.62	3.16	3.23
	V@R at 5%	−1.3%	−1.3%	−1.3%	−1.3%	−1.3%
	CV@R at 5%	−1.9%	−1.8%	−1.7%	−1.7%	−1.6%
2019	Annual. Average	17.8%	21.9%	25.8%	23.4%	25.9%
	Annual. Std Dev	10.1%	10.9%	10.1%	10.9%	10.2%
	Skewness	−0.55	−0.38	−0.30	−0.38	−0.24
	Kurtosis	4.84	4.80	5.70	4.77	5.26
	V@R at 5%	−1.1%	−1.2%	−1.0%	−1.1%	−1.0%
	CV@R at 5%	−1.5%	−1.6%	−1.5%	−1.6%	−1.4%
2020	Annual. Average	3.5%	5.7%	7.8%	14.0%	28.9%
	Annual. Std Dev	25.8%	26.3%	26.3%	29.1%	27.6%
	Skewness	−0.74	−0.47	−0.49	−0.87	−0.72
	Kurtosis	11.22	10.01	10.07	11.75	13.14
	V@R at 5%	−2.6%	−3.0%	−3.0%	−3.0%	−2.5%
	CV@R at 5%	−4.3%	−4.5%	−4.5%	−4.9%	−4.3%
2021	Annual. Average	13.0%	23.9%	22.3%	32.5%	37.4%
	Annual. Std Dev	11.0%	12.1%	11.9%	12.8%	12.6%
	Skewness	−0.81	−0.45	−0.50	−0.88	−0.87
	Kurtosis	6.61	5.46	5.73	7.09	7.58
	V@R at 5%	−1.1%	−1.1%	−1.2%	−1.2%	−1.1%
	CV@R at 5%	−1.8%	−1.7%	−1.8%	−1.9%	−1.9%
2022	Annual. Average	−6.8%	3.7%	3.6%	4.9%	4.6%
	Annual. Std Dev	16.7%	19.0%	18.8%	20.6%	19.6%
	Skewness	0.25	0.33	0.39	0.26	0.40
	Kurtosis	4.25	4.95	5.20	4.77	5.49
	V@R at 5%	−1.7%	−1.9%	−1.9%	−2.0%	−1.9%
	CV@R at 5%	−2.3%	−2.6%	−2.5%	−2.7%	−2.5%
2023	Annual. Average	10.8%	17.5%	19.1%	19.7%	24.0%
	Annual. Std Dev	10.4%	10.8%	8.6%	10.8%	9.4%
	Skewness	−0.40	−0.18	0.07	−0.21	−0.09
	Kurtosis	3.73	3.22	3.70	2.99	3.10
	V@R at 5%	−1.1%	−1.2%	−0.8%	−1.1%	−0.9%
	CV@R at 5%	−1.5%	−1.5%	−1.1%	−1.4%	−1.2%

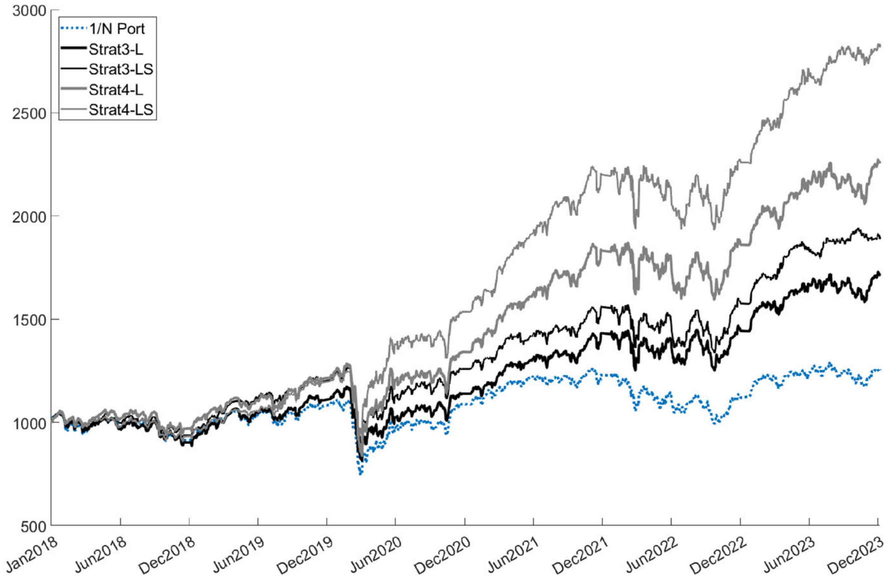

Fig. 6 Strategy 3 and Strategy 4 with full rebalancing. The dotted blue line represents the $1/N$ portfolio, i.e. the benchmark. The black lines represent Strat3 (thick line is Strat3-L and thin line is Strat3-LS). The gray lines represent Strat4 (thick line is Strat4-L and thin line is Strat4-LS)

ibrate the portfolio, both Strat3-W and Strat4-W outperform the $1/N$ portfolio in all the considered years. Moreover, the risk of the strategies is aligned with that of the benchmark. Strat3-W seems to be slightly less conservative than Strat4-W, as it generates more extreme returns.

5 Conclusion

In this paper, we aimed to demonstrate whether implied volatility and state price density can provide valuable information to provide a basis for efficiently implementing a market timing strategy or a portfolio selection strategy for a risk-neutral investor. We tested the market timing strategy on a set of indexes, comparing the results with a simple buy-and-hold investment in the index itself. The portfolio selection strategies were evaluated against the benchmark of an equally weighted portfolio. Several sub-strategies were considered to explore possible alternative ways to exploit the information observed in the SPDs.

For the market timing strategies, we tried two types of implementation: Strat1, which considers the areas below the SPD, and Strat2, which considers the peak of the SPD. Both, are applied either as long-only (-L) or as long-short (-LS) investment. After collecting all the results, we observe the following:

- Strat1 and Strat2 perform quite differently according to the considered underlying index.
- Strat1 is always better then Strat2 in terms of both return and risk.
- In most cases, the strategy beats the underlying index, and the profit generated by the strategy is sometimes extremely large compared with the buy-and-hold investment.
- The wealth produced by Strat1-LS almost always has better statistics than the wealth produced by Strat1-L.

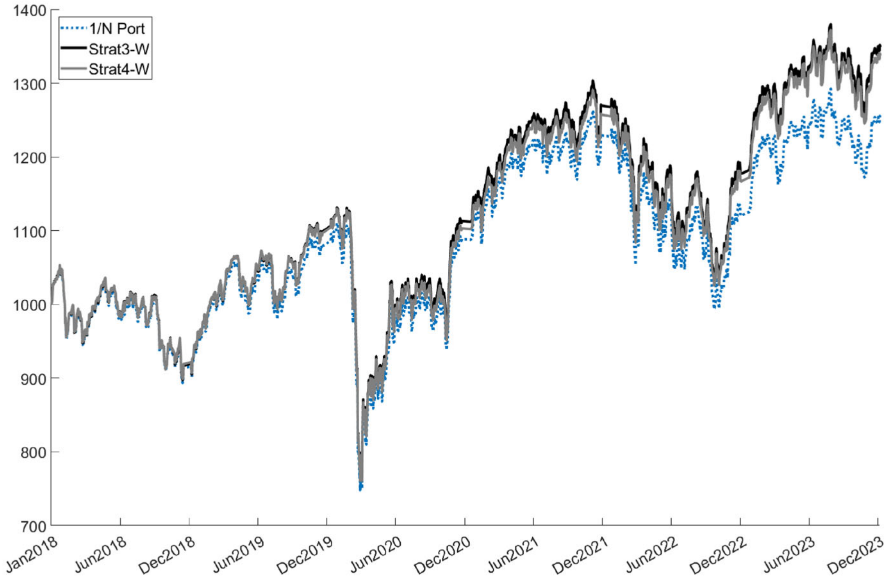

Fig. 7 Strategy 3 and Strategy 4 with limited rebalancing. The dotted blue line represents the $1/N$ portfolio, i.e. the benchmark. The black line represents Strat3-W. The gray line represents Strat4-W

We can conclude that, as far as the market timing strategies are concerned, the area below the SPD is more informative than the peak of the SPD. Indeed, the forecast capability of the whole curve seems to be stronger than the peak point alone. In this study, the transaction costs were not considered. However, a portfolio manager can partially quantify their impact on the strategies, considering the information reported on the tables about the number of days in which the investment changes from a long position to a short position, or vice versa, for which some transaction fees must be paid. In some cases, the strategies require rebalancing only few times per year; in others, rebalancing is required once per week, on average.

For the portfolio selection strategies, we again exploited the same information as for the market timing strategies but in a different way; indeed, Strat3 considers the area below the SPD and Strat4 the peak of the SPD. These strategies use the information to generate a ranking of the available indexes. Then, the portfolio is composed using a long-only strategy (-L), a long-short strategy (-LS), or a weighted (-W) strategy. The first strategy suggests buying the top-ranked indexes, the second to buy the top-ranked indexes and to short sell the bottom-ranked ones, and the third to overweight the top-ranked indexes and to underweight the bottom-ranked ones. After collecting all the results, we observe the following:

- Strat3 generates a wealth path with better statistics and, in the -L and -LS cases, dominates even the equally weighted portfolio, according to SSD.
- All the implemented strategies beat the benchmark over the six years considered in the analysis.
- In addition, the -W case generates a higher average return than the benchmark and has a similar volatility and tail risk.

We can conclude that, regarding the portfolio selection strategies, the area below the SPD is more informative than the peak of the SPD. In addition, in this case, the transaction costs would not have too great an impact because the strategies usually mimic the benchmark, and

Table 6 Statistics on the daily return of the 1/N Portfolio and of the two portfolio strategies that allow a limited rebalancing of the allocation

		1/N Port	Strat 3-W	Strat 4-W
2018	Annual. Average	−8.2%	−7.8%	−7.6%
	Annual. Std Dev	11.4%	11.4%	11.4%
	Skewness	−0.74	−0.75	−0.75
	Kurtosis	4.05	4.03	4.05
	V@R at 5%	−1.3%	−1.3%	−1.3%
	CV@R at 5%	−1.9%	−1.9%	−1.9%
2019	Annual. Average	17.8%	19.2%	18.8%
	Annual. Std Dev	10.1%	10.0%	10.1%
	Skewness	−0.55	−0.56	−0.57
	Kurtosis	4.84	4.75	4.82
	V@R at 5%	−1.1%	−1.1%	−1.1%
	CV@R at 5%	−1.5%	−1.5%	−1.5%
2020	Annual. Average	3.5%	3.9%	3.2%
	Annual. Std Dev	25.8%	25.8%	25.9%
	Skewness	−0.74	−0.71	−0.72
	Kurtosis	11.22	11.37	11.38
	V@R at 5%	−2.6%	−2.6%	−2.6%
	CV@R at 5%	−4.3%	−4.4%	−4.4%
2021	Annual. Average	13.0%	14.0%	13.9%
	Annual. Std Dev	11.0%	11.0%	11.0%
	Skewness	−0.81	−0.82	−0.83
	Kurtosis	6.61	6.68	6.68
	V@R at 5%	−1.1%	−1.2%	−1.2%
	CV@R at 5%	−1.8%	−1.8%	−1.8%
2022	Annual. Average	−7.5%	−5.7%	−5.6%
	Annual. Std Dev	16.7%	16.8%	16.8%
	Skewness	0.26	0.27	0.27
	Kurtosis	4.25	4.27	4.30
	V@R at 5%	−1.7%	−1.6%	−1.7%
	CV@R at 5%	−2.3%	−2.3%	−2.3%
2023	Annual. Average	10.8%	13.3%	13.5%
	Annual. Std Dev	10.4%	10.2%	10.3%
	Skewness	−0.40	−0.36	−0.35
	Kurtosis	3.73	3.66	3.66
	V@R at 5%	−1.1%	−1.1%	−1.1%
	CV@R at 5%	−1.5%	−1.5%	−1.5%

for Strat3-W and Strat4-W, the rebalancing costs should be even lower because the positions are never completely closed or reopened. This analysis could be extended in several ways:

- By analyzing a stocks instead of indexes portfolio, i.e. dividend-paying assets;
- By extending the number of assets and checking the computing time of the whole process;
- By testing the model's adoption for both an active and a passive investment fund.

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Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest and that the authors have nothing to disclose.

Appendix A Figures and tables for results of Sect. 4.1

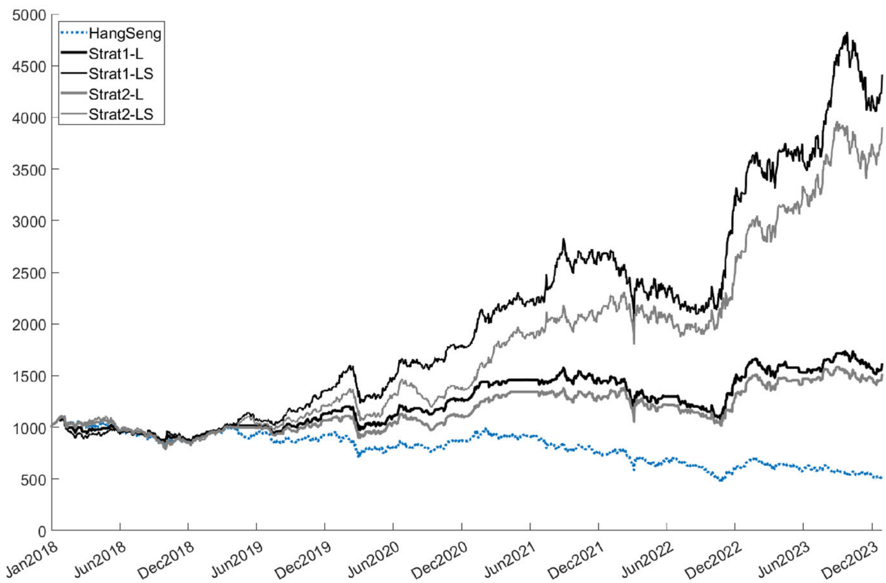

Fig. 8 Strategy 1 and Strategy 2 results for HangSeng. The dotted blue line represents HangSeng, i.e. the benchmark. The black lines represent Strat1 (thick line is Strat1-L and thin line is Strat1-LS). The gray lines represent Strat2 (thick line is Strat2-L and thin line is Strat2-LS)

Table 7 Statistics on the daily return of the HangSeng Index and of the four market-timing strategies

		HangSeng	Strat 1-L	Strat 1-LS	Strat 2-L	Strat 2-LS
2018	Annual. Average	−13.2%	−11.6%	−10.0%	−13.9%	−14.5%
	Annual. Std Dev	19.2%	15.6%	19.2%	17.0%	19.2%
	Skewness	−0.32	−0.62	−0.25	−0.54	−0.35
	Kurtosis	4.12	7.73	4.14	6.08	4.11
	V@R at 5%	−2.2%	−1.8%	−2.2%	−2.2%	−2.2%
	CV@R at 5%	−2.8%	−2.7%	−2.7%	−2.8%	−2.8%
2019	Annual. Average	8.8%	26.0%	43.2%	22.5%	36.3%
	Annual. Std Dev	15.5%	11.9%	15.3%	12.2%	15.4%
	Skewness	−0.04	0.29	0.05	0.31	0.14
	Kurtosis	4.47	8.15	4.52	7.52	4.45
	V@R at 5%	−1.8%	−1.0%	−1.4%	−1.1%	−1.4%
	CV@R at 5%	−2.3%	−1.7%	−2.1%	−1.7%	−2.0%
2020	Annual. Average	−3.9%	12.5%	28.8%	5.5%	14.8%
	Annual. Std Dev	22.7%	19.4%	22.6%	20.2%	22.7%
	Skewness	−0.32	−0.35	−0.17	−0.29	−0.10
	Kurtosis	5.22	6.93	5.32	6.01	5.26
	V@R at 5%	−2.3%	−2.2%	−2.2%	−2.2%	−2.2%
	CV@R at 5%	−3.6%	−3.2%	−3.4%	−3.2%	−3.4%
2021	Annual. Average	−13.6%	14.3%	42.2%	16.6%	46.8%
	Annual. Std Dev	19.6%	13.7%	19.4%	14.3%	19.4%
	Skewness	−0.36	−0.32	0.01	−0.29	−0.01
	Kurtosis	3.64	6.68	3.72	5.85	3.74
	V@R at 5%	−2.1%	−1.5%	−1.8%	−1.6%	−1.8%
	CV@R at 5%	−2.9%	−2.2%	−2.5%	−2.2%	−2.5%
2022	Annual. Average	−11.8%	6.1%	24.1%	7.2%	26.3%
	Annual. Std Dev	32.0%	27.7%	32.0%	27.1%	32.0%
	Skewness	0.85	1.36	0.90	1.43	0.87
	Kurtosis	5.89	8.91	5.64	9.44	5.64
	V@R at 5%	−3.0%	−2.5%	−2.9%	−2.4%	−2.9%
	CV@R at 5%	−4.0%	−3.4%	−3.6%	−3.3%	−3.7%
2023	Annual. Average	−12.1%	9.8%	31.7%	13.0%	38.1%
	Annual. Std Dev	21.7%	16.4%	21.6%	16.6%	21.6%
	Skewness	0.35	0.42	−0.06	0.59	0.08
	Kurtosis	3.14	5.33	3.11	5.70	3.05
	V@R at 5%	−2.1%	−1.8%	−2.1%	−1.7%	−2.1%
	CV@R at 5%	−2.5%	−2.3%	−2.7%	−2.2%	−2.6%

Table 8 Number of total trading days for HangSeng, days in which the strategy suggests to stay long, days in which the strategy suggests to stay short, number of changes from a long position to a short position or vice-versa

	Total	Strat1			Strat2		
		Long	Short	Changes	Long	Short	Changes
2018	255	154	101	75	182	73	70
2019	254	156	98	55	170	84	58
2020	257	182	75	54	207	50	48
2021	256	126	130	68	143	113	77
2022	255	164	91	80	171	84	71
2023	258	150	108	86	147	111	98

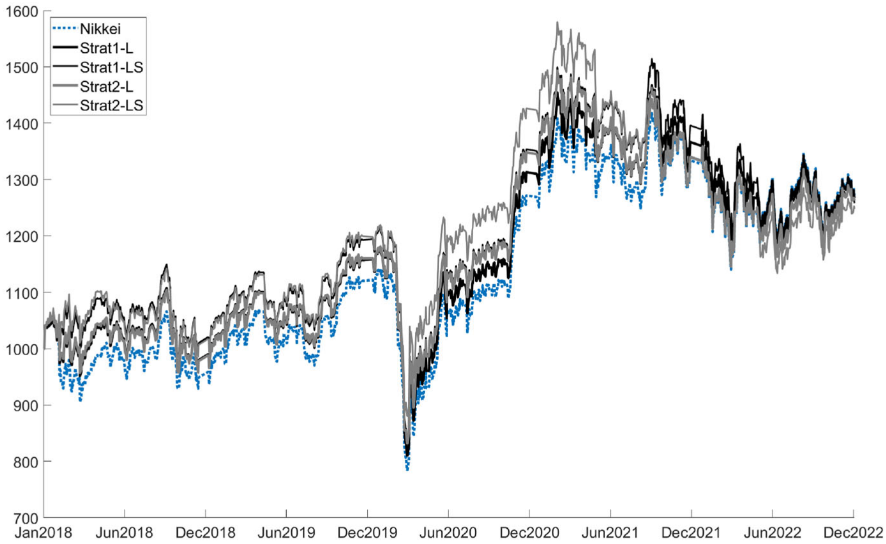

Fig. 9 Strategy 1 and Strategy 2 results for Nikkei225. The dotted blue line represents Nikkei225, i.e. the benchmark. The black lines represent Strat1 (thick line is Strat1-L and thin line is Strat1-LS). The gray lines represent Strat2 (thick line is Strat2-L and thin line is Strat2-LS)

Table 9 Statistics on the daily return of the Nikkei225 Index and of the four market-timing strategies

		Nikkei225	Strat 1-L	Strat 1-LS	Strat 2-L	Strat 2-LS
2018	Annual. Average	−2.8%	0.4%	3.6%	0.2%	3.2%
	Annual. Std Dev	17.5%	17.1%	17.5%	17.1%	17.5%
	Skewness	−0.83	−0.84	−0.74	−0.84	−0.74
	Kurtosis	5.72	6.11	5.79	6.12	5.79
	V@R at 5%	−2.0%	−1.9%	−1.9%	−1.9%	−1.9%
	CV@R at 5%	−3.0%	−2.9%	−2.9%	−2.9%	−2.9%
2019	Annual. Average	16.8%	16.8%	16.8%	17.1%	17.5%
	Annual. Std Dev	13.3%	13.3%	13.3%	13.3%	13.3%
	Skewness	−0.15	−0.15	−0.15	−0.16	−0.16
	Kurtosis	4.37	4.37	4.37	4.38	4.37
	V@R at 5%	−1.5%	−1.5%	−1.5%	−1.5%	−1.5%
	CV@R at 5%	−2.0%	−2.0%	−2.0%	−2.0%	−2.0%
2020	Annual. Average	15.7%	15.7%	15.7%	18.2%	20.7%
	Annual. Std Dev	25.5%	25.5%	25.5%	25.4%	25.5%
	Skewness	0.36	0.36	0.36	0.36	0.35
	Kurtosis	7.74	7.74	7.74	7.86	7.73
	V@R at 5%	−2.5%	−2.5%	−2.5%	−2.4%	−2.4%
	CV@R at 5%	−3.9%	−3.9%	−3.9%	−3.9%	−3.9%
2021	Annual. Average	6.3%	5.4%	4.5%	0.7%	−4.9%
	Annual. Std Dev	17.9%	17.5%	17.9%	17.6%	17.9%
	Skewness	−0.21	−0.21	−0.18	−0.22	−0.21
	Kurtosis	3.55	3.68	3.54	3.67	3.52
	V@R at 5%	−2.0%	−2.0%	−2.0%	−2.0%	−2.0%
	CV@R at 5%	−2.5%	−2.5%	−2.5%	−2.5%	−2.6%
2022	Annual. Average	−2.4%	−5.3%	−8.2%	−3.7%	−4.9%
	Annual. Std Dev	19.9%	19.8%	19.9%	19.7%	19.9%
	Skewness	0.08	0.07	0.06	0.05	0.03
	Kurtosis	3.37	3.41	3.37	3.43	3.37
	V@R at 5%	−2.1%	−2.1%	−2.2%	−2.1%	−2.2%
	CV@R at 5%	−2.7%	−2.7%	−2.7%	−2.7%	−2.7%
2023	Annual. Average	24.9%	12.5%	0.0%	23.6%	22.3%
	Annual. Std Dev	15.6%	8.0%	15.7%	9.4%	15.6%
	Skewness	−0.06	0.27	−0.09	0.50	−0.09
	Kurtosis	2.94	9.08	2.91	7.47	2.94
	V@R at 5%	−1.7%	−0.7%	−1.7%	−0.9%	−1.7%
	CV@R at 5%	−2.0%	−1.3%	−2.1%	−1.4%	−2.0%

Table 10 Number of total trading days for Nikkei225, days in which the strategy suggests to stay long, days in which the strategy suggests to stay short, number of changes from a long position to a short position or vice-versa

	Total	Strat1			Strat2		
		Long	Short	Changes	Long	Short	Changes
2018	248	236	12	14	241	7	12
2019	247	247	0	0	245	2	3
2020	246	246	0	0	245	1	3
2021	245	241	4	6	242	3	6
2022	244	241	3	5	241	3	6
2023	305	142	163	33	157	148	40

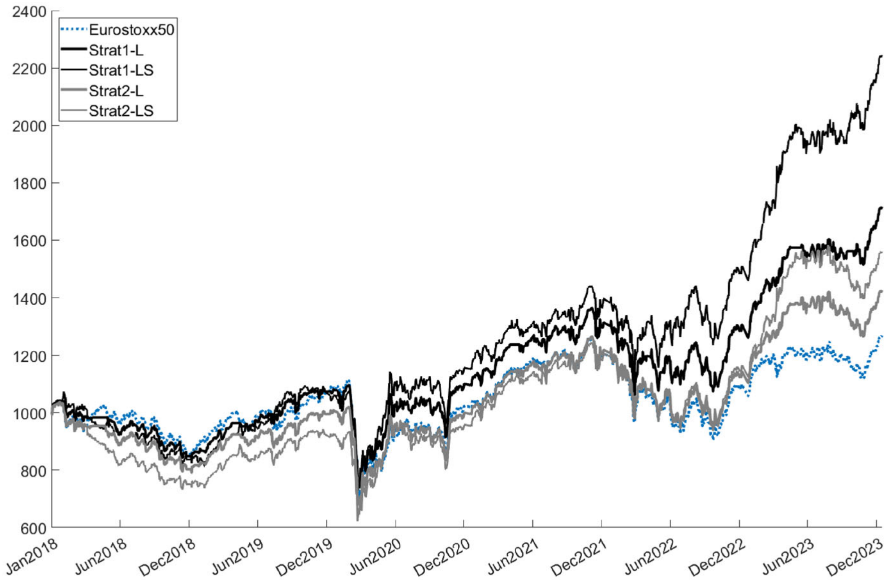

Fig. 10 Strategy 1 and Strategy 2 results for EuroStoxx50. The dotted blue line represents EuroStoxx50, i.e. the benchmark. The black lines represent Strat1 (thick line is Strat1-L and thin line is Strat1-LS). The gray lines represent Strat2 (thick line is Strat2-L and thin line is Strat2-LS)

Table 11 Statistics on the daily return of the EuroStoxx50 Index and of the four market-timing strategies

		EuroStoxx50	Strat 1-L	Strat 1-LS	Strat 2-L	Strat 2-LS
2018	Annual. Average	−14.4%	−15.5%	−16.5%	−20.8%	−27.3%
	Annual. Std Dev	13.3%	11.7%	13.3%	12.3%	13.3%
	Skewness	−0.39	−0.72	−0.47	−0.61	−0.48
	Kurtosis	3.71	4.89	3.67	4.27	3.59
	V@R at 5%	−1.5%	−1.5%	−1.5%	−1.5%	−1.5%
	CV@R at 5%	−2.0%	−1.9%	−2.0%	−2.0%	−2.0%
2019	Annual. Average	23.5%	23.9%	24.2%	21.9%	20.2%
	Annual. Std Dev	12.9%	10.6%	12.9%	11.4%	12.9%
	Skewness	−0.55	−0.23	0.07	−0.60	−0.42
	Kurtosis	5.37	6.99	5.09	7.32	5.27
	V@R at 5%	−1.5%	−1.1%	−1.3%	−1.2%	−1.4%
	CV@R at 5%	−2.0%	−1.6%	−1.8%	−1.9%	−2.0%
2020	Annual. Average	−0.3%	6.5%	13.3%	4.6%	9.4%
	Annual. Std Dev	31.9%	29.8%	31.9%	30.0%	31.9%
	Skewness	−0.90	−0.97	−0.69	−0.95	−0.68
	Kurtosis	11.02	13.72	11.11	13.41	11.08
	V@R at 5%	−3.7%	−3.4%	−3.5%	−3.4%	−3.5%
	CV@R at 5%	−5.2%	−4.9%	−4.9%	−4.9%	−4.9%
2021	Annual. Average	16.9%	18.1%	19.3%	20.2%	23.6%
	Annual. Std Dev	14.8%	13.7%	14.8%	13.8%	14.7%
	Skewness	−0.53	−0.55	−0.42	−0.57	−0.46
	Kurtosis	6.35	7.99	6.34	7.86	6.41
	V@R at 5%	−1.6%	−1.3%	−1.4%	−1.3%	−1.4%
	CV@R at 5%	−2.2%	−2.1%	−2.1%	−2.1%	−2.1%
2022	Annual. Average	−7.8%	0.9%	9.7%	−4.8%	−1.7%
	Annual. Std Dev	23.5%	22.6%	23.5%	23.1%	23.5%
	Skewness	0.31	0.34	0.29	0.31	0.28
	Kurtosis	5.77	6.50	5.71	6.07	5.75
	V@R at 5%	−2.3%	−2.2%	−2.2%	−2.3%	−2.3%
	CV@R at 5%	−3.4%	−3.3%	−3.3%	−3.4%	−3.4%
2023	Annual. Average	17.6%	30.1%	42.7%	24.9%	32.2%
	Annual. Std Dev	14.0%	12.1%	13.8%	12.6%	13.9%
	Skewness	−0.48	−0.14	0.11	−0.15	0.14
	Kurtosis	4.59	4.52	4.45	4.09	4.40
	V@R at 5%	−1.5%	−1.3%	−1.3%	−1.3%	−1.4%
	CV@R at 5%	−2.1%	−1.7%	−1.8%	−1.8%	−1.8%

Table 12 Number of total trading days for EuroStoxx50, days in which the strategy suggests to stay long, days in which the strategy suggests to stay short, number of changes from a long position to a short position or vice-versa

	Total	Strat1			Strat2		
		Long	Short	Changes	Long	Short	Changes
2018	254	180	74	54	194	60	57
2019	253	185	68	57	197	56	46
2020	252	214	38	47	226	26	35
2021	250	206	44	48	215	35	38
2022	249	223	26	34	230	19	26
2023	245	203	42	20	217	28	26

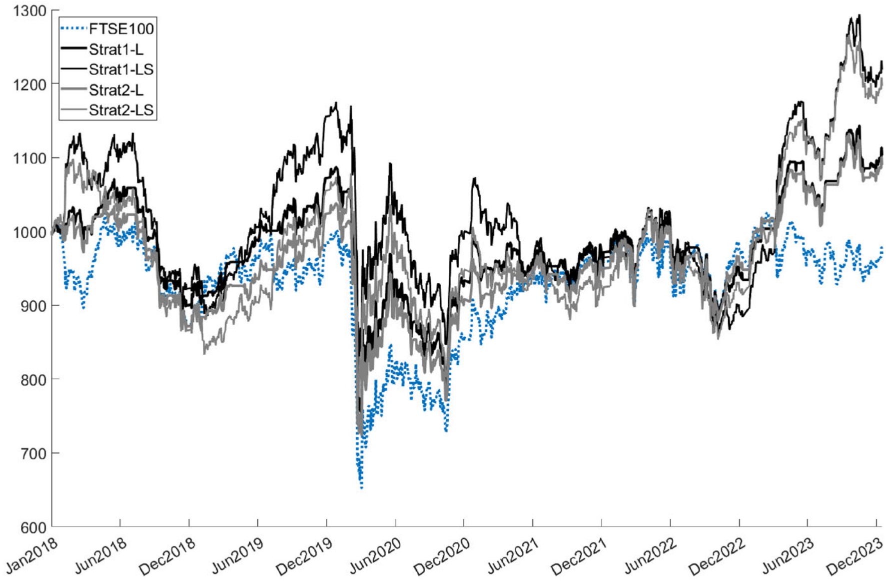

Fig. 11 Strategy 1 and Strategy 2 results with FTSE100. The dotted blue line represents FTSE100, i.e. the benchmark. The black lines represent Strat1 (thick line is Strat1-L and thin line is Strat1-LS). The gray lines represent Strat2 (thick line is Strat2-L and thin line is Strat2-LS)

Table 13 Statistics on the daily return of the FTSE100 Index and of the four market-timing strategies

		FTSE100	Strat 1-L	Strat 1-LS	Strat 2-L	Strat 2-LS
2018	Annual. Average	−12.5%	−9.8%	−7.1%	−12.9%	−13.3%
	Annual. Std Dev	12.3%	10.1%	12.3%	10.6%	12.3%
	Skewness	−0.36	−0.67	−0.28	−0.43	−0.08
	Kurtosis	4.07	6.00	4.12	5.74	4.14
	V@R at 5%	−1.3%	−1.2%	−1.3%	−1.3%	−1.3%
	CV@R at 5%	−1.8%	−1.7%	−1.8%	−1.7%	−1.7%
2019	Annual. Average	12.5%	17.7%	22.9%	16.4%	20.2%
	Annual. Std Dev	11.6%	8.9%	11.6%	9.3%	11.6%
	Skewness	−0.40	−0.36	−0.16	−0.38	−0.17
	Kurtosis	5.17	9.49	5.19	8.31	5.17
	V@R at 5%	−1.0%	−0.8%	−0.9%	−0.8%	−0.9%
	CV@R at 5%	−1.7%	−1.2%	−1.5%	−1.3%	−1.6%
2020	Annual. Average	−10.0%	−10.3%	−10.5%	−8.4%	−6.8%
	Annual. Std Dev	29.1%	27.8%	29.1%	27.9%	29.1%
	Skewness	−0.77	−0.82	−0.65	−0.81	−0.65
	Kurtosis	10.33	12.13	10.34	11.98	10.35
	V@R at 5%	−3.3%	−3.2%	−3.2%	−3.2%	−3.2%
	CV@R at 5%	−4.7%	−4.6%	−4.6%	−4.6%	−4.6%
2021	Annual. Average	12.6%	5.1%	−2.4%	5.7%	−1.3%
	Annual. Std Dev	12.6%	11.4%	12.7%	11.7%	12.7%
	Skewness	−0.34	−0.41	−0.29	−0.39	−0.30
	Kurtosis	6.40	8.44	6.27	7.74	6.28
	V@R at 5%	−1.1%	−1.0%	−1.1%	−1.1%	−1.1%
	CV@R at 5%	−1.9%	−1.8%	−2.0%	−1.8%	−1.9%
2022	Annual. Average	0.6%	−2.8%	−6.3%	0.9%	1.3%
	Annual. Std Dev	16.3%	15.6%	16.3%	15.8%	16.3%
	Skewness	−0.29	−0.30	−0.24	−0.31	−0.28
	Kurtosis	5.22	5.99	5.19	5.79	5.22
	V@R at 5%	−1.8%	−1.8%	−1.9%	−1.8%	−1.8%
	CV@R at 5%	−2.5%	−2.5%	−2.5%	−2.5%	−2.5%
2023	Annual. Average	2.3%	17.0%	31.7%	14.4%	26.5%
	Annual. Std Dev	11.5%	8.5%	11.3%	9.2%	11.4%
	Skewness	−0.70	0.23	0.67	0.09	0.65
	Kurtosis	6.54	6.56	6.25	5.44	6.27
	V@R at 5%	−1.2%	−0.9%	−1.0%	−1.0%	−1.0%
	CV@R at 5%	−1.8%	−1.3%	−1.4%	−1.4%	−1.4%

Table 14 Number of total trading days for FTSE100, days in which the strategy suggests to stay long, days in which the strategy suggests to stay short, number of changes from a long position to a short position or vice-versa

		Strat1			Strat2		
	Total	Long	Short	Changes	Long	Short	Changes
2018	254	170	84	58	173	81	50
2019	253	165	88	46	175	78	40
2020	252	218	34	34	219	33	36
2021	250	199	51	46	209	41	38
2022	250	208	42	33	215	35	32
2023	249	146	103	39	182	67	30

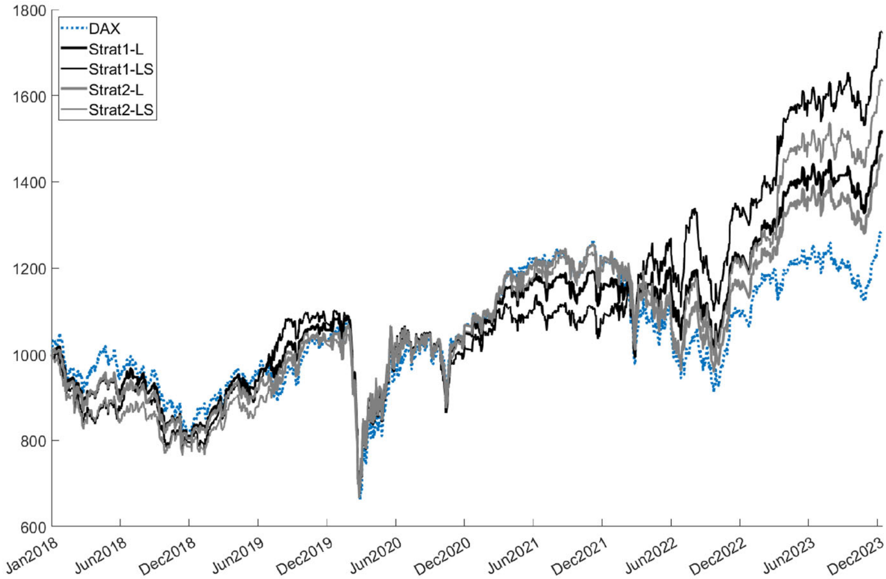

Fig. 12 Strategy 1 and Strategy 2 results for DAX. The dotted blue line represents DAX, i.e. the benchmark. The black lines represent Strat1 (thick line is Strat1-L and thin line is Strat1-LS). The gray lines represent Strat2 (thick line is Strat2-L and thin line is Strat2-LS)

Table 15 Statistics on the daily return of the DAX Index and of the four market-timing strategies

		DAX	Strat 1-L	Strat 1-LS	Strat 2-L	Strat 2-LS
2018	Annual. Average	−18.0%	−19.7%	−21.4%	−20.3%	−22.7%
	Annual. Std Dev	15.1%	13.7%	15.1%	14.2%	15.1%
	Skewness	−0.32	−0.29	−0.04	−0.33	−0.17
	Kurtosis	3.33	4.20	3.41	3.94	3.36
	V@R at 5%	−1.7%	−1.5%	−1.6%	−1.6%	−1.6%
	CV@R at 5%	−2.2%	−2.1%	−2.1%	−2.1%	−2.1%
2019	Annual. Average	23.9%	26.9%	29.9%	25.3%	26.6%
	Annual. Std Dev	13.9%	12.0%	13.8%	12.3%	13.8%
	Skewness	−0.31	−0.17	−0.10	−0.16	−0.06
	Kurtosis	4.99	6.09	4.93	5.63	4.89
	V@R at 5%	−1.6%	−1.5%	−1.6%	−1.5%	−1.6%
	CV@R at 5%	−2.0%	−1.8%	−1.9%	−1.8%	−1.9%
2020	Annual. Average	8.2%	4.2%	0.1%	8.4%	8.6%
	Annual. Std Dev	32.8%	32.0%	32.8%	32.1%	32.8%
	Skewness	−0.57	−0.51	−0.39	−0.53	−0.43
	Kurtosis	11.12	11.99	11.08	11.81	11.11
	V@R at 5%	−3.7%	−3.7%	−3.7%	−3.7%	−3.7%
	CV@R at 5%	−5.2%	−5.0%	−5.0%	−5.0%	−5.0%
2021	Annual. Average	14.2%	11.7%	9.2%	15.1%	15.9%
	Annual. Std Dev	14.1%	13.3%	14.2%	13.7%	14.1%
	Skewness	−0.38	−0.38	−0.29	−0.39	−0.35
	Kurtosis	5.69	6.97	5.63	6.28	5.70
	V@R at 5%	−1.6%	−1.5%	−1.5%	−1.5%	−1.5%
	CV@R at 5%	−2.1%	−2.1%	−2.1%	−2.1%	−2.1%
2022	Annual. Average	−8.1%	7.2%	22.5%	−3.0%	2.1%
	Annual. Std Dev	23.4%	22.2%	23.3%	22.9%	23.4%
	Skewness	0.49	0.52	0.39	0.50	0.46
	Kurtosis	6.26	7.39	6.15	6.71	6.21
	V@R at 5%	−2.2%	−2.2%	−2.2%	−2.2%	−2.2%
	CV@R at 5%	−3.1%	−3.1%	−3.1%	−3.1%	−3.1%
2023	Annual. Average	18.1%	23.8%	29.5%	25.1%	32.2%
	Annual. Std Dev	13.1%	11.8%	13.0%	12.0%	13.0%
	Skewness	−0.52	−0.16	0.16	−0.14	0.19
	Kurtosis	4.60	3.95	4.36	3.75	4.34
	V@R at 5%	−1.3%	−1.3%	−1.3%	−1.3%	−1.3%
	CV@R at 5%	−1.9%	−1.6%	−1.7%	−1.6%	−1.6%

Table 16 Number of total trading days for DAX, days in which the strategy suggests to stay long, days in which the strategy suggests to stay short, number of changes from a long position to a short position or vice-versa

	Total	Strat1			Strat2		
		Long	Short	Changes	Long	Short	Changes
2018	254	202	52	71	218	36	49
2019	253	188	65	72	209	44	60
2020	252	225	27	39	236	16	25
2021	251	212	39	54	228	23	38
2022	249	216	33	42	228	21	30
2023	244	219	25	20	232	12	18

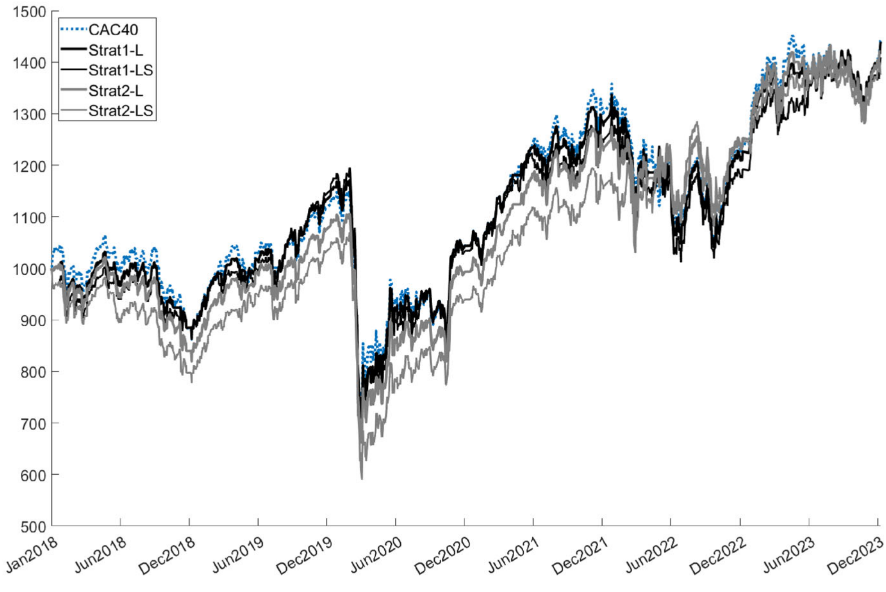

Fig. 13 Strategy 1 and Strategy 2 results for CAC40. The dotted blue line represents CAC40, i.e. the benchmark. The black lines represent Strat1 (thick line is Strat1-L and thin line is Strat1-LS). The gray lines represent Strat2 (thick line is Strat2-L and thin line is Strat2-LS)

Table 17 Statistics on the daily return of the CAC40 Index and of the four market-timing strategies

		CAC40	Strat 1-L	Strat 1-LS	Strat 2-L	Strat 2-LS
2018	Annual. Average	−11.3%	−11.2%	−11.1%	−16.3%	−21.4%
	Annual. Std Dev	13.5%	12.9%	13.5%	13.2%	13.4%
	Skewness	−0.35	−0.42	−0.37	−0.36	−0.31
	Kurtosis	3.92	4.49	3.92	4.16	3.89
	V@R at 5%	−1.5%	−1.5%	−1.5%	−1.5%	−1.5%
	CV@R at 5%	−2.0%	−2.0%	−2.0%	−2.0%	−2.0%
2019	Annual. Average	24.7%	25.9%	27.0%	25.9%	27.1%
	Annual. Std Dev	13.2%	12.2%	13.2%	12.7%	13.2%
	Skewness	−0.69	−0.71	−0.54	−0.71	−0.62
	Kurtosis	5.44	6.85	5.41	6.18	5.45
	V@R at 5%	−1.6%	−1.2%	−1.4%	−1.2%	−1.4%
	CV@R at 5%	−2.1%	−2.0%	−2.0%	−2.0%	−2.1%
2020	Annual. Average	−2.1%	−3.8%	−5.5%	−3.5%	−4.8%
	Annual. Std Dev	32.4%	31.7%	32.4%	31.7%	32.4%
	Skewness	−0.85	−0.96	−0.95	−0.96	−0.96
	Kurtosis	10.28	11.06	10.25	10.96	10.26
	V@R at 5%	−3.7%	−3.7%	−3.8%	−3.7%	−3.8%
	CV@R at 5%	−5.3%	−5.3%	−5.4%	−5.3%	−5.4%
2021	Annual. Average	25.1%	23.5%	21.9%	23.6%	22.2%
	Annual. Std Dev	13.9%	13.3%	13.9%	13.4%	13.9%
	Skewness	−0.84	−0.82	−0.66	−0.82	−0.67
	Kurtosis	6.93	7.80	6.78	7.73	6.79
	V@R at 5%	−1.4%	−1.3%	−1.3%	−1.3%	−1.3%
	CV@R at 5%	−2.1%	−2.0%	−2.0%	−2.0%	−2.0%
2022	Annual. Average	−2.7%	−2.7%	−2.6%	2.4%	7.6%
	Annual. Std Dev	22.2%	21.0%	22.2%	21.2%	22.2%
	Skewness	0.35	0.42	0.37	0.37	0.29
	Kurtosis	6.06	7.27	6.06	7.12	6.03
	V@R at 5%	−2.2%	−2.1%	−2.3%	−2.1%	−2.2%
	CV@R at 5%	−3.1%	−3.1%	−3.1%	−3.1%	−3.1%
2023	Annual. Average	14.8%	15.6%	16.4%	14.0%	13.1%
	Annual. Std Dev	13.9%	13.5%	13.9%	13.6%	13.9%
	Skewness	−0.55	−0.58	−0.53	−0.56	−0.51
	Kurtosis	4.49	4.97	4.50	4.84	4.46
	V@R at 5%	−1.4%	−1.4%	−1.4%	−1.4%	−1.4%
	CV@R at 5%	−2.0%	−2.0%	−2.0%	−2.0%	−2.0%

Table 18 Number of total trading days for CAC40, days in which the strategy suggests to stay long, days in which the strategy suggests to stay short, number of changes from a long position to a short position or vice-versa

	Total	Strat1			Strat2		
		Long	Short	Changes	Long	Short	Changes
2018	254	224	30	24	242	12	16
2019	253	207	46	44	223	30	34
2020	251	236	15	26	241	10	20
2021	250	223	27	14	228	22	11
2022	248	219	29	10	224	24	18
2023	245	222	23	28	228	17	18

References

Benko, M., Fengler, M., Härdle, W., & Kopa, M. (2007). On extracting information implied in options. *Computational Statistics, 22*(4), 543–553.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *The journal of political economy, 81*(3), 637–654.

Brockhaus, O., Farkas, M., Ferraris, A., et al. (2000). *Equity Derivatives and Market Risk Models*. London: Risk.

Brunner, B., & Hafner, R. (2003). Arbitrage-free estimation of the risk-neutral density from the implied volatility smile. *Journal of Computational Finance, 7*(1), 75–106.

Cont, R., Da Fonseca, J., et al. (2002). Dynamics of implied volatility surfaces. *Quantitative finance, 2*(1), 45–60.

Corrado, C. J., & Su, T. (1997). Implied volatility skews and stock return skewness and kurtosis implied by stock option prices. *The European Journal of Finance, 3*(1), 73–85. https://doi.org/10.1080/135184797337543

DeMiguel, V., Plyakha, Y., Uppal, R., & Vilkov, G. (2013). Improving portfolio selection using option-implied volatility and skewness. *Journal of Financial and Quantitative Analysis, 48*(6), 1813–1845.

Fan, J., & Gijbels, I. (1996). *Local Polynomial Modelling and its Applications: Monographs on Statistics and Applied Probability 66*. London: CRC Press.

Fan, J., & Yao, Q. (2003). *Nonlinear Time Series*. Berlin: Springer.

Fengler, M. R. (2012) Option data and modeling bsm implied volatility. In: Duan, J. C., Härdle, W., & Gentle, J. (eds) Handbook of Computational Finance. Springer Handbooks of Computational Statistics. Springer, Berlin, Heidelberg.

Fengler, M. R. (2006). *Semiparametric Modeling of Implied Volatility*. Berlin: Springer.

Fengler, M. R., Härdle, W. K., & Villa, C. (2003). The dynamics of implied volatilities: A common principal components approach. *Review of Derivatives Research, 6*(3), 179–202.

Fengler, M. R., & Hin, L. Y. (2015). Semi-nonparametric estimation of the call-option price surface under strike and time-to-expiry no-arbitrage constraints. *Journal of Econometrics, 184*(2), 242–261.

Figlewski, S. (2018). Risk-neutral densities: A review. *Annual Review of Financial Economics, 10*(1), 329–359.

García-Machado, J. J., & Rybczyński, J. (2017). How Spanish options market smiles in summer: An empirical analysis for options on ibex-35. *The European Journal of Finance, 23*(2), 153–169. https://doi.org/10.1080/1351847X.2015.1067634

Glaser, J., & Heider, P. (2012). Arbitrage-free approximation of call price surfaces and input data risk. *Quantitative Finance, 12*(1), 61–73.

Hafner, R., & Wallmeier, M. (2007). Volatility as an asset class: European evidence. *The European Journal of Finance, 13*(7), 621–644. https://doi.org/10.1080/13518470701380142

Hardle, W. (1990). *Applied Nonparametric Regression*. Cambridge: Cambridge Univ Press.

Homescu, C. (2011). Implied volatility surface: Construction methodologies and characteristics. Available at SSRN 1882567

Ielpo, F., & Guillaume, S. (2010). Mean-reversion properties of implied volatilities. *The European Journal of Finance, 16*(6), 587–610. https://doi.org/10.1080/1351847X.2010.481463

Jackwerth, J. C. (2004). *Option-implied Risk-neutral Distributions and Risk Aversion*. Charlotteville (USA): Research Foundation of AIMR.

Jang, H., & Lee, J. (2019). Generative Bayesian neural network model for risk-neutral pricing of American index options. *Quantitative Finance, 19*(4), 587–603.

Kempf, A., Korn, O., & Saßning, S. (2015). Portfolio optimization using forward-looking information. *Review of Finance, 19*(1), 467–490.
Kim, N., & Lee, J. (2013). No-arbitrage implied volatility functions: Empirical evidence from Kospi 200 index options. *Journal of Empirical Finance, 21*, 36–53.
Kopa, M., Vitali, S., Tichỳ, T., & Hendrych, R. (2017). Implied volatility and state price density estimation: Arbitrage analysis. *Computational Management Science, 14*(4), 559–583.
Kosolapova, M., Hanke, M., & Weissensteiner, A. (2023). Estimating time-varying risk aversion from option prices and realized returns. *Quantitative Finance, 23*(1), 1–17.
Kostakis, A., Panigirtzoglou, N., & Skiadopoulos, G. (2011). Market timing with option-implied distributions: A forward-looking approach. *Management Science, 57*(7), 1231–1249.
Ludwig, M. (2015). Robust estimation of shape-constrained state price density surfaces. *The Journal of Derivatives, 22*(3), 56–72.
Merton, R. C. (1973). Rational theory of option pricing. *Bell Journal of Economics and Management Science, 4*, 141–183.
Shimko, D. (1993). Bounds of probability. *Risk, 6*(4), 33–37.
Silvia, M. (2010). Option-based forecasts of volatility: An empirical study in the Dax-index options market. *The European Journal of Finance, 16*(6), 561–586. https://doi.org/10.1080/13518471003640134
Skiadopoulos, G., Hodges, S., & Clewlow, L. (2000). The dynamics of the s&p 500 implied volatility surface. *Review of Derivatives Research, 3*(3), 263–282.
Tompkins, R. G. (2001). Implied volatility surfaces: Uncovering regularities for options on financial futures. *The European Journal of Finance, 7*(3), 198–230. https://doi.org/10.1080/13518470110040375
Vitali, S., Kopa, M., & Giana, G. (2023). Implied volatility smoothing at covid-19 times. *Computational Management Science, 20*(1), 32.
Vitali, S., Kopa, M., & Tichỳ, T. (2017). State price density estimation for options with dividend yields. *Central European Review of Economic Issues, 20*, 81–90.