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A CONSERVATIVE DISCONTINUOUS TARGET VOLATILITY STRATEGY

Abstract

The asset management sector is constantly looking for a reliable investment strategy, which is able to keep its promises. One of the most used approaches is the target volatility strategy that combines a risky asset with a risk-free trying to maintain the portfolio volatility constant over time. Several analyses highlight that such target is fulfilled on average, but in periods of crisis, the portfolio still suffers market's turmoils. In this paper, the authors introduce an innovative target volatility strategy: the discontinuous target volatility. Such approach turns out to be more conservative in high volatility periods. Moreover, the authors compare the adoption of the VIX Index as a risk measure instead of the classical standard deviation and show whether the former is better than the latter. In the last section, the authors also extend the analysis to remove the risk-free assumption and to include the correlation structure between two risky assets. Empirical results on a wide time span show the capability of the new proposed strategy to enhance the portfolio performance in terms of standard measures and according to stochastic dominance theory.

Keywords target volatility strategy, asset allocation, implied volatility, VIX, stochastic dominance

JEL Classification G11

INTRODUCTION

Nowadays, many investors are looking for simple allocation mechanisms to protect their portfolio from significant losses due to bearish markets. Meanwhile, they are interested in investments, which participate to the attractive profits of equity indices. Volatility is often at the center of most investment decisions (see Sears, 2013) and investors cannot avoid it anymore. Thus, many portfolio strategies have emerged to help in protecting against surges in volatility. A possible approach is to use derivative to hedge the riskiest positions. However, it is possible to control the risk exposition also via asset allocation. In this sense, a well known and intuitive strategy is to diversify the portfolio among asset classes with low correlation. However, a simple diversification between global equity and fixed income, for example, could bring a not satisfying risk reduction. For instance, considering the common strategy where 60% is invested in equity and 40% in fixed income, Qian (2011) shows that the 85% of the portfolio risk arises from the equity component. This implies that investors may not be achieving the desired level of protection.

Recently, in order to tackle this issue, several investment banks and asset management companies developed an investment mechanism called volatility target approach, which became very popular among practitioners. This method describes how to create and to rebalance a portfolio in order to keep its volatility constant. The target volatility mechanism is one of the risk-based dynamic asset allocation approaches that currently exist in the market of portfolio management (see, e.g., Perold and Sharpe, 1988; Black and Perold, 1992; Bertrand and Prigent, 2001; Herold et al., 2007; Ho et al., 2010). The volatility targeting identifies as primal objective the portfolio volatility, since the portfolio allocation is determined exclusively by comparing the volatility of its components with a target level. Thus, the strategy requires to overweight riskier assets in low risk periods and to move to safer assets in periods of high risk. This approach takes advantage of the negative relationship between volatility and returns, as well as of the persistence of volatility. Indeed, considering, for instance, the S&P 500, many works prove the inverse relationship between its returns and its volatility measured by the VIX Index (see Dash and Moran, 2005; Cipollini and Manzini, 2007; Whaley, 2008; Joy, 2010; Arak, 2013; Sloyer and Tolkin, 2008). Therefore, the target volatility approach is an effective way to reduce the fat tails of the distributions without incurring the cost associated with traditional tail hedging techniques.

Considering a portfolio composed by two assets, a risky asset and a safe asset, the volatility targeting strategy requires to identify three elements: the risk measure adopted to quantify the portfolio risk, the maximum portfolio leverage, and a risk measure target value. Most of the strategies in literature (see Albeverio et al., 2013) use an equity index as risk asset, and a risk-free rate as safe asset. Moreover, they consider as risk measure the historical volatility and use a constant target.

Our goal is twofold. On the one hand, we want to empirically prove that a forward looking volatility measure, namely the implied volatility, is more suitable to be adopted as a risk measure within a target volatility approach. On the other hand, we propose an evolution of the classical target volatility strategy used in literature. The new approach requires to set an additional risk measure target greater than the first one. Such target represents an alarm level. Then, if the portfolio risk measure is below the lower target, we utilize the classical target volatility approach, if it is between the two targets, we adopt a fixed portfolio allocation, if it is above the upper target, we move all the portfolio to the risk-free.

In general, the volatility targeting strategy can be applied with a constant frequency rebalancing or with a rebalancing buffer. The first suggests to change the allocation periodically (daily, weekly, monthly) and has been analyzed by Morrison and Tadrowski (2013), Marra (2014) and Collie et al. (2011). The rebalancing buffer has been proposed in practical applications (see CME Group, 2011; SSGA, 2016), and requires to update the allocation only if the portfolio reallocation with respect to the last rebalancing is greater than a predetermined percentage. In this way, marginal changes in the portfolio allocation are avoided and we obtain a remarkable reduction of the rebalancing activity.

Moreover, following Banerjee et al. (2016), we propose a more realistic portfolio allocation removing the risk-free assumption on the safe asset and simply considering two assets, a high-risk asset and a low-risk asset. Thus, both the classical target volatility strategy and the discontinuous target volatility strategy are adjusted to take into account the volatility of both assets and their correlation.

We test the two allocation strategies combining the two different risk measures, standard deviation and implied volatility, and combining the two rebalancing rules. The quality of the obtained portfolios is measured with well-known performance measures and according to stochastic dominance theory.

1. METHOD

The main idea of the target volatility asset allocation approach is to define the optimal portion of the portfolio which should be invested in a risky asset and in a risk-free. The goal is to keep a chosen portfolio risk measure constant over time. Then, during periods when the risk measure is over the target, the portfolio exposure to the risky asset decreases, while during periods when the risk measure is below the target, the exposure to the risky asset increases.

Given a risk measure ρ, which accounts the risk of the risky asset, a classical dynamic volatility targeting strategy consists in the following steps:

- define a target risk measure $\tilde{\rho}$, a maximum leverage λ and a rebalancing frequency;
- at a rebalancing date, calculate the value of ρ for the risky asset;
- calculate the portfolio portion to be invested in the risky asset by equating the portfolio volatility to the target using the following formula:

$$\rho x_e = \tilde{\rho}, \tag{1}$$

where x_e is the portion invested in the risky asset; then, imposing the maximum leverage, we obtain:

$$x_e = \min\left(\frac{\tilde{\rho}}{\rho}; \lambda\right), \tag{2}$$

and, finally, the risk-free allocation is given by

$$x_f = 1 - x_e.$$

The discontinuous volatility strategy assumes that we differentiate the portfolio strategy by comparing the risk measure with a pre-alarm target $\tilde{\rho}_1$ and with an alarm target $\tilde{\rho}_2$, with $\tilde{\rho}_1 < \tilde{\rho}_2$, and we compute the portion invested in the risky asset according to the following scheme:

– if $\rho \le \tilde{\rho}_1$ then, we adopt the standard volatility targeting strategy, i.e.

$$x_e = \min\left(\frac{\tilde{\rho}_1}{\rho}; \lambda\right), \tag{3}$$

– if $\tilde{\rho}_1 < \rho \le \tilde{\rho}_2$ then, we invest in the risk asset a constant percentage γ, i.e.

$$x_e = \frac{\tilde{\rho}_2}{\tilde{\rho}_1} \cdot \gamma, \tag{4}$$

– if $\rho > \tilde{\rho}_2$ we move all the portfolio in the safe asset, i.e.

$$x_e = 0, \quad x_f = 1. \tag{5}$$

In subsections 2.1 and 2.2, our aim is to compare these two different approaches using alternatively the historical volatility or the implied volatility as a risk measure and to observe the performance of the resulting portfolios.

A further extension is the remotion of the risk-free assumption. The hypothesis of the absence of a risk-free asset is nowadays very strong in many markets. Therefore, following the suggestion of Banerjee et al. (2016), we extend the proposed methodologies to consider a portfolio composed by two assets, a high-risk asset and a low-risk one, and their correlation structure. In this framework, when we remove the hypothesis that the portfolio volatility is related only to the risky asset and when we adopt the standard deviation as a risk measure, we compute the portfolio volatility with the standard approach considering the covariance between the two portfolio components. Therefore, in a target volatility perspective, we equal the portfolio volatility to a target

$$\sqrt{\sigma_1^2 x_e^2 + \sigma_2^2 (1 - x_e)^2 + 2x_e (1 - x_e) \cdot cov_{1,2}} = \tilde{\rho}, \tag{6}$$

$$x_e = \frac{\sigma_2^2 - cov_{1,2} + \sqrt{A}}{\sigma_1^2 + \sigma_2^2 - 2cov_{1,2}}, \tag{7}$$

where

$$A = cov_{1,2}^2 - \sigma_1^2 \sigma_2^2 + \sigma_1^2 \tilde{\rho}^2 + \sigma_2^2 \tilde{\rho}^2 - 2\tilde{\rho}^2 cov_{1,2}.$$

σ_1 is the volatility of the high risk assets, σ_2 is the volatility of the low risk assets and $cov_{1,2}$ is the covariance between them. Clearly, in order to impose a maximum leverage, we should define the allocation in the risky asset as:

$$x_e = \min\left(\frac{\sigma_2^2 - cov_{1,2} + \sqrt{A}}{\sigma_1^2 + \sigma_2^2 - 2cov_{1,2}}; \lambda\right). \quad (8)$$

On the contrary, when we use the implied volatility $\left(\sigma_{impl}\right)$ as a risk measure for the high-risk asset, we cannot consider the covariance anymore, and, then, the formula to define the allocation in the high-risk asset becomes:

$$x_e = \min\left(\frac{\sigma_2^2 + \sqrt{B}}{\sigma_{impl}^2 + \sigma_2^2}; \lambda\right), \quad (9)$$

where

$$B = -\sigma_{impl}^2 \cdot \sigma_2^2 + \sigma_{impl}^2 \cdot \tilde{\rho}^2 + \sigma_2^2 \cdot \tilde{\rho}^2.$$

In subsection 2.3 we show the results of removing the assumption of the risk-free and, therefore, using (8) and (9) (according to the chosen risk measure) both for the classical target volatility strategy and the discontinuous target volatility strategy.

2. RESULTS

In order to compare the described strategies, we divide the whole analysis according to the chosen rebalancing policy: a constant frequency rebalancing or a rebalancing buffer. As constant rebalancing frequency we use weekly rebalancing scheme, since Marra (2014) shows that weekly rebalancing frequency allows for some drift in the realized volatility (between 8% and 12%), while, at the same time, provides a reliable protection in a rapidly rising volatility environment. In the case of rebalancing buffer, SSGA (2016) proposes a 10% buffer, while CME Group (2011) proposes a safer 5%. We adopt 10% cumulative rebalancing buffer, i.e., each day we compute the portion, which should be allocated in the risky asset, but we actually change the portfolio allocation, when such portion is greater than or equal to 10% with respect to the last rebalancing.

Following Albeverio et al. (2013), we adopt the S&P 500 Index as a risky asset, but instead of a 3% annual risk-free rate, we consider the 3-month Treasury Bill rate as a risk-free asset. For both assets, we consider daily time series from March 1990 to March 2016.

As a historical risk measure we consider the standard deviation (STD), while as an implied volatility measure we consider the VIX Index, as suggested by Giese (2010 a,b), which proves that it exploits all the features of the implied volatility of the S&P 500. For the computation of the historical volatility, Albeverio et al. (2013) use, in each rebalancing date, the estimation over the previous year, while we estimate over the previous month.

In the case of the classical targeting volatility strategy (TVS), as a volatility target Albeverio et al. (2013) set 12%, while we adopt 10% following CME Group (2011), Morrison and Tadrowski (2013), Marra (2014) and Banerjee et al. (2016). For the discontinuous volatility targeting strategy (DTVP), in order to establish reasonable targets $\tilde{\rho}_1$ and $\tilde{\rho}_2$, we analyze different settings and we finally fix as targets $\tilde{\rho}_1 = 25\%$ and $\tilde{\rho}_2 = 35\%$, and as intermediate proportion $\gamma = 0.5$.

To avoid an aggressive strategy during bear market, the leverage λ is limited to 100%.

Therefore, for each rebalancing method, we compare the performance of each strategy, the classical and the innovative one, adopting alternatively specific volatility measures, the standard deviation or the VIX. We name the strategy as follows: the classical target volatility strategy (TVS) based on standard deviation (TVS STD), the TVS based on VIX (TVS VIX), the discontinuous target volatility approach (DTVS) based on standard deviation (DTVS STD) and the DTVS based on VIX (DTVS VIX).

We sketch the algorithm to achieve the optimal strategy and the wealth process followed by the selected portfolio for a given initial wealth of the portfolio $w_{t_0,p}$:

- Step 1 – In the initial day, we compute the risk measure ρ of the risky asset which could be the historical volatility computed on the past 20 observations and, then, annualized, or the quotation of the VIX index.

Then, we define the allocation of the wealth $w_{t_0,p}$ between risky asset $w_{t_0,e}$ and safe asset $w_{t_0,f}$ according to the weights of the risk asset x_e and the safe asset x_f computed using either (2) (for TVS STD and TVS VIX) or (3)-(4)-(5) (for DTVS STD and DTVS VIX), i.e. $w_{t_0,e} = w_{t_0,p} \cdot x_e$ and $w_{t_0,f} = w_{t_0,p} \cdot x_f$.

- Step 2 – At the end of day t_k, we consider the daily return that we observe in the market and we define the wealth that we have in each asset, i.e. $w_{t_k,e} = w_{t_{k-1},e} \cdot r_{t_k,e}$ and $w_{t_k,f} = w_{t_{k-1},f} \cdot r_{t_k,f}$, and the total wealth, i.e. $w_{t_k,p} = w_{t_k,e} + w_{t_k,f}$.

- Step 3 – If t_k is a rebalancing day and, according to the rebalancing approach adopted, the rebalancing condition is satisfied, we compute the risk measure ρ of the risky asset and we update the weights of the risky asset x_e and the safe asset x_f, and we reallocate the wealth: $w_{t_k,e} = w_{t_k,p} \cdot x_e$ and $w_{t_k,f} = w_{t_k,p} \cdot x_f$.

- Step 4 – Repeat from Step 2 until t_k reaches the final horizon, March 2016.

To analyze the performance of the strategy, we compute the return of the portfolio for each day t_k as weighted average of the returns of the two assets, i.e.,

$$r_{t_k,p} = \frac{r_{t_k,e} \cdot w_{t_{k-1},e} + r_{t_k,f} \cdot w_{t_{k-1},f}}{w_{t_{k-1},p}}.$$

In the following subsections, we compare the portfolio wealth processes $w_{t_k,p}$ obtained with the different approaches and we compute the statistics on the portfolio return processes $r_{t_k,p}$. The annualized return corresponds to the annualized average of the process $r_{t_k,p}$, i.e., $E\left(r_{t_k,p}\right) \cdot 252$, while the maximum drawdown is the minimum observed daily return, i.e., $\min\left[r_{t_k,p}\right]$; the portfolio annualized volatility is computed every day by computing the annualized volatility of the process $r_{t_k,p}$ over the previous 20 days, i.e.,

$$\sigma_{t_k,p} = \sqrt{252} \cdot \sqrt{\frac{\sum_{i=1}^{20}\left(r_{t_{k-i},p} - E\left[r_{t_{k-s},p} \,|\, 1 \le s \le 20\right]\right)^2}{20}},$$

as a maximum volatility we consider its maximum, i.e., $\max\left[\sigma_{t_k,p}\right]$. The performance is measured with the Sharpe Ratio (see Sharpe, 1994), and with the Rachev Ratio (see Biglova et al., 2004). For the Sharpe Ratio, we consider a risk-free rate equal to zero in order to make a fair comparison between all strategies and we compute it with the annualized average return and the average volatility as follows:

$$SR\left(r_{t_k,p}\right) = \frac{E\left(r_{t_k,p}\right) \cdot 252}{E\left(\sigma_{t_k,p}\right)}, \tag{10}$$

where $E\left(\sigma_{t_k,p}\right)$ is the average value of the volatility $\sigma_{t_k,p}$. The Rachev Ratio defined as:

$$RR\left(r_{t_k,p}, \alpha, \beta\right) = \frac{CVaR_\beta\left(-r_{t_k,p}\right)}{CVaR_\alpha\left(r_{t_k,p}\right)}, \tag{11}$$

where Conditional Value-at-Risk $(CVaR)$ is a coherent risk measure defined as:

$$CVaR_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_q(X)\,dq, \tag{12}$$

with the Value-at-Risk (VaR) defined as:

$$\begin{aligned} VaR_q(X) &= -F_X^{-1}(q) = \\ &= -\inf\left\{x | P(X \le x) > q\right\}, \end{aligned} \tag{13}$$

and $F_X^{-1}(q)$ being the quantile function of distribution X. Since we have to compute the $CVaR$ on empirical observations, we use a consistent estimator of the $CVaR$ given by:

$$CVaR_\alpha(X) = \frac{-1}{[\alpha M]}\sum_{i=1}^{[\alpha M]} X_{i:M}, \tag{14}$$

where M is the number of historical observations of X, $[\alpha M]$ is the integer part of αM, and $X_{i:M}$ is the ith observation of X ordered by increasing values.

In our analysis, we computed the Rachev Ratio on the empirical distribution of $r_{t_k,p}$ with $\alpha = \beta = 0.05$. Each portfolio is, therefore, compared with a pure equity strategy and the common 60/40 strategy.

2.1. Constant frequency rebalancing case

In Table 1, we compare the results of the different strategies considering a constant frequency rebalancing. We consider the classical target volatility strategy (TVS) based on standard deviation (TVS STD), the TVS based on VIX (TVS VIX), the discontinuous target volatility approach (DTVS) based on standard deviation (DTVS STD) and the DTVS based on VIX (DTVS VIX).

From Table 1, we can notice that, for both risk measures, the DTVS portfolio has higher Sharpe Ratio and higher Rachev Ratio than the TVS. Moreover, all strategy beat both the 60/40 strategy and the S&P 500. Now we will focus deeply on every TVS and how it behaves with respect to the pure equity strategy.

Figure 1 presents comparative results for the TVS STD compared to the S&P 500. We can observe that TVS STD has a volatility (dotted black line) coherent with the target (10%) without any spike, while we notice many of them in the pure equity strategy. For the right side, we observe that this strategy, during the entire period, invested on average 75% in S&P 500, with a maximum allocation in equity of 100%, as imposed by equation 2, and a minimum value of 11.65% during the crisis. This strategy in 43.52% of cases had a volatility greater than the target one (10%) and just once it was over 20%. On average, the rolling volatility is 9.924%, which is very close to the target. The average annualized return is 6.193%. One of the most relevant values about this strategy is its maximum volatility: 20.104% versus the 79.843% of the pure equity strategy.

Figure 2 presents comparative results between TVS VIX and S&P 500. In this case, the volatility of the TVS VIX is, on average, lower with respect to the TVS STD one and still without any pick even in unstable periods. Regarding the right side of Figure 2, the TVS VIX equity exposure highlights that this strategy is more conservative than the TVS STD one. Indeed, comparing Figure 1 and Figure 2, we can see that TVS VIX exposure only twice reaches the maximum allocation of 100%. This strategy invested, on average, 56.65% in S&P 500 with a minimum value of 13.19%. The average rolling volatility is 7.398%, which is sensibly below the target 10%. This behavior is justified by Giese (2010 a, b), who affirms that the realized long-term volatility of each index is below its target level due to the fact that implied volatilities are used to forecast future volatility, which are (on average) higher than realized volatilities. From this result, it is clear that implied volatility induces a more conservative investment strategy in the long term. Figure 3 shows the performance of DTVS STD portfolio versus S&P 500. Thanks to the alarm threshold fixed at 35%, the pre-alarm threshold at 25% and the middle interval ratio at 0.5, the strategy is very conservative in case of volatile periods and, in particular, in case of stock market shocks. We can notice that the exposure to equity becomes several times equal to zero in correspondence with market's crashes.

The average rolling volatility is 9.507%, which is slightly less than the one obtained with the TVS STD.

Table 1. Portfolio statistics for constant frequency rebalancing case

	TVS STD	TVS VIX	DTVS STD	DTVS VIX	60/40	S&P 500
Annualized return	6.193%	5.003%	6.070%	4.49%	6.04%	8.27%
Average volatility	9.924%	7.398%	9.507%	6.60%	9.09%	15.17%
Maximum draw-down	–4.491%	–3.45%	–4.491%	–3.45%	–5.45%	–9.03%
Maximum volatility	20.104%	17.440%	19.359%	13.89%	46.74%	79.84%
Sharpe ratio	0.624	0.676	0.638	0.680	0.665	0.545
Rachev ratio	1.087	1.094	1.100	1.112	1.014	1.016

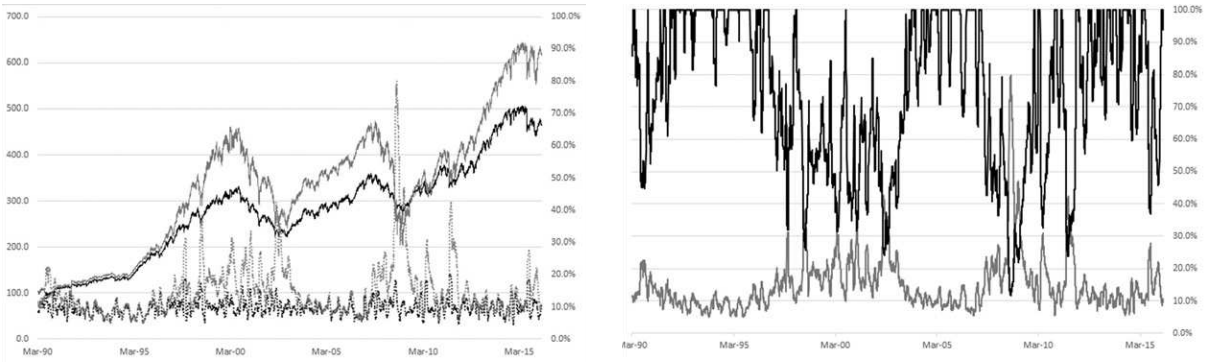

Figure 1. Comparative results for the TVS STD portfolio and the S&P 500

Note: The left side presents the path of TVS STD (solid black line) and of S&P 500 (solid gray line), and the TVS STD rolling standard deviation (dotted black line) and the S&P rolling standard deviation (dotted gray line). The right side presents how the equity exposure of the TVS STD (solid black line) changes with respect to the volatility of the S&P 500 (solid gray line).

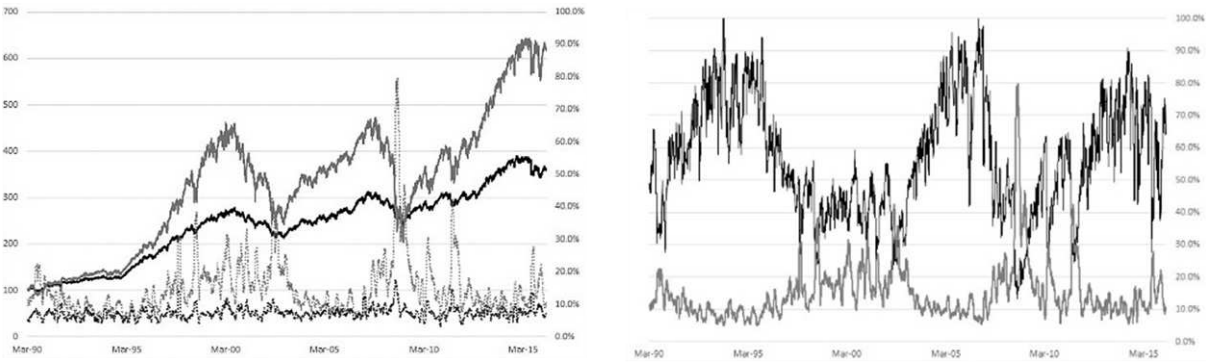

Figure 2. Comparative results for the TVS VIX portfolio and the S&P 500

Note: The left side presents the path of TVS VIX (solid black line) and of S&P 500 (solid gray line), and the TVS VIX rolling standard deviation (dotted black line) and the S&P rolling standard deviation (dotted gray line). The right side presents how the equity exposure of the TVS VIX (solid black line) changes with respect to the volatility of the S&P 500 (solid gray line).

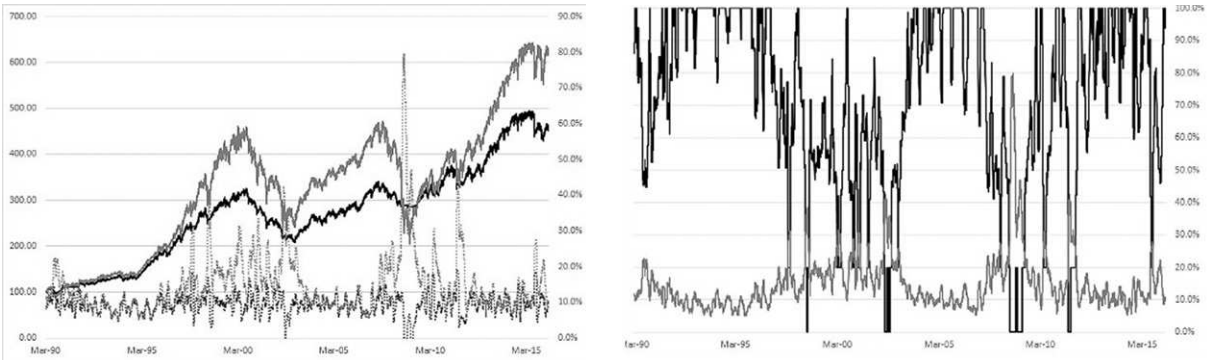

Figure 3. Comparative results for the DTVS STD portfolio and the S&P 500

Note: The left side presents the path of DTVS STD (solid black line) and of S&P 500 (solid gray line), and the DTVS STD rolling standard deviation (dotted black line) and the S&P rolling standard deviation (dotted gray line). The right side presents how the equity exposure of the DTVS STD (solid black line) changes with respect to the volatility of the S&P 500 (solid gray line).

Figure 4 shows that DTVS VIX is even more conservative than DTVS STD, as well as TVS VIX is more conservative than TVS STD. Indeed, the pre-alarm and alarm intervals are touched much more often and, then, more often the portfolio moves to the risk-free asset. Therefore, the average rolling volatility is 6.6%, much less than the target.

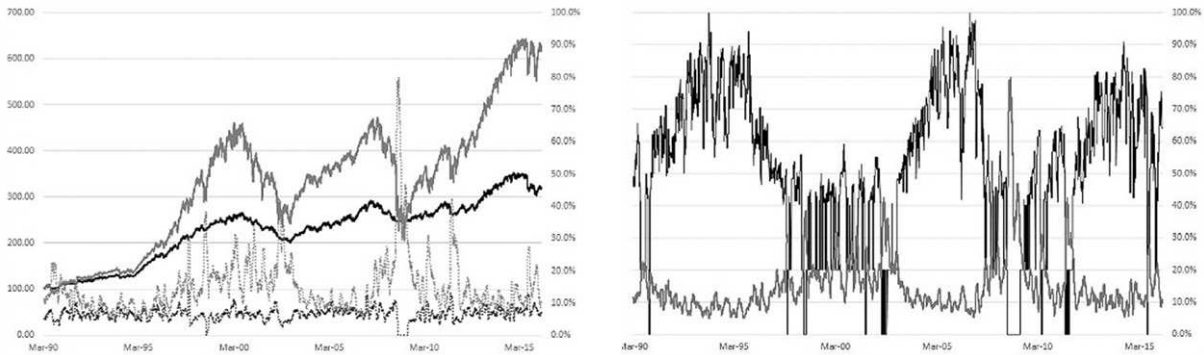

Figure 4. Comparative results for the DTVS VIX portfolio and the S&P 500

Note: The left side presents the path of DTVS VIX (solid black line) and of S&P 500 (solid gray line), and the DTVS VIX rolling standard deviation (dotted black line) and the S&P rolling standard deviation (dotted gray line). The right side presents how the equity exposure of the DTVS VIX (solid black line) changes with respect to the volatility of the S&P 500 (solid gray line).

Comparing the four strategies it is clear that the discontinuous target volatility approach is more conservative and allows to better protect the portfolio in case of crisis. The VIX risk measure produces better results than STD in terms of Sharpe Ratio and Rachev Ratio for all strategies. As analyzed by Giese (2010 b), the portfolio using VIX has a lower average volatility than the portfolio using STD. But, as already stated, the lower volatility does not imply lower performance. Then, considering the case with constant frequency rebalancing, the new approach performs better than the classical one, and with the adoption of VIX as a risk measure, the results are further enhanced. These results are confirmed also by the stochastic dominance tests performed in subsection 2.4.

2.2. Rebalancing buffer case

Following the works of CME Group (2011) and SSGA (2016), we show results applying the rebalancing buffer instead of a constant weekly rebalancing. Thus, the portfolio is reviewed on a daily basis, but allocation adjustments are made only in the event that the cumulative reallocation with respect to previous rebalancing exceeds a predetermined level.

In Table 2, we report the statistics of the portfolios obtained applying the proposed strategies in a rebalancing buffer framework. We have the classical target volatility strategy based on standard deviation (TVS STD), the TVS based on VIX (TVS VIX), the discontinuous approach based on standard deviation (DTVS STD) and the discontinuous approach based on VIX (DTVS VIX).

In Figure 5, we compare the TVS STD with the S&P 500. From the right side, we observe that this strategy during the entire period invested on average 74.26% in S&P 500, with a maximum allocation in equity of 100%, and a minimum value of 15.71%. We can still observe that TVS STD has a volatility coherent with the target (9.92%) with-

Table 2. Portfolio statistics for rebalancing buffer case

	TVS STD	TVS VIX	DTVS STD	DTVS VIX	60/40	S&P 500
Annualized return	6.109%	5.000%	5.794%	4.34%	6.04%	8.27%
Average volatility	9.917%	7.331%	9.423%	6.27%	9.09%	15.17%
Maximum drawdown	–4.541%	–3.112%	–4.541%	–3.11%	–5.45%	–9.03%
Maximum volatility	19.478%	17.062%	18.549%	12.26%	46.74%	79.84%
Sharpe ratio	0.616	0.681	0.614	0.693	0.665	0.545
Rachev Ratio	1.090	1.091	1.101	1.108	1.014	1.016

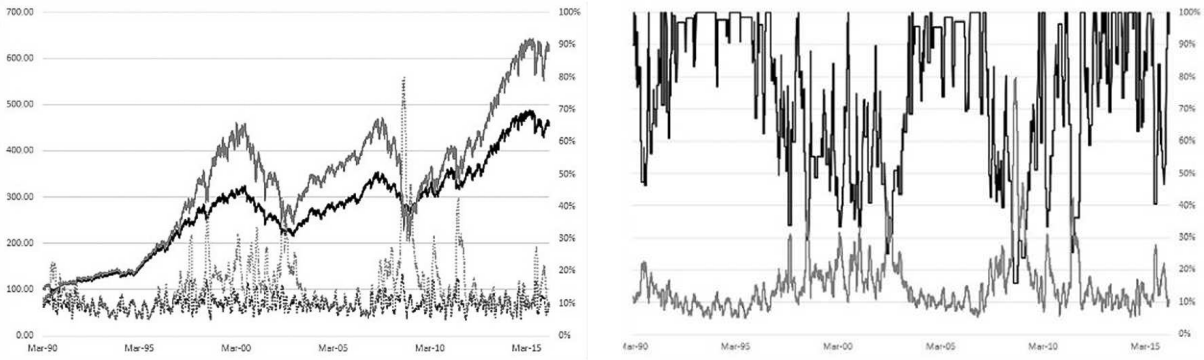

Figure 5. Comparative results for the TVS STD portfolio and the S&P 500

Note: The left side presents the path of TVS STD (solid black line) and of S&P 500 (solid gray line), and the TVS STD rolling standard deviation (dotted black line) and the S&P rolling standard deviation (dotted gray line). The right side presents how the equity exposure of the TVS STD (solid black line) changes with respect to the volatility of the S&P 500 (solid gray line).

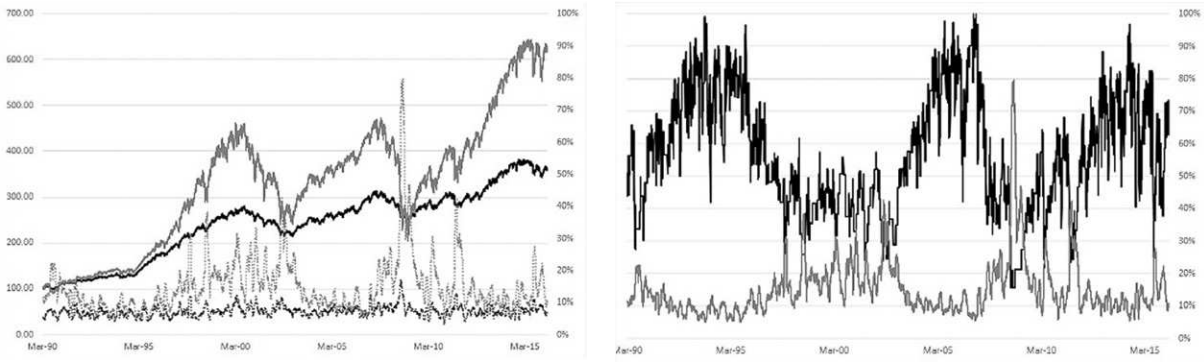

Figure 6. Comparative results for the TVS VIX portfolio and the S&P 500

Note: The left side presents the path of TVS VIX (solid black line) and of S&P 500 (solid gray line), and the TVS VIX rolling standard deviation (dotted black line) and the S&P rolling standard deviation (dotted gray line). The right side presents how the equity exposure of the TVS VIX (solid black line) changes with respect to the volatility of the S&P 500 (solid gray line).

out any spikes. This strategy 44.56% of times had a volatility greater than the target and never over 20%. The average annualized return is 6.109%, its maximum volatility 19.478%. Every value is in line with the same strategy in the periodical case: the main difference here is the number of rebalancing. With the rebalancing buffer, we have 619 rebalancing cases, while with the weekly rebalancing 1371 cases, but the lower rebalancing does not influence the performance of the strategy.

In Figure 6, we compare the TVS VIX still with the S&P 500. The volatility of the TVS VIX is still on average lower with respect to the TVS STD: 7.331% versus 9.917% (in the periodical rebalancing was: 7.398% versus 9.024%). This strategy invested on average 56.68% of the portfolio in S&P 500 with a minimum value of 15.64%. The Sharpe and Rachev Ratios are, respectively, 0.681 and 1.091 (in the periodical case they were 0.676 and 1.094). In this case, the number of rebalancing cases is 1246 versus 1371 of the same strategy, but with the periodical setting. We still have very similar values and performance, but with less number of rebalancing.

In Figure 7, we compare the DTVS STD with the S&P 500. DTVS STD strategy has an average volatility of 9.423% with an average equity exposure of 72.38%. This strategy, thanks to the alarm threshold, in tense periods, totally moves to the risk-free. In particular, 222 times it has 0% invested in the equity (more than 3% over the entire period). It shows a Sharpe Ratio of 0.614 and a Rachev ratio of 1.101 with respect to 0.638 and 1.100 of the constant frequency rebalancing case. These values are

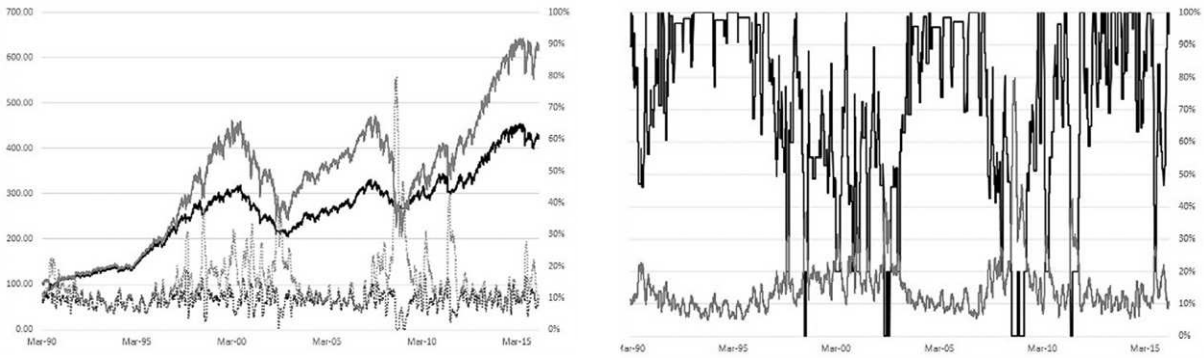

Figure 7. Comparative results for the DTVS STD portfolio and the S&P 500

Note: The left side presents the path of DTVS STD (solid black line) and of S&P 500 (solid gray line), and the DTVS STD rolling standard deviation (dotted black line) and the S&P rolling standard deviation (dotted gray line). The right side presents how the equity exposure of the DTVS STD (solid black line) changes with respect to the volatility of the S&P 500 (solid gray line).

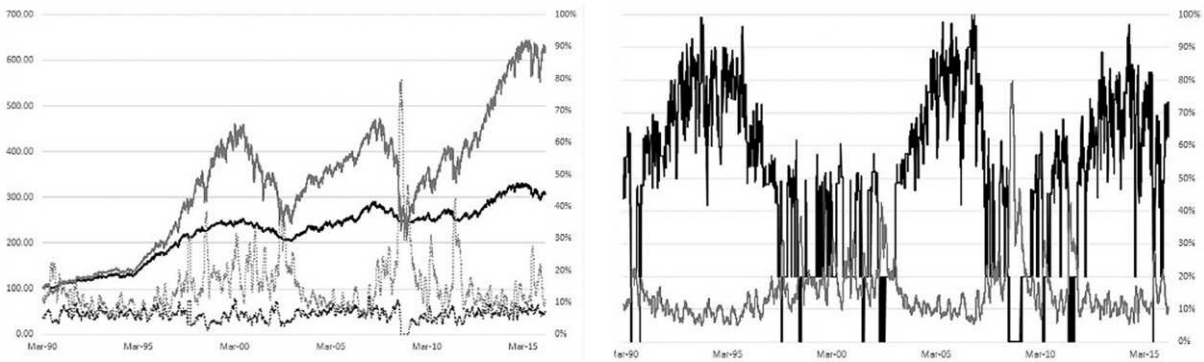

Figure 8. Comparative results for the DTVS VIX portfolio and the S&P 500

Note: The left side presents the path of DTVS VIX (solid black line) and of S&P 500 (solid gray line), and the DTVS VIX rolling standard deviation (dotted black line) and the S&P rolling standard deviation (dotted gray line). The right side presents how the equity exposure of the DTVS VIX (solid black line) changes with respect to the volatility of the S&P 500 (solid gray line).

obtained with just 593 rebalancing cases: the rebalancing buffer in this case implicates half the total number of cases the portfolio is adjusted.

In Figure 8, we have the DTVS VIX strategy and the mere equity strategy based on S&P 500. DTVS VIX strategy has an average volatility of 6.27%, quite far from the 10% imposed as target. The average equity exposure is 51.98% and 299 times it shows an exposure of 0%, i.e. 4.36% of days along the entire period. It shows a Sharpe Ratio of 0.623 and a Rachev Ratio of 1.108 with respect 0.680 and 1.112 in the constant frequency rebalancing case. As far as the number of rebalancing cases is concerned, in this strategy we have 1084 rebalancing cases with respect to 1371 cases of the weekly setting: we have a reduction of 20% in the total number. As in the constant frequency rebalancing, the strategies which perform better are those which adopt VIX as a risk measure rather than the standard deviation, and the discontinuous target volatility performs always better than the standard approach in terms of Rachev Ratio. Such results are further confirmed by stochastic dominance tests performed in subsection 2.4.

2.3. Rebalancing buffer case removing risk-free assumption

In this subsection, we remove the assumption that the 3-month Treasury Bill rate can be considered a risk-free rate and we take into account its volatility and its correlation with S&P 500. Since the rebalancing buffer case shows better results in case of risk-free assumption, we perform the analysis only with rebalancing buffer to highlight if it is possible to further enhance the results and we compare the two strategies (TVS and DTVS), each with both risk measure (STD and VIX). As explained with formulae (8) and (9), the correlation is used

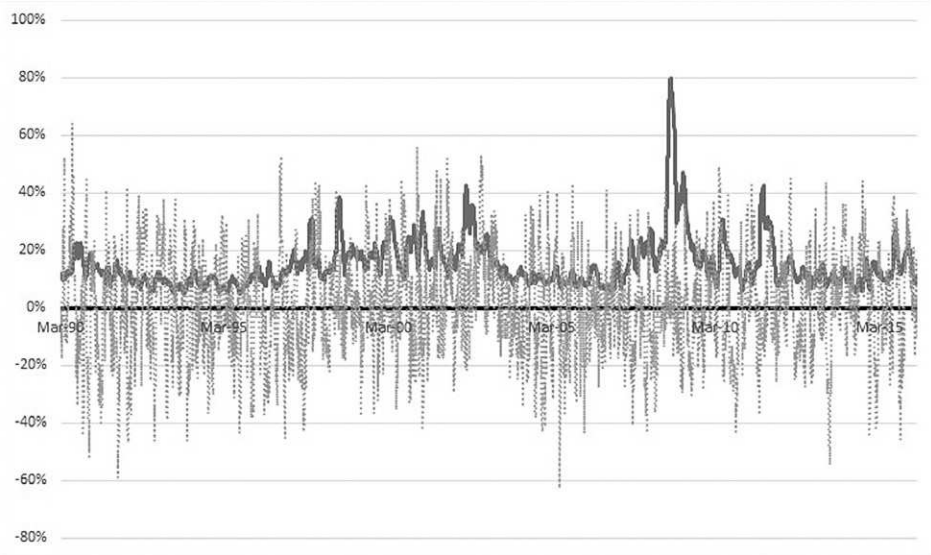

Figure 9. Volatility structure of S&P 500 and 3-month Treasury Bill rate

Note: Volatility of the S&P 500 (solid gray line), volatility of the 3-month Treasury Bill rate (solid black line), and their rolling correlation (dotted gray line).

only when standard deviation is adopted as a risk measure. The volatility of the two assets and their correlation is reported in Figure 9. The 3-month Treasury Bill rate volatility seems to be constant, because it is almost equal to zero. The rolling correlation computed in each day of the previous month appears very instable touching peaks of 60% and –60%.

In Table 3, we show the results of the four strategies. Since the average volatility of the 3-month Treasury Bill is almost close to zero, the strategy which uses VIX results must be very similar to the previous analysis, because the portfolio volatility is still almost completely determined by the risky asset component. Thus, the TVS VIX has a slightly higher Sharpe Ratio (0.682 vs 0.681) and the same Rachev Ratio (1.091), while the DTVS VIX has lower Sharpe Ratio (0.689 vs 0.693), but a higher Rachev Ratio (1.110 vs 1.108). Considering the standard deviation case, even if the volatility of the 3-month Treasury Bill is very low, the correlation plays an important role and produces some tangible differences. Again, the TVS STD strategy has lower Sharpe Ratio (0.602 vs 0.616), but a higher Rachev Ratio (1.098 vs 1.090) when correlation is taken into account. Similarly, the DTVS STD portfolio has lower Sharpe Ratio (0.600 vs 0.614), but a higher Rachev Ratio (1.105 vs 1.101). In general, the Sharpe Ratio worsens, but the Rachev Ratio improves when we remove the risk-free hypothesis and we adjust the allocation strategy to consider both asset volatilities. Moreover, such approach is much closer to reality, especially in the last years when the risk-free assumption reveals to be a chimera rather than a matter of fact. However, the differences between the risk-free assumption case and the no risk-free assumption case

Table 3. Portfolio statistics for rebalancing buffer case removing risk-free assumption

	TVS STD	TVS VIX	DTVS STD	DTVS VIX	60/40	S&P 500
Annualized return	6.378%	5.002%	6.064%	4.314%	6.04%	8.27%
Average volatility	10.595%	7.332%	10.101%	6.261%	9.09%	15.17%
Maximum drawdown	–4.925%	–3.112%	–4.925%	–3.112%	–5.45%	–9.03%
Maximum volatility	19.475%	17.062%	18.809%	12.259%	46.74%	79.84%
Sharpe Ratio	0.602	0.682	0.600	0.689	0.665	0.545
Rachev Ratio	1.098	1.091	1.105	1.110	1.014	1.016

Table 4. Dominance relations between the strategies and the S&P 500 index

			Rebalancing buffer no risk-free				Rebalancing buffer				Constant rebalancing					
			TVS	TVS	DTVS	DTVS	TVS	TVS	DTVS	DTVS	TVS	TVS	DTVS	DTVS	60/40	S&P500
			STD	VIX	STD	VIX	STD	VIX	STD	VIX	STD	VIX	STD	VIX		
Rebalancing buffer no risk-free	TVS	STD	n.c.													SSD
	TVS	VIX	SSD	n.c.	SSD		SSD	TSD	SSD		SSD	SSD	SSD		SSD	SSD
	DTVS	STD	TSD		n.c.											SSD
	DTVS	VIX	SSD	TSD	SSD	n.c.	SSD	TSD	SSD	TSD	SSD	TSD	SSD	SSD	SSD	SSD
Rebalancing buffer	TVS	STD	SSD		TSD		n.c.									SSD
	TVS	VIX	SSD		SSD		SSD	n.c.	SSD		SSD	SSD	SSD		SSD	SSD
	DTVS	STD	TSD		SSD		TSD		n.c.							SSD
	DTVS	VIX	SSD	TSD	SSD		SSD	TSD	SSD	n.c.	SSD	TSD	SSD	SSD	SSD	SSD
Constant rebalancing	TVS	STD	SSD		SSD						n.c.					SSD
	TVS	VIX	SSD		SSD		SSD		SSD		SSD	n.c.	SSD		SSD	SSD
	DTVS	STD	SSD		SSD						SSD		n.c.			SSD
	DTVS	VIX	SSD		SSD		SSD		SSD		SSD	TSD	SSD	n.c.	SSD	SSD
	60/40														n.c.	SSD
	S&P500															n.c.

are rather small, as proven also by the stochastic dominance tests performed in subsection 2.4. The small difference is due to the fact that in a large portion of the considered period, the low-risk asset has such a low volatility that is very close to be a risk-free rate.

2.4. Stochastic dominance relation test

In this subsection, we extend the comparison between the portfolios according to the stochastic dominance theory. In particular, we verify if there exist some stochastic dominance relations, namely the first order (FSD), second order (SSD) and third order stochastic dominance (TSD) (see Müller and Stoyan, 2002; Davidson and Duclos, 2000; Kopa and Post, 2015; Kopa et al., 2016). The motivation of the following part is due to the fact that in previous analysis, the Sharpe Ratio and the Rachev Ratio did not produce a clear evidence about what an investor should prefer, since the values of the Ratios were very close and the return of the S&P 500 was always higher than the other proposed strategies. Therefore, we use the stochastic dominance relations, because they represent the preference for a large class of investors: a stochastic dominance relation implies that the dominant portfolio is preferred to the dominated one for a specific class of investors according to the order of the stochastic dominance. In particular, the FSD represents the non-satiated investor preference, the SSD represents the non-satiated and risk-averse investor preference, the TSD represents the non-satiated, risk-averse and skewness-loving investor preference. A first analysis involves the whole period from March 1990 to March 2016. In this period, mainly because of the bear rally of the S&P 500 in the first months of 2016, we do not notice any type of relations. Therefore, we check stochastic dominance relations removing the beginning of 2016 from the series. Indeed, we still consider a very long period from March 1990 to December 2015. For this time span, we observe the relations reported in Table 4. No strategy dominates another strategy in the FSD sense. All the target volatility strategies, no matter if classical or discontinuous and no matter if with standard deviation or VIX, dominate in the SSD sense the S&P 500. All strategies, which adopt the VIX as a risk measure, dominate the corresponding strategy, which adopts standard deviation. Moreover, each strategy, which adopts the VIX as a risk measure, dominates all other strategies, which adopt standard deviation. Discontinuous target volatility strategies, which adopt VIX, dominate the classical strategies, which adopt VIX in the TSD sense. In general, the discontinuous target volatility with VIX and with rebalancing buffer dominates all other strategies either in SSD or in TSD

sense. These results confirm that the adoption of the VIX Index as a risk measure and the discontinuous target volatility strategies are much preferable to other strategies not only according to standard performance measure, i.e., Sharpe Ratio and Rachev Ratio, but also according to the stochastic dominance theory, which involves much larger classes of investors.

CONCLUSION

In this paper, we present and compare several portfolio allocation strategies. In particular, we apply two types of target volatility strategies, the classical target volatility strategy and the discontinuous target volatility strategy, for different risk measures and different rebalancing methods. According to both the Sharpe Ratio and the Rachev Ratio, the portfolios using VIX have better performance than the portfolios built using the standard deviation. Similarly, the portfolios obtained with the discontinuous target volatility perform better than the portfolios obtained with the classical target volatility strategy. Moreover, as far as the rolling volatility is concerned, the portfolios produced adopting VIX as a risk measure are more conservative than the ones built using standard deviation. Similarly, the noncontinuous target volatility turns out to be more conservative than the classical strategy.

In general, each volatility targeting strategy brings better performance with respect to a classic 60/40 strategy or pure equity strategy. Indeed, an asset allocation scheme that targets volatility can be expected to produce positive results in terms of performance, when compared to standard approaches. Incorporating volatility targeting into the allocation decision improves the efficiency of the portfolio: one can typically get more return for the same amount of realized volatility, as compared to the results of a rigid allocation decision.

Comparing the constant frequency rebalancing and the rebalancing buffer, we remark that the resulting portfolios appear to be quite similar, but, in the analysis, we do not include any transaction cost. In case we would include some transaction costs, the rebalancing buffer would definitely perform better than the constant frequency rebalancing, because the number of rebalancing cases is definitely lower. The constant frequency rebalancing is weekly, then, we have 1371 rebalancing cases, while with the rebalancing buffer, we have on average approximately 1100 rebalancing cases when we consider VIX, and 600 rebalancing cases when we consider standard deviation. The choice of presenting results without transaction costs is devoted to make a fair comparison not assuming any particular type and level for the transaction costs.

This work does not intend the stylized case study to be the solution to asset allocation. But, in view of the results of our testing, based on historical performance, and in view of the fact that traditional asset allocation models have, to some extent, failed to meet investor expectations, incorporating some level of explicit volatility targeting in asset allocation portfolios would be a welcome new feature that might better align investor expectations with investment reality. In this sense, the proposed innovative discontinuous target volatility strategy protects the portfolio in case of market shocks much better that the classical strategy, because, instead of reducing the equity exposure, it completely moves the portfolio to the risk-free. The stochastic dominance tests confirm that the rebalancing buffer setting is preferable to the constant frequency rebalancing, and if we include transaction costs, the results would be even stronger. Moreover, the discontinuous target volatility approach with VIX risk measure dominates the cases with standard deviation and the strategies with standard target volatility approach.

Moreover, we follow the suggestion made by Banerjee et al. (2016) and we assume that the risky assets and the safe assets are correlated, since the safe asset is not considered risk-free anymore, but only a low volatility security. The results show that we achieve a slightly better portfolio in terms of Rachev Ratio comparing with the risk-free assumption case and, again, the discontinuous target volatility outper-

forms the classical target volatility approach. The stochastic dominance tests confirm the quality of the discontinuous target volatility strategy approach with VIX even if the results between rebalancing buffer with and without risk-free assumption are very similar.

Nevertheless, the volatility targeting mechanism is not free of drawbacks: the strategy works well in specific market environments such as a falling market accompanied by high volatility levels or a rising market accompanied by low volatility levels. The volatility targeting mechanism is one of the rule-based dynamic asset allocation approaches. As such, it may cause significant losses in case of non-standard market environments, e.g., in case of falling and low volatile markets, which might lead to a significantly increased equity exposure within the volatility targeting portfolio. Therefore, the target volatility approach may not be sufficient to solely define the portfolio management decisions and should be combined with other asset allocation strategies. The challenges that remain have mostly to do with how to integrate this approach into familiar framework of asset allocation.

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