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Pension fund management with hedging derivatives, stochastic dominance and nodal contamination

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Abstract

The main goal of a pension fund manager is sustainability. We propose an Asset and Liability Management model structured as a multi-stage stochastic programming problem adopting a discrete scenario tree and a multi-objective function. Among other constraints, we consider the second-order stochastic dominance with respect to a benchmark portfolio. To protect the pension fund from shocks, we test the inclusion of hedge financial contracts in the form of put options and, moreover, we stress the portfolio introducing a new scenario tree contamination technique, namely the nodal contamination. Numerical results show that we can efficiently manage the pension fund satisfying several targets such as liquidity, returns, sponsor's extraordinary contribution and funding gap. Moreover, we test the sensitivity with respect to put option strikes and to the stochastic dominance constraints. Finally, we demonstrate the effect of the scenario tree contamination.

Keywords: Stochastic programming, Portfolio selection, Sensitivity analysis, Asset and Liability Management, Pension fund, Hedging derivatives, Contamination

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1. Introduction

Pension funds are typically managed via Asset and Liability Management (ALM) models which have been deeply explored in last fifty years. One of the first ALM model is BONDS model proposed in Bradley and Crane (1972, 1980). The authors would help the bond portfolio manager to take decision in an uncertain environment, but they do not explore the liabilities side in depth. Thus, it is usually considered as an asset model. At the end of the eighties and the beginning of the nineties, the MIDAS model has been developed and several other ALM models have been proposed, cf. Kusy and Ziemba (1986). In Dempster and Ireland (1988) one of the first asset and liability model focused on the immunization of the liability side is described. The model is formulated to be applied to a real electricity company and to handle its debt management properly, see also Dempster and Ireland (1989, 1991). A typical approach is to adopt a multistage stochastic formulation such as the milestone Russell-Yasuda Kasai Model described in Cariño et al. (1994) and then analyzed in Cariño and Ziemba (1998a) and Cariño et al. (1998b). In the same years Mulvey and Zenios proposed interesting and complete analysis of the multiperiod stochastic problem applied to ALM problems considering the fixed income investment, see e.g. Mulvey (1994a,b), Nielsen and Zenios (1996) and Zenios (1995). Later on, Mulvey's research on the Tower Perrin scenario generation system evolved into the well known Towers Perrin-Tillinghast ALM model, see Mulvey et al. (2000). Recent and innovative formulations of the ALM problem are proposed in Consigli et al. (2011); Consigli and di Tria (2012); Consigli and Moriggia (2014).

Standard ALM models can be applied to represent a pension fund management problem only once specific features are included. Pflug and Świetanowski (1999) proposed a ALM model for pension funds paying a particular attention to both the asset and the liability side modeling. To address the pension problem Consigli and Dempster (1998a,b) introduced the CALM model which is specifically designed to handle the pension fund management problem covering a long-term period and considering as liabilities different types of pension contracts. A similar approach can be found in Dempster et al. (2003). The pension fund features are mainly related to two principal characteristics. The first is the distinction between defined contribution and define benefit pension fund. In a define contribution pension fund the final pension benefit is unknown at the beginning and it will be the results of the contribution investment, i.e. the pensioner bears the risk; in a define benefit pension fund the final benefit is known since the beginning of the pension contract, i.e. the pension fund sponsor bears the risk. A specific focus on the defined benefit pension fund can be found in Dert (1998). The second distinction relies on the considered pension pillar. There exist three pension pillars. The first is the state pension system, the second is based on the worker category and/or on the employee's employer, the third is composed of private insurance contracts. For instance, the InnoALM model proposed in Ziemba (2007) and Geyer and Ziemba (2008) considers a second pillar pension fund since it is built to manage the pension fund of an electricity company. Our work develops within the define benefit framework and considers a second pillar pension fund.

Summarizing, the suitability of multistage approach to deal with ALM problems has been proved correct on multiple occasions during the last twenty years. Nevertheless, in Mulvey et al. (2006) the authors suggest again a multiperiod model to increase the understanding of risk and reward in a long-term horizon framework for pension plans and other long-term investors, see also Mulvey et al. (2007, 2008). Comprehensive collections are in Ziemba and Mulvey (1998) and Zenios and Ziemba (2006, 2007).

Recent studies differentiate the pension fund problem considering not only the pension fund manager problem, i.e. the ALM problem, but also the issuer problem, see e.g. Vitali et al. (2017), and the individual pension problem, see e.g. Consigli (2007), Consigli et al. (2012) and Kopa et al. (2018).

ALM for pension fund must consider the evolution of the asset universe according to regulatory updates, cf. McKendall et al. (1994); Davis (2000); Guiso et al. (2002); Hardy (2003); Gollier (2008); Broeders et al. (2009).

Following this path, our aim is to propose an extension of the model proposed in Consigli et al. (2017) which is based on the Property & Casualty insurance fund model described in Consigli et al. (2011) and Consigli and Moriggia (2014). The first improvement is to enlarge the asset universe with a set of hedging contracts in the form of put options. As a matter of fact, during the last years the pension fund portfolios are stressed because of the financial crisis. Due to such crisis, the equity market and alternative investments, e.g. real estate, exceed their common level of riskiness. Therefore, we include hedging financial contracts to give to the pension fund manager the possibility to protect the position on both Public Equity and Real Estate. Another fundamental extension of Consigli et al. (2017) that we investigate in this paper is the effect of the second-order stochastic dominance constraints with respect to a benchmark portfolio.

The notion of stochastic dominance was introduced in statistics more than 50 years ago and it was firstly applied to economics and finance in Quirk and Saposnik (1962), Hadar and Russell (1969) and Hanoch and Levy (1969). Later on, the second-order stochastic dominance constraints were applied to static stochastic programs in Dentcheva and Ruszczynski (2003) and Luedtke (2008) and to portfolio efficiency analysis, see e.g. Post (2003), Kuosmanen (2004), Dupačová and Kopa (2012) and Kopa and Post (2015). Similarly, the first-order stochastic dominance constraints were used in Kuosmanen (2004), Dentcheva and Ruszczyński (2004) and Dupačová and Kopa (2014). In multistage stochastic programming, the second-order stochastic dominance constraints were applied to asset-liability modeling in Yang et al. (2010) and in an individual pension allocation problem in Kopa et al. (2018).

As a matter of fact, we prove that the portfolio obtained applying the model in Consigli et al. (2017) suffers extremely unexpected events since the protection of the portfolio is achieved uniquely by diversifying the allocation. Therefore, we propose a twofold improvement to reduce these drawbacks. In the former, we enhance the formulation of the problem including the second-order stochastic dominance constraints to prove that the optimal portfolio is able to dominate the currently implemented portfolio monitoring the tails of the final wealth distribution in a better way. In the latter, we consider a different asset universe allowing the portfolio to invest in hedging contracts to be able to protect itself from shocks on the main risky assets. Thus, on the one hand, our aim is to provide evidences that derivative contracts, if properly used, can improve the risk/reward profile of a pension fund ensuring the portfolio to be protected by market shocks. On the other hand, we observe whether stochastic dominance enhances the asset evolution dynamic guaranteeing a better shape of the final wealth distribution.

Finally, we develop and apply a new stress testing technique suitable for multistage scenario trees, namely the nodal contamination. The nodal contamination allows for incorporating unexpected shocks or changes in scenarios' evolution and can be seen as a tool for sensitivity analysis. This type of contamination differs from the one considered in Dupačová and Kopa (2012) and Dupačová and Kopa (2014) in a subtle way. The advantage of the nodal contamination is that it does not increase the size of the scenario tree and, hence, the contaminated problems are of the same computational complexity as the original problem. Assuming two stress situations (increasing volatility and a negative shock) we employ the contamination to analyse the sensitivity of optimal portfolios.

To the best of our knowledge, the second order stochastic dominance and the hedging derivatives have never been adopted jointly in a multistage stochastic optimization model. In particular, this is new for the case of an ALM model for a pension fund management. Therefore, the contribution of this work is to analyze the reciprocal impact of these two instruments in the definition of the optimal strategies. Moreover, we invent a new kind of stress testing based on the so-called nodal contamination approach. Finally, we implement it in the pension fund ALM model to stress the optimal strategy with some unexpected scenarios.

The paper is structured as follows. Section 2 introduces the formulation of the ALM model including a put option pricing and the second-order stochastic dominance constraints. It is followed by a discussion on scenario generation and the nodal contamination in Section 3. Section 4 presents the numerical results. In particular, from Section 4.1 to Section 4.4 we discuss the findings for all the proposed cases, in Section 4.5 we discuss further results based on a sensitivity analysis with respect to the parameter settings and the benchmark choice. In Section 4.6 we perform an out-of-sample analysis to confirm the quality of the model. The paper is concluded in Section 5.

2. Model description

Following Consigli et al. (2017), the stochastic tree is differentiated between decisional nodes and intermediate nodes. In the decisional nodes buying and selling are allowed, while in intermediate nodes no multiple branching originates and we account pension payments and portfolio income. The intermediate stages are every year. The decisional stages are denoted $t_h, h = 0, ..., H$, where $t_0 = 0$ represents the initial stage, then we account five further decisional stages at time 1, 2, 3, 5 and 10, and the final horizon at $t_H = 20$ years. The scenario tree is represented with the nodal notation and contains the asset coefficients both for the price returns and for the income returns. For each node n we denote the set of its children $C(n)$ and define $t(n)$ as the corresponding stage time.

Dissimilarly to Consigli et al. (2017), where the liabilities are composed by different classes of populations, we consider only the already pensioned people. Therefore, we assume that the pension fund wants to ensure its sustainability in case there will be no more new contracts in the future and so we deal with the so-called runoff case. In such extreme framework, as time goes by, the number of pensioners decreases and, consequently, in each node n also the net liability flows Λ_n^{NET} decrease. The pension fund has still to pay the net liabilities with the financial asset incomes and/or negotiating directly the assets. Therefore, the liquidity gap and the ALM risk proposed in Consigli et al. (2017) have been adjusted considering the new liability flows and their duration. Moreover, contrary to the original model that adopts a portfolio replication approach, we define in each node n the Defined Benefit Obligation (DBO) D_n as the present value of the future payments in all children nodes weighted by the probability of each node.

$$D_n = \sum_{m \in C(n)} \left[e^{-r_{n,m}(t(m)-t(n))} \cdot \mathbb{E}\left[\Lambda_m^{NET} | t(m) = t_h\right], \forall t_h > t(n) \right] \tag{1}$$

The maximum surviving horizon for pensioners is 50 years. Then, according to Consigli et al. (2017), we take into account the final conditions in 20-year final horizon because the fund continues for other 30 years.

2.1. Hedging contract evaluation

In each node of the tree, the price of the option could not be estimated using the classic Black and Scholes (B&S) formula since we deal with a discrete amount of possible paths for the underlying and, moreover, it does not follow a Brownian Motion process, as required in the B&S model. Therefore, we adopt the following pricing approach to compute the price $p_{i,n}$ of the i put option given the price $S_{j,n}$ of the underlying j in node n: we observe directly on the tree the average payoff of the children nodes n^+, for a fixed strike return κ, and we discount that average with the risk free rate r_{n,n^+} associated to the cash account:

$$p_{i,n} = e^{-r_{n,n^+}(t(n^+)-t(n))} \cdot \mathbb{E}\left[\max(\kappa S_{j,n} - S_{j,n^+}, 0)\right] \tag{2}$$

The expectation for node n is then a conditional expectation given by its children n^+ and their probability. The value of κ represents the protection level offered by the option: a value of $\kappa = 1$ implies a protection for any negative return, similarly, for instance, a value of $\kappa = 0.97$ offers a protection for any return worst than -3%. Then, the return $\rho_{i,n}$ of the i put option in each node n is computed considering the payoff of the current node and the purchase price on the ancestor node n^-:

$$\rho_{i,n} = \begin{cases} \frac{\max(\kappa S_{j,n^-} - S_{j,n}, 0) - p_{i,n^-}}{p_{i,n^-}}, & \text{if } p_{i,n^-} \neq 0 \\ -1, & \text{otherwise} \end{cases} \tag{3}$$

According to the adopted pricing formulation (2), the price $p_{i,n}$ of a put option is equal to zero if and only if in all children nodes n^+ the put option is out-of-the-money. Thus, if $p_{i,n^-} = 0$ the return $\rho_{i,n}$ in each node n (children of n^-) is -100%, i.e. any amount of money invested in that put option will be lost.

Once the returns have been computed, the put options are used in the model as any other asset. The pension fund manager can buy the option and income the payoff at maturity, if positive. An additional constraint for this asset is that the sales are forbidden. This means that the pension fund manager cannot use the options to speculate and get an extra income by their intra-stage price change but can only use them in the hedging sense, i.e. buying a protection on a certain horizon against an unexpected fall of the underlying asset value.

2.2. Objective formulation

As proposed in Consigli et al. (2017), the objective function is a representation of a multicriteria approach that synthesizes a short-term risk control, a medium-term profitability and a long-term sustainability. The objective function aims to minimize the expected shortfall of a set K of variables Y_k with respect to a specific threshold $\bar{Y}_k$. The k variable is accounted only in a specific stage t^k. The expected shortfalls are combined associating a weight λ_k to each objective variable, with $\sum_{k=1}^{K} \lambda_k = 1$. Therefore, the objective function is:

$$\min \left\{ \sum_{k=1}^{K} \lambda_k \cdot \mathbb{E}[\bar{Y}_k - Y_{k,n} | Y_{k,n} < \bar{Y}_k], \quad \forall n | t(n) = t^k \right\} \tag{4}$$

In the objective function, following the definitions given in the original model, we consider four target variables $Y_{k,n}$:

$Y_{1,n}$: a joint measure of the ALM risk and of the liquidity gap,

$Y_{2,n}$: a measure of the return adjusted by the risk,

$Y_{3,n}$: the cumulative sponsor contribution,

$Y_{4,n}$: the difference between the DBO and the portfolio value.

The stages in which we account these variables are $t^1 = 1$, $t^2 = 3$, $t^3 = 10$, $t^4 = 20$ and we assign the weights $\lambda_1 = 10\%, \lambda_2 = 30\%, \lambda_3 = 40\%$ and $\lambda_4 = 20\%$.

2.3. Stochastic dominance

The basic definition of the second-order stochastic dominance (SSD) relation is as follows: A random variable A SSD dominates a random variable B if the integrated cumulative distribution function of A is below that of B. Equivalently, the second-order stochastic dominance holds if and only if no risk averse decision maker prefers B to A. Since the random variables in our model are discrete with equiprobable realizations it is useful to formulate the SSD conditions using a double stochastic matrix as proposed in Kuosmanen (2004). In particular, if we define $\mathbf{w}_{t_h}$ the vector of the optimal portfolio wealth realizations occurring in all nodes at stage t_h and, similarly, we define $\mathbf{w}^B_{t_h}$ the vector of a benchmark portfolio wealth realizations occurring in all nodes at stage t_h. The optimal portfolio SSD dominates the benchmark portfolio at stage t_h if and only if

$$\mathbf{w}_{t_h} \geq \mathbf{Q} \cdot \mathbf{w}^B_{t_h} \tag{5}$$

for some matrix $\mathbf{Q}$ which is double stochastic, i.e. satisfies the following conditions:

$$\sum_i Q_{i,j} = 1 \tag{6}$$

$$\sum_j Q_{i,j} = 1. \tag{7}$$

and the elements of $\mathbf{Q}$ have to belong to the interval $[0,1]$, so each row and each column represents a convex combination.

For validation reasons, we also solve the model substituting the SSD constraint with the Expected Value (EV) constraint in which we require that the expected value of the wealth obtained by the optimal portfolio is greater than the expected wealth obtained by the benchmark portfolio, i.e.

$$\mathbb{E}\left[\mathbf{w}_{t_h}\right] \geq \mathbb{E}\left[\mathbf{w}^B_{t_h}\right] \tag{8}$$

In such way, we check whether the control of the whole distribution, i.e. the SSD constraint, adds a tangible improvement to the simpler expected value requirement. The SSD constraint (5) and the EV constraint (8) could be considered in any stage $t_h, h > 0$.

3. Scenario Generation and Nodal Contamination

In order to focus on the consequences of the second-order stochastic dominance constraints and on the impact of the inclusion of the hedging contracts, we reduce the asset universe to 15 assets split in 5 classes: Cash, $i = 1$; Bonds, $i = 2, ..., 9$; Real Estate, $i = 10$; Public Equity, $i = 11$; and Put Options, $i = 12, ..., 15$. For each asset class we define an allocation upper bound according to the pension fund policy and regulatory constraints, see Table 1.

Table 1: Asset universe adopted for the portfolio problem with asset, related asset type, corresponding Index, allocation upper bound and proportion of the initial portfolio used in the model.

<table>
<tr><th>i</th><th>Asset</th><th>Asset Type</th><th>Index/Underlying</th><th>Upper bound</th><th>Initial Portfolio</th></tr>
<tr><td>1</td><td>Bank Account</td><td>Cash</td><td>EURIBOR 3-month</td><td>30%</td><td>15%</td></tr>
<tr><td>2</td><td>1-3 years</td><td>Treasury bond</td><td>Euro-Aggregate 1-3year</td><td rowspan="8">100%</td><td>6.25%</td></tr>
<tr><td>3</td><td>3-5 years</td><td>Treasury bond</td><td>Euro-Aggregate 3-5year</td><td>6.25%</td></tr>
<tr><td>4</td><td>5-7 years</td><td>Treasury bond</td><td>Euro-Aggregate 5-7year</td><td>6.25%</td></tr>
<tr><td>5</td><td>7-10 years</td><td>Treasury bond</td><td>Euro-Aggregate 7-10year</td><td>6.25%</td></tr>
<tr><td>6</td><td>10+ years</td><td>Treasury bond</td><td>Euro-Aggregate 10+year</td><td>6.25%</td></tr>
<tr><td>7</td><td>Securitized</td><td>Securitized bond</td><td>Euro-Aggregate: Securitized</td><td>6.25%</td></tr>
<tr><td>8</td><td>Investment Grade</td><td>Corporate bond</td><td>Euro Corporate ex Subordinated 1% Cap</td><td>6.25%</td></tr>
<tr><td>9</td><td>High Yield</td><td>Corporate bond</td><td>Euro HY B and above</td><td>6.25%</td></tr>
<tr><td>10</td><td>Real Estate</td><td>Real Estate</td><td>GPR General Europe</td><td>20%</td><td>10%</td></tr>
<tr><td>11</td><td>MSCI</td><td>Public Equity</td><td>MSCI Europe</td><td>50%</td><td>25%</td></tr>
<tr><td>12</td><td>Put 1-year</td><td>Derivatives</td><td>Real Estate</td><td rowspan="4">10%</td><td>0%</td></tr>
<tr><td>13</td><td>Put 2-year</td><td>Derivatives</td><td>Real Estate</td><td>0%</td></tr>
<tr><td>14</td><td>Put 1-year</td><td>Derivatives</td><td>Public Equity</td><td>0%</td></tr>
<tr><td>15</td><td>Put 2-year</td><td>Derivatives</td><td>Public Equity</td><td>0%</td></tr>
</table>

The Treasury bonds are represented by five different maturity buckets. The Corporate bonds are subdivided into investment grade (rating higher than Baa3 Moody's or BBB- Standard&Poor's) and high yield (rating in the interval (Ba1, B3) Moody's or (BB+, B-) Standard&Poor's), see (Bertocchi et al., 2013, Chapter 5). To address the aim of this paper, we adjust the asset universe to include hedging contracts, namely four put options. Two of them are written on Real Estate and have 1-year and 2-year maturity, respectively; the other two have the same maturities and have the Public Equity as the underlying asset. The maturities are chosen according to the tree discretization and to the market liquidity.

3.1. Scenario Generation

As shown in Table 1, each asset has an associated underlying index. For each index we have historical series of 17 years of quarterly price and income returns, from the beginning of 1999 till the end of 2015. All series have been downloaded by Datastream. We assume that each process of price return can be described as a linear regression of all asset price returns and of two main macroeconomic variables: Gross Domestic Product (GDP) and Consumer Price Index (CPI). Regressors are initially included with 5 lags and we also assume that they can be included with lag 0 in a hierarchical sense, that means that according to the order in Table 1 variable i can depend on variables $j, j < i$ at the same time, so that, excluding simultaneous equations, we obtain the following general model for the price return of asset i:

$$\rho_{i,t} = \beta_{i,0} + \sum_{j=1}^{i-1} \beta_{i,j,0}\rho_{j,t} + \sum_{j=1}^{11}\sum_{l=1}^{5} \beta_{i,j,l}\rho_{j,t-l} + \sum_{v=1,2}\sum_{l=1}^{5} \gamma_{i,v,l} w_{v,t-l} + \varepsilon_{i,t}, \ \ i = 1, ..., 11$$

where $w_{v,t}$ is the price return of the macroeconomic variable v at time t. The same process is needed also to estimate the regression for the macroeconomic variables:

$$w_{v,t} = \alpha_{v,0} + \sum_{i=1}^{11}\sum_{l=0}^{5} \alpha_{v,i,l}\rho_{i,t-l} + \sum_{j=1,2}\sum_{l=1}^{5} \kappa_{v,j,l} w_{j,t-l} + \xi_{v,t}, \ \ v = 1,2$$

Then, for each linear regression, we proceed iteratively. At each step we estimate the β, γ, α and κ coefficients, we remove the most non-significant one and we estimate again until all coefficients are statistically significant. Finally, if the associated R^2 statistics is large enough, we assume that the model is reliable to be used for further estimations, otherwise we estimate only the regression $\rho_{i,t} = \beta_{i,0} + \varepsilon_{i,t}$ (or $w_{v,t} = \alpha_{v,0} + \xi_{v,t}$ for the macroeconomic variables) and we assume that the underlying process is a geometric Brownian motion having $\mu = \beta_{i,0}$ and $\sigma = \sigma(\varepsilon_{i,t})$ (or $\mu = \alpha_{v,0}$ and $\sigma = \sigma(\xi_{v,t})$ for the macroeconomic variables). Once the models have been defined for all assets and macroeconomic variables, we adopt a Monte Carlo approach to generate the tree nodal values for price returns. For all assets except the cash we proceed in a similar way to define the tree nodal values of the income returns. Moreover, we generate in each node the yield-curve for nominal interest rate using the Svensson model adopted by the European Central Bank, see Svensson (1994). Consequently, using the simulated CPI, we generate also the real interest rate curves for each node.

Then, using (2) and (3), we determine the price and the return of the put options in each node fixing the strike at $\kappa = 1$. In the last part of the empirical section we propose also a sensitivity analysis with respect to the level of κ.

The branching structure of the stochastic tree is 8-4-2-2-2-2-2, i.e. the root node has 8 children, each of them has 4 children, etc. Then, the tree is composed of 512 scenarios which grow over the 7 stages. Such choice is consistent with the number of scenarios in Consigli et al. (2017) that is adopted in practice by an insurance company that manages a pension fund and, thus, it is considered as representative of the degree of uncertainty faced by the portfolio manager. Comparing to Consigli et al. (2017), we reduce the branching in the first stage to compensate the complexity induced by the stochastic dominance constraints.

3.2. Nodal Contamination

One of the purpose of this article is to prove that the put options are a suitable instrument to provide protection against unexpected turmoil of the portfolio assets. Using the scenarios generated with the procedure described above, we verify the hedging features of the put options in standard market conditions. Then, we stress the behavior of the underlying assets in case of high volatility or in case of crisis. To do so, the scenario based on the econometric estimation performed on the period 1999-2015 are used as basis of a further extension that adopt the concept of contamination to produce stressed scenarios. The contamination procedure requires to have an original scenario tree and an alternative scenario tree built under other (usually more extreme) hypothesis. Then, for a set of stages C, we merge the nodal coefficients of the two trees to redefine the value of the asset returns. Given the return $\rho_{i,n}$ of the original tree, the return $\rho^c_{i,n}$ of the alternative tree and a level θ of contamination, the new asset return $\rho^*_{i,n}$ is computed as

$$\rho^*_{i,n} = (1-\theta)\cdot\rho_{i,n} + \theta\cdot\rho^c_{i,n}, \forall n | t(n) \in C \tag{9}$$

This new contamination technique, namely the nodal contamination, differs from the ones proposed in literature, e.g. in Dupačová and Kopa (2012, 2014). Indeed, it changes neither the number of scenarios nor the associated probability and, therefore, it is more suitable for those applications where equiprobable scenarios are needed. Moreover, the alternative tree can be generated according to a new distribution which can be either estimated on new data or assumed. Further, the level θ of nodal contamination allows to test the sensitivity of the solution to the magnitude of the contamination in a simple way and independently by the complexity of the problem which is the same of the uncontaminated one because the proposed technique does not require a nested optimization.

In our analysis, we test the proposed method under two different assumptions.

The former keeps the same econometric estimations performed on the whole period 1999-2015 and doubles the volatility. The latter redefines a new model for the asset and macroeconomic variable processes considering as historical series only the crisis years 2007-2008. We refer to this two approaches as the contamination by volatility (CbV) and the contamination by crisis (CbC). Thus, on the one hand, such approach allows to test the reaction of the portfolio to different types of shocks; on the other hand, we must carefully choose the stages to contaminate in order to create a reasonable settings for our problem. We decide to contaminate two stages under the hypothesis of a sort of prolonged turmoil period and in order to observe the concrete effect of the nodal contamination we pick the first and second stages, i.e. t_1 and t_2. Such choice is driven by two motivations. Firstly, we pick the stages close to the initial stage in order to make more visible the impact on the here-and-now (H&N) optimal solution otherwise the relation between the contamination and the optimal solution would be more diluted; secondly, the choice of the length of two years seems appropriate to reproduce a deep crisis and observe the consequent effects. In Figure 1 and 2, we observe the impact of the nodal contamination on two representative assets: Cash and Real Estate. For each, we propose on the first row the original tree, on the second row the nodal contamination at level $\theta = 0.5$ for CbV (left) and for CbC (right), on the third row the nodal contamination at level $\theta = 0.9$ for CbV (left) and for CbC (right). The CbV simply enlarges the branches of the tree, while the CbC completely changes the shape of the scenarios. In particular, during the crisis we observe extremely high and constant EURIBOR 3-month interest rate because of the loss of trust in the bank system. Jointly, risky assets have huge losses.

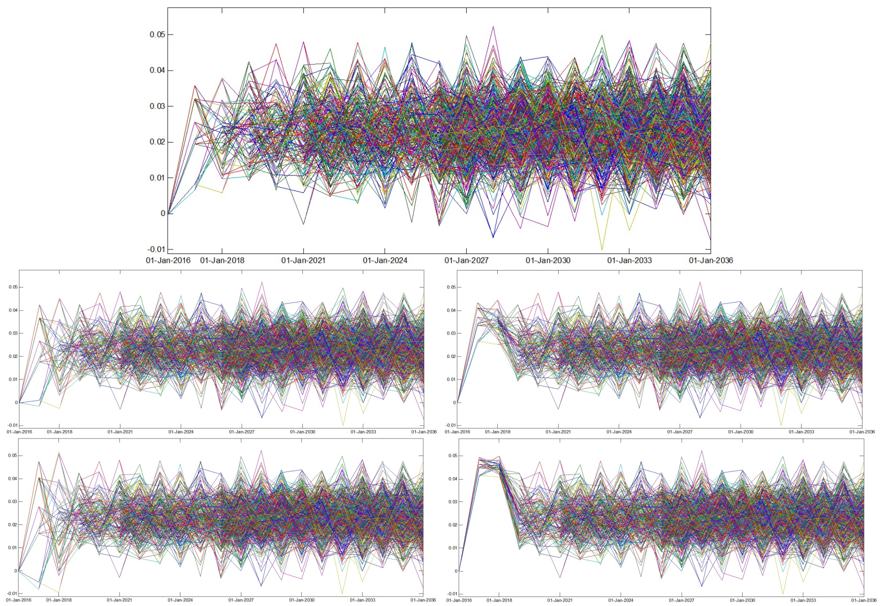


Figure 1: Tree scenarios for Cash. In the top the original tree; on the second row the CbV (left) and the CbC (right) for $\theta = 0.5$; on the third row the CbV (left) and the CbC (right) for $\theta = 0.9$

4. Setting and results

In this section we describe the specific setting of the model and we propose the main results to prove the improvement achieved with the SSD constraints and with the put options.

Furthermore, the benchmark portfolio used in (5) and (8) is built requiring a 10% over performance with respect to the wealth obtained keeping constant the allocation of the initial portfolio where the sponsor guarantees that it never becomes negative. The initial portfolio has a nominal total value of EUR 500 000 and it is composed assuming an allocation for each asset class equal to half of its upper bound as required by the pension fund manager, cf. Table 1. For classes with more than one asset, we allocate uniformly in every asset. Then, we allocate 15% in

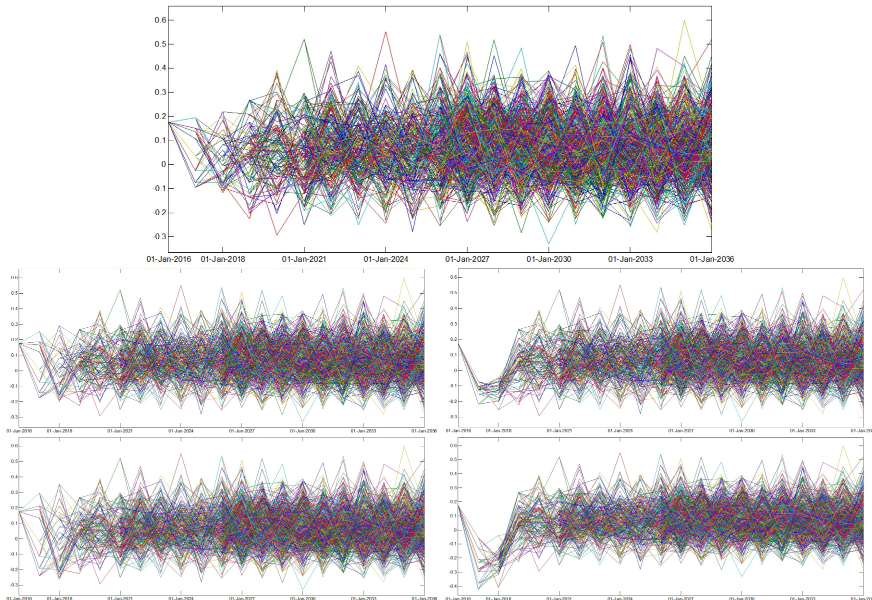


Figure 2: Tree scenarios for Real Estate. In the top the original tree; on the second row the CbV (left) and the CbC (right) for $\theta = 0.5$; on the third row the CbV (left) and the CbC (right) for $\theta = 0.9$

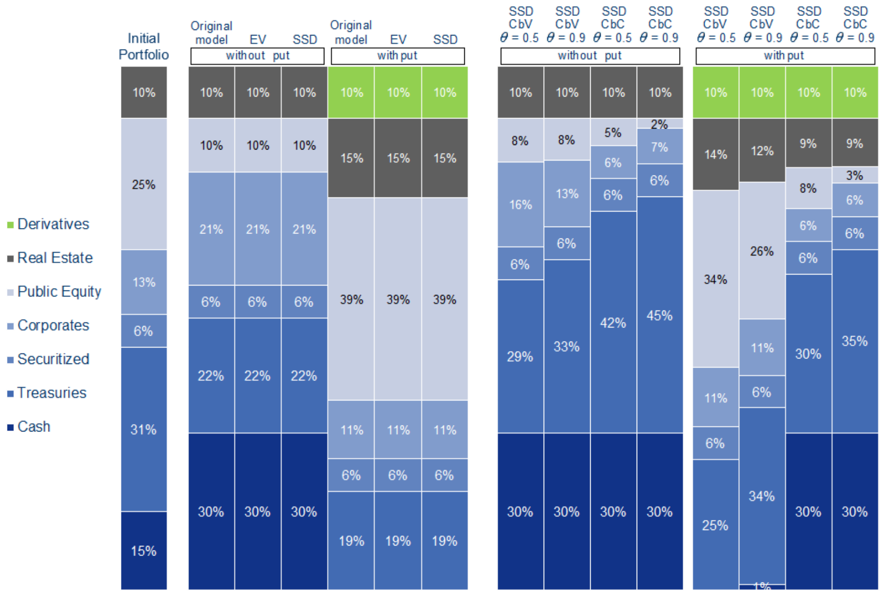
Cash, 50% in Bonds uniformly distributed among the assets (6.25% each), 10% in Real Estate and 25% in Public Equity. Note that the initial portfolio does not invest in put option since our purpose is to prove that the inclusion of the hedging contracts is able to improve the performance of a no-option portfolio. Therefore, $\mathbf{w}_{t_h}^{B}$ is obtained by the evolution of the initial portfolio in each node given by the returns of the assets and the payment of the liabilities, then setting to zero the negative values assuming an injection of liquidity by the pension fund sponsor and finally increasing all the values by 10%.

We produce results for three cases: the original model, the model with the expected value constraint (8) on the last stage, the model with the second-order stochastic dominance constraints (5)-(7) on the last stage. For each, we test either the portfolio without put options and the case with hedging contracts. The results are reported in Figure 3 in terms of the optimal H&N allocations and of the statistics of the final wealth, and in Figures 4 and 6 the whole distributions of the achieved final wealth are depicted. In Figure 3 we label the analyzed cases with a letter from (a) to (n) for a better readability.

All the results are given by the implementation of a linear programming model solved by CPLEX 12.1.0 in GAMS, with an Intel(R) Core(TM) i7-4510U CPU 2.60GHz with 8.00GB RAM running Windows 10. Input management, parameter and coefficient computations, and output analysis are performed in MATLAB R2013b. CPLEX generates the original model with 845 656 single equations, 921 655 single variables, 4 201 484 non zero elements and generates the model with stochastic dominance constraints with 847 192 single equations, 1 183 799 single variables, 4 979 724 non zero elements. For the latter case, the solvers takes less than one hour to find the optimal solution which is compatible with the pension fund manager requirements. Problem instances with larger dimension gave similar results but inducing memory issues.

4.1. Without contamination and without derivatives

In Figure 3(a), the H&N solution of the original model without put increases the allocation in Cash and in Corporate, decreasing the concentration in Treasury and Public Equity with respect to the initial portfolio. Such strategy evolves to achieve a final wealth of EUR 252 380 (in average) keeping under control the left tail with $AV@R$ equal to EUR 50 625 and with $V@R$ equal to EUR 79 667. Indeed, the final wealth distribution is highly concentrated around the threshold identified by the objective variable on the last stage, see the left side of Figure 4. Nevertheless, the final wealth suffers some extreme events and the left tail of the distribution is remarkably fat. Adding the EV constraint the left tail does not change with respect to the original model, cf. $AV@R$ and $V@R$ in Figures 3(a) and 3(b). In



	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)
Mean	2.52	2.56	2.58	2.46	2.56	2.59	2.62	2.66	2.12	1.99	2.61	2.66	2.11	1.94
Std Dev	1.44	1.46	1.49	1.13	1.30	1.32	1.60	1.66	1.27	1.17	1.37	1.41	1.11	0.96
V@R	0.80	0.80	0.80	1.51	1.51	1.51	0.71	0.61	0.36	0.35	1.55	1.50	0.51	0.44
AV@R	0.51	0.51	0.51	0.95	0.95	0.95	0.42	0.35	0.19	0.18	0.97	1.00	0.30	0.22

Figure 3: Comparison among the optimal H&N allocations of all tested models with statistics (hundreds of thousands of EUR) related on the final wealth achieved with each strategy

particular, the portfolio final wealth, even starting from the same H&N solution, reaches a higher expected value (from EUR 252 380 to EUR 255 502) increasing also the standard deviation (from EUR 144 265 to EUR 146 020) on the right side of the distribution (higher final wealth). With the SSD constraint the same behavior is further strengthened (the expected final wealth increases again by 1%), cf. Figures 3(b) and 3(c).

4.2. *Without contamination and with derivatives*

Introducing put options in the asset universe, see Figure 3(d), the H&N solution of the original model exploits such opportunity and allocates the whole admitted

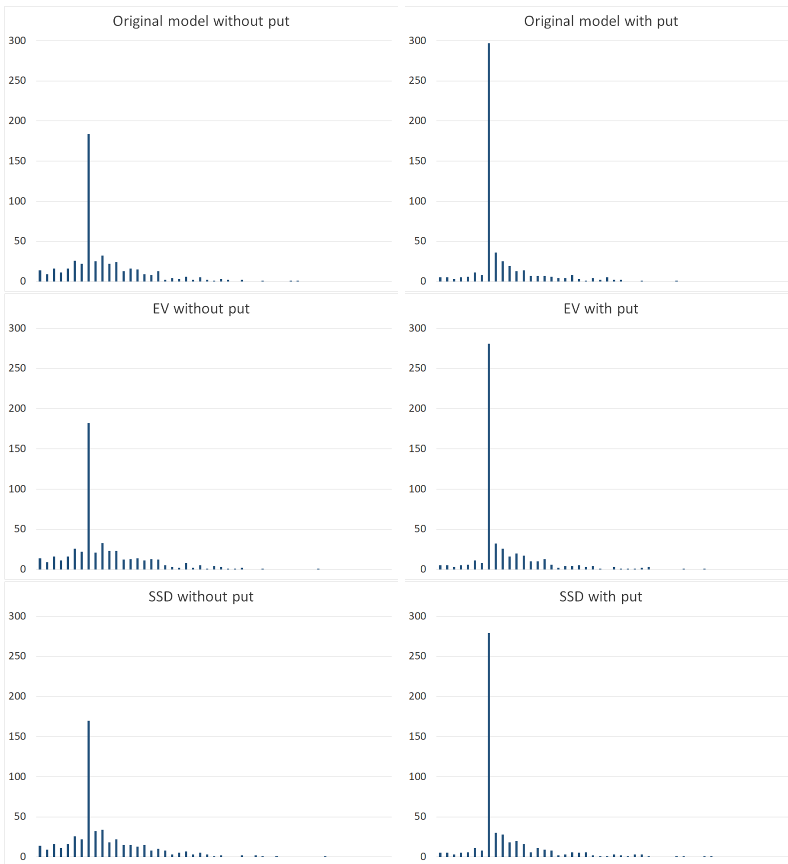
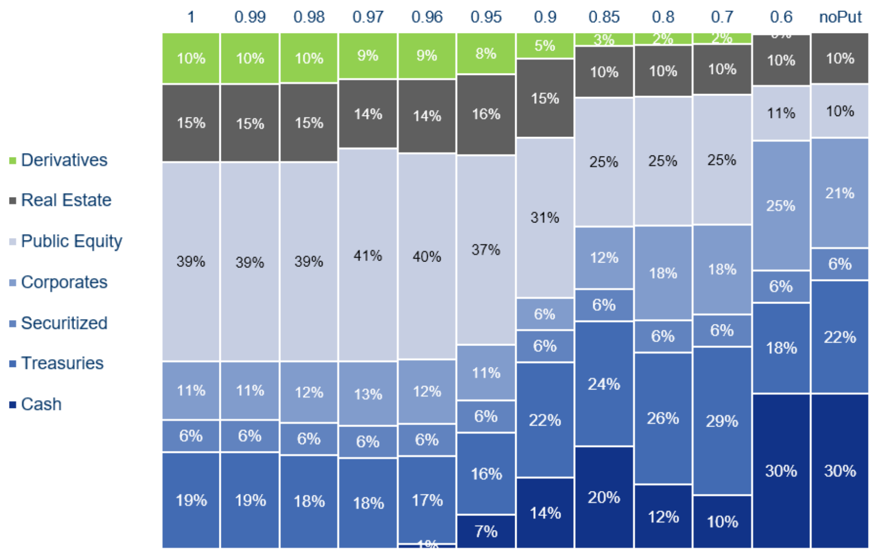


Figure 4: Final wealth distributions for the cases without contamination

10% of the portfolio in the hedging contracts. The protection regards only the 1-year maturity and is equally divided in put options on Real Estate and on Public Equity. Simultaneously, since the two most risky assets are now covered by the hedging positions, the solution increases the allocation in Real Estate and Public Equity. Another effect of the hedging is the reduction of the shortfall of the objective variables. The impact of the put option costs decreases the expected final wealth from EUR 252 380 to EUR 246 322. Therefore, we observe a safer solution, with $AV@R$ almost doubled from EUR 50 652 to EUR 94 642, but also more conservative with standard deviation decreased from EUR 144 265 to EUR 112 757. Such behavior is highlighted in Figure 4 where the left tails of the distributions found exploiting the put are extremely thinner than the cases without options. The EV solution maintains a reduced standard deviation, but reaches the same expected final wealth EUR 255 502 of the no-option case as required by the EV constraint itself, cf. Figures 3(e) and 3(b). The SSD solution keeping a low risk strategy, is even able to further increase the expected value of the final wealth despite the cost related to the put options, from EUR 257 893 to EUR 258 507, cf. Figures 3(f) and 3(c).

To summarize, the inclusion of the derivatives allows for a safer solution for all three cases. Using the original model the drawback is a lower expected final wealth probably due to the cost of the hedging positions; using the EV constraint the portfolio reaches the same expected final wealth; using the SSD constraint the solution achieves a even higher expected final wealth.

Focusing on the put options used in the optimal portfolio, the 2-year options are used only in optimal allocation of next stages, whereas the 1-year maturity options constitute 10% of the H&N optimal portfolio. Pricing all these options, we fixed strike κ equal to 1 in (2) and (3), i.e. the option protects from any performance under 0%. In Figure 5, we test for the SSD case the sensitivity of the optimal



	1	0.99	0.98	0.97	0.96	0.95	0.9	0.85	0.8	0.7	0.6	noPut
Mean	2.59	2.56	2.57	2.57	2.57	2.63	2.59	2.59	2.60	2.63	2.59	2.58
Std Dev	1.32	1.26	1.32	1.33	1.33	1.55	1.47	1.50	1.50	1.55	1.59	1.49
V@R	1.51	1.52	1.47	1.56	1.53	1.47	1.39	1.15	1.15	1.08	0.94	0.80
AV@R	0.95	0.93	0.95	0.98	0.98	0.91	0.88	0.70	0.68	0.58	0.50	0.51

Figure 5: Comparison among the H&N allocations for different put strikes considering the SSD case and statistics (hundreds of thousands of EUR) related on the final wealth achieved with each strategy

H&N solution with respect to the strike κ of the put options. In particular, we fix $\kappa = 1, 0.99, 0.98, 0.97, 0.96, 0.95, 0.90, 0.85, 0.8, 0.7, 0.6$, i.e. a protection against a loss greater than 0%, 1%, 2%, 3%, 4%, 5%, 10%, 15%, 20%, 30%, 40%, respectively, and observe a progressive reduction in the option allocation since their protection power reduces. Jointly, the allocation in Cash increases and the whole portfolio tends to converge to the optimal allocation of the case without put options. The statistics on the final wealth distribution show that riskiness increases both in terms of $AV@R$ and $V@R$, and according to the standard deviation, while the expected final wealth increases because the hedging cost decreases. All values finally converge to the case without derivatives.

4.3. *With contamination, without derivatives*

In this Section, we perform several analysis to study the capability of the optimal portfolio to contain the market risk also during turmoil periods. Hence, we use the contaminated scenario trees presented in Section 3.2. All results are produced imposing the SSD on the last stage. In Figures 3(g) and 3(h), we consider the optimal portfolios using the CbV at level $\theta = 0.5$ and $\theta = 0.9$ without put. Comparing with the SSD case in absence of contamination and without put, cf. Figure 3(c), we notice that the expected final wealth increases from EUR 257 893 to EUR 261 701 and to EUR 266 411, the volatility increases as well from EUR 148 856 to EUR 160 327 and to EUR 166 379, while the left tail worsens: the $V@R$ decreases from EUR 79 667 to EUR 70 945 and to EUR 60 900, and the $AV@R$ decreases from EUR 50 652 to EUR 42 292 and to EUR 35 165. The H&N allocation slightly reduces the investment in Corporates and in Public Equity in favor of the Treasuries. We remark a similar behavior in the CbC cases without put, cf. Figures 3(i) and 3(j). Again, the left tail worsens. Moreover, also the expected final wealth decreases because most of the assets experience huge losses during the crisis and even if the H&N solution allocates more than 80% in the less risky assets (Cash, Treasuries and Securitized), still the portfolio extremely suffers the first two-year crises and is not able to recover in the next 18 years. In all cases, the final wealth distributions highlight a remarkable effect when some contaminated scenarios are considered, cf. left side of Figure 6. Therefore, in the next Section, we include the hedging derivatives to reduce these losses.

4.4. *With contamination and with derivatives*

After noticing that the portfolio reflects the turmoil of the markets, especially when the crisis contamination is considered, we test whether the presence of the put options protects the portfolio against the market risk.

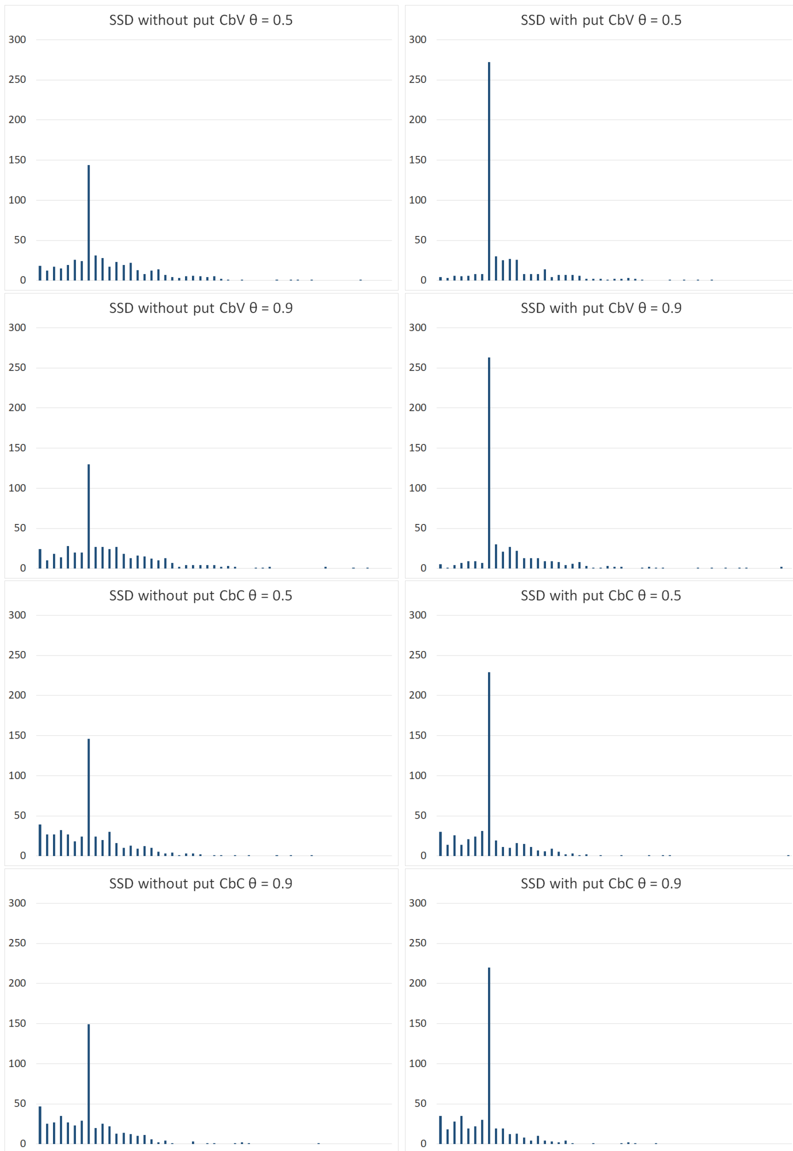


Figure 6: Final wealth distributions with contamination

Comparing the results of the CbV, cf. Figures 3(k) and 3(l) with Figure 3(f), it is interesting to notice that the solution obtained when CbV is considered is better than the one obtained without contamination. In particular, the expected final wealths are greater: EUR 261 607 and EUR 266 077 against EUR 258 507; and the $AV@R$ is greater as well: EUR 97 139 and EUR 99 818 against EUR 94 642. This is because the perturbation generated by CbV enlarges the returns on both sides, i.e. the negative returns become more negative and the positive ones become more positive. Nevertheless, the derivatives protect the negative outcomes and their higher cost is more than compensated by the extra gain given by the positive returns. Comparing Figures 3(k) and 3(l) with Figures 3(g) and 3(h) respectively, we notice that the expected final wealth of the cases with contamination and with put remains approximately the same of the cases with contamination and without put options. The volatility reduces to EUR 137 025 and EUR 140 552 from EUR 160 327 and EUR 166 379. Also the left tail improves significantly: the $V@R$ increases to EUR 154 612 and EUR 149 653 from EUR 70 945 and EUR 60 900, and the $AV@R$ increases to EUR 97 139 and EUR 99 818 from EUR 42 292 and EUR 35 165. Thus, the put options add a huge benefit to the portfolio in terms of reduction of the risk and, in particular, in terms of protection of the left tail of the distribution, see Figure 6. Moreover, since now the Real Estate and the Public Equity positions are hedged, the optimal solution allocates a large portion into these two assets in order to benefit from the larger positive returns because the negative ones are covered by the put options.

While in the CbV case, the presence of the derivatives allows the portfolio to invest more in Real Estate and Public Equity, in the CbC case the hedging instruments are used to cover the existing positions but the optimal portfolio does not increases the two most risky exposures, cf. Figures 3(m) and 3(n) with Figures 3(k) and 3(l). This is because the negative returns are not balanced by some very positive ones as in the CbV case. Now the scenarios in the first two years are all very pessimistic and, since for turnover reasons the pension fund manager is forced to keep these assets, the optimal solution suggests to reduce the riskiest asset allocation and hedge them as much as possible. However, cf. Figures 3(m) and 3(n) with Figures 3(i) and 3(j), the use of the put options is able to improve significantly the left tail: the $V@R$ increases to EUR 50 746 and EUR 44 218 from EUR 35 533 and EUR 34 882, and the $AV@R$ increases to EUR 30 326 and EUR 22 246 from EUR 19 392 and EUR 18 414. Also the volatility reduces to EUR 110 825 and EUR 96 423 from EUR 117 111 and EUR 137 025, while the expected final wealth decreases. Comparing these results with the case without contamination, cf. Figures 3(m) and 3(n) with Figure 3(f), the optimal H&N allocation is significantly more conservative and the investment in Public Equity, Real Estate and Corporates is extremely reduced. Nevertheless, the final wealth distribution is greatly riskier both in terms of left tail and in terms of expected final wealth, see Figure 6.

4.5. Further results

Similar results can be obtained adopting the naïve portfolio as benchmark portfolio, i.e. the $1/N$ allocation where N is the number of assets, see DeMiguel et al. (2009). In this article, we preferred to propose a comparison with the initial portfolio benchmark which is the one the pension fund manager beliefs in and, therefore, the more interesting to stochastically dominate.

We test also the case in which the upper bound for the derivatives allocation is relaxed till 100% but we keep an upper bound of 10% on each single put. The results show that in all cases (no-contamination, CbV and CbC) the solution is only slightly better, and the H&N allocation in derivatives never exceeds 11%, not even in the CbC with $\theta = 0.9$. This means that the optimal allocation is mainly defined to fulfill other constraints (diversification, liquidity, turnover, etc.). Similar results are obtained relaxing also the single put upper bounds to 100%, the total allocation increases to 15% in the CbV case and 13% in the CbC case.

4.6. Out-of-sample analysis

In order to study the performance of the proposed portfolios, we conduct an out-of-sample analysis considering the realizations of the portfolio assets along the eight quarters in 2016-2017. In these quarters, severe shocks occurred in the markets: at the beginning of 2016 the Chinese market slowdowns and this shock waves across the world penalizing also the European Equity market; in June 2016 the Brexit referendum provokes a huge turmoil in the European bond market and an escape from the Treasury market in eurozone in the subsequent six months. Such contest represents a hard period to test our strategies out-of-sample. However, some interesting results show up. Starting with the H&N allocations defined previously and with a initial wealth of 100, we observe the wealth evolution of the portfolios and we focus in particular on the wealth achieved at the end of the first year, according to the rebalancing policy as required by the pension fund manager.

Figure 7 shows the case without contamination, both with put options and without put options. Since the optimal H&N solutions of the original model, of the EV and of the SSD cases are the same (cf. Sections 4.1 and 4.2), the out-of-sample wealth evolution of the three cases is the same. All portfolios reach a higher final wealth than the benchmark in both cases: with and without put options. In particular the portfolios that exploit the put options outperform remarkably the benchmark. This is true considering also the intermediate horizon of one year (Q4 2016).

Figure 8 shows the two cases with CbV (at level 0.5 and 0.9) both with put options and without put options. The two portfolios that exploit the put options outperform the benchmark from Q4 2016. During the first two quarters of 2016 the

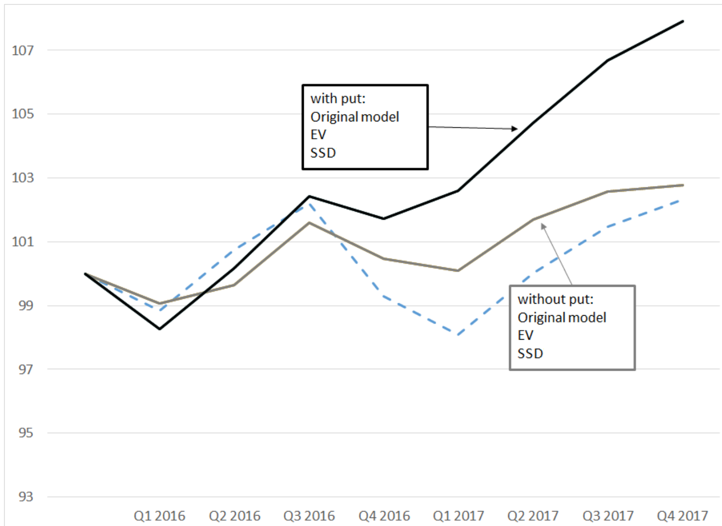


Figure 7: Out-of-sample analysis for the case with no-contamination. The dashed line represents the benchmark wealth evolution.

markets do not show a relevant volatility and then all the strategies are dominated by the benchmark. However, during the Treasury bond crisis of Q4 2016 and Q1 2017, the portfolios with options are able to exploit the increasing volatility and recover until they dominate the benchmark, while the portfolios without put options are overexposed on the Treasury investments and suffer relevantly.

Figure 9 shows the two cases with CbC (at level 0.5 and 0.9) both with put options and without put options. All the portfolios dominate the benchmark during Q1 and Q2 2016 because they are constructed exactly to face the turmoil induced by the Chinese crisis: a loss in the Equity market and a fly-to-safety in direction of Treasury investments like for the crisis in 2007-2008. However, all of them suffer remarkably when the opposite situation realized during the Treasury market crisis. Finally, none of them is able to beat the benchmark.

Comparing the results reported in Table 2, we can remark some interesting ev-

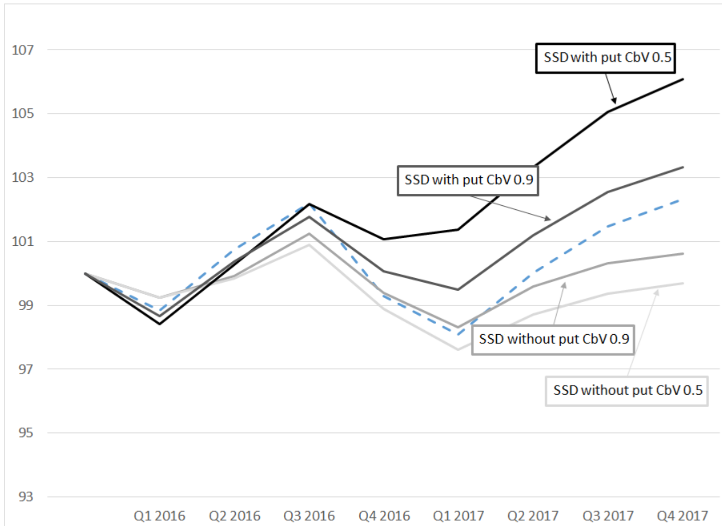


Figure 8: Out-of-sample analysis for the case with CbV, either at level 0.5 and 0.9. The dashed line represents the benchmark wealth evolution.

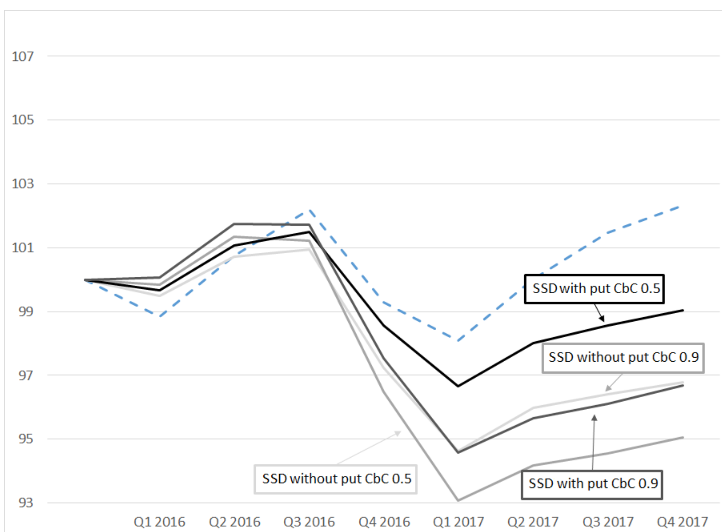


Figure 9: Out-of-sample analysis for the case with CbC, either at level 0.5 and 0.9. The dashed line represents the benchmark wealth evolution.

idences. At the short horizon (Q1 2016), the contamination is working properly and the relative strategies are able to dominate the benchmark. This is given by the occurrence of the Chinese crisis that is similar to the one assumed for the contamination and then the portfolios are protected against such conditions. In a longer period (Q4 2016 and Q4 2017), the market evolves to a different type of crisis, the low-risk investors escape from eurozone Treasury market, and then the contamination does not capture this behavior. Therefore, only the portfolios constructed under the CbV case are able to beat the benchmark thanks to the put options. On the other hand, all strategies defined in the no-contamination framework outperform the benchmark.

The main contribution of this analysis is to stress that the nodal contamination technique influences strongly the structure of the optimal portfolio and, therefore, that the portfolio could be not able to effectively adapt in case of market turmoils quite far from the contamination assumptions. Moreover, in presence of crisis, the put option protection gives a benefit larger than its cost in most of the cases.

5. Conclusion

The paper proposed and analysed a new Asset Liability Management model in the form of multi-criteria multi-stage stochastic program. Extending Consigli et al. (2017), we proposed three main improvements: the use of put option within the asset universe, the inclusion of stochastic dominance constraints and the application of a new contamination technique, the nodal contamination. The main contribution of the paper is to study their joint and reciprocal impact. The hedging contracts allow the portfolio to improve the optimal objective value reducing the shortfall. Such results are achieved through a dynamic and well diversified strategy. Moreover, the definition of a proper benchmark portfolio and the inclusion

Table 2: Out-of-sample report of the wealth achieved with the suggested portfolios at three representative horizon: one quarter (Q1 2016), one year (Q4 2016) and two years (Q4 2017). In bold the values that outperform the benchmark.

			Q1 2016	Q4 2016	Q4 2017
no-contamination		Benchmark	98.8	99.3	102.3
	without put	Original model	**99.1**	**100.5**	**102.8**
		EV	**99.1**	**100.5**	**102.8**
		SSD	**99.1**	**100.5**	**102.8**
	with put	Original model	98.5	**102.0**	**108.2**
		EV	98.5	**102.0**	**108.2**
		SSD	98.5	**102.0**	**108.2**
CbV 0.5	without put	SSD	**99.2**	**99.4**	100.6
	with put	SSD	98.7	**101.3**	**106.3**
CbV 0.9	without put	SSD	**99.2**	98.9	99.7
	with put	SSD	**98.9**	**100.3**	**103.5**
CbC 0.5	without put	SSD	**99.5**	97.2	96.8
	with put	SSD	**99.8**	98.7	99.2
CbC 0.9	without put	SSD	**99.9**	96.5	95.1
	with put	SSD	**100.2**	97.7	96.8

of a constraint regarding its expected final wealth proved that it is possible to improve the expected final wealth with respect to the original model. Moving from this result, we exploit the instruments provided by the stochastic dominance theory including the SSD constraint that pushed the portfolio final wealth even higher and proved that we can beat the benchmark not only in expectation but dominating its whole distribution. Then, we introduced the nodal contamination to analyse whether the portfolio is protected and able to face also undesired events such as an unexpected increase of the volatility or a huge negative shock on the markets. The model proved to react well to these scenarios but without derivatives the shortfall is quite relevant. Using the hedging contracts, the behavior improves both in the CbV case and in the CbC case. In particular, in the CbV case, the put options redesign the allocation moving back a large portion of the portfolio to the riskiest assets; contrary, in the CbC case, the options are again used to hedge the positions in the Real Estate and Public Equity but there is not a corresponding increase in their allocations. The out-of-sample analysis shows that, most of the time, the optimal portfolio suggested by the model is able to beat the benchmark portfolio. Special attention must be paid when choosing the type of contamination since an extreme contamination could produce a portfolio that hardly adapt to a different market framework. To summarize, if we use the put options in the model and no-contamination is applied then the optimal portfolio out-of-sample outperforms the benchmark no matter if stochastic dominance constraints are used or not.

In conclusion, we proved that the model proposed in Consigli et al. (2017) can be effectively improved adopting the second-order stochastic dominance and enlarging the asset universe to include also hedging derivatives. Moreover, we highlight the quality of the nodal contamination to stress the scenarios and observe the behavior of the portfolio during turmoil periods. Especially in such periods, the benefit introduced by the derivatives is widely appreciated.

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Bibliography

Bertocchi, M., Consigli, G., D'Ecclesia, R., Giacometti, R., Moriggia, V., Ortobelli, S., 2013. Euro bonds: Markets, infrastructure and trends. World Scientific Books.

Bradley, S. P., Crane, D. B., 1972. A dynamic model for bond portfolio management. Management Science 19 (2), 139–151.

Bradley, S. P., Crane, D. B., 1980. Managing a bank bond portfolio over time. Stochastic Programming"(MAH Dempster, ed.), Academic Press, London, 449–471.

Broeders, D., Chen, A., Koos, B., 2009. An institutional evaluation of pension funds and life insurance companies. De Nederlandsche Bank.

Cariño, D. R., Kent, T., Myers, D. H., Stacy, C., Sylvanus, M., Turner, A. L., Watanabe, K., Ziemba, W. T., 1994. The Russell-Yasuda Kasai model: An asset/liability model for a Japanese insurance company using multistage stochastic programming. Interfaces 24 (1), 29–49.

Cariño, D. R., Myers, D. H., Ziemba, W. T., 1998b. Concepts, technical issues, and uses of the Russell-Yasuda Kasai financial planning model. Operations research 46 (4), 450–462.

Cariño, D. R., Ziemba, W. T., 1998a. Formulation of the Russell-Yasuda Kasai financial planning model. Operations Research 46 (4), 433–449.

Consigli, G., 2007. Individual asset liability management for individual investors. In: Zenios, S. A., Ziemba, W. T. (Eds.), Handbook of Asset and Liability Management: Applications and case studies. North-Holland Finance Handbook Series, Elsevier, pp. 752–827.

Consigli, G., Dempster, M. A. H., 1998a. The CALM stochastic programming model for dynamic asset-liability management. Worldwide asset and liability modeling 10, 464.

Consigli, G., Dempster, M. A. H., 1998b. Dynamic stochastic programming for asset-liability management. Annals of Operations Research 81, 131–162.

Consigli, G., di Tria, M., 2012. Optimal long-term property and casualty ALM with risk capital control. Asset-Liability Management for Financial Institutions 1, 137–150.

Consigli, G., di Tria, M., Gaffo, M., Iaquinta, G., Moriggia, V., Uristani, A., 2011. Dynamic portfolio management for property and casualty insurance. In: Stochastic Optimization Methods in Finance and Energy. Springer, pp. 99–124.

Consigli, G., Iaquinta, G., Moriggia, V., di Tria, M., Musitelli, D., 2012. Retirement planning in individual asset-liability management. IMA Journal of Management Mathematics 23 (4), 365–396.

Consigli, G., Moriggia, V., 2014. Applying stochastic programming to insurance portfolios stress-testing. Quantitative Finance Letters 2 (1), 7–13.

Consigli, G., Moriggia, V., Benincasa, E., Landoni, G., Petronio, F., Vitali, S., di Tria, M., Skoric, M., Uristani, A., 2017. Optimal multistage defined-benefit pension fund management. In: Consigli, G., Stefani, S., Zambruno, G. (Eds.), Recent Advances in Commmodity and Financial Modeling: Quantitative methods in Banking, Finance, Insurance, Energy and Commodity markets. Springer's International Series in Operations Research and Management Science.

Davis, P. E., 2000. Portfolio regulation of life insurance companies and pension funds. Economics.

DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? Review of Financial Studies 22 (5), 1915–1953.

Dempster, M. A. H., Germano, M., Medova, E. A., Villaverde, M., 2003. Global asset liability management. British Actuarial Journal 9 (01), 137–195.

Dempster, M. A. H., Ireland, A. M., 1988. MIDAS: An expert debt management advisory system. In: Data, expert knowledge and decisions. Springer, pp. 116–127.

Dempster, M. A. H., Ireland, A. M., 1989. Object-oriented model integration in MIDAS [Manager's Intelligent Debt Advisory System]. In: System Sciences, 1989. Vol. III: Decision Support and Knowledge Based Systems Track, Proceedings of the Twenty-Second Annual Hawaii International Conference on. Vol. 3. IEEE, pp. 612–620.

Dempster, M. A. H., Ireland, A. M., 1991. Object-oriented model integration in a financial decision support system. Decision Support Systems 7 (4), 329–340.

Dentcheva, D., Ruszczynski, A., 2003. Optimization with stochastic dominance constraints. SIAM Journal on Optimization 14 (2), 548–566.

Dentcheva, D., Ruszczyński, A., 2004. Semi-infinite probabilistic optimization: first-order stochastic dominance constrain. Optimization 53 (5-6), 583–601.

Dert, C. L., 1998. A dynamic model for asset liability management for defined benefit pension funds. Worldwide asset and liability modeling 10, 501–536.

Dupačová, J., Kopa, M., 2012. Robustness in stochastic programs with risk constraints. Annals of Operations Research 200 (1), 55–74.

Dupačová, J., Kopa, M., 2014. Robustness of optimal portfolios under risk and stochastic dominance constraints. European Journal of Operational Research 234 (2), 434–441.

Geyer, A., Ziemba, W. T., 2008. The Innovest Austrian pension fund financial planning model InnoALM. Operations Research 56 (4), 797–810.

Gollier, C., 2008. Intergenerational risk-sharing and risk-taking of a pension fund. Journal of Public Economics 92 (5), 1463–1485.

Guiso, L., Haliassos, M., Jappelli, T., 2002. Household portfolios. MIT press.

Hadar, J., Russell, W. R., 1969. Rules for ordering uncertain prospects. The American Economic Review, 25–34.

Hanoch, G., Levy, H., 1969. The efficiency analysis of choices involving risk. The Review of Economic Studies, 335–346.

Hardy, M., 2003. Investment guarantees: modeling and risk management for equity-linked life insurance. Vol. 215. John Wiley & Sons.

Kopa, M., Moriggia, V., Vitali, S., 2018. Individual optimal pension allocation under stochastic dominance constraints. Annals of Operations Research 260 (1-2), 255–291.

Kopa, M., Post, T., 2015. A general test for ssd portfolio efficiency. OR Spectrum 37 (3), 703–734.

Kuosmanen, T., 2004. Efficient diversification according to stochastic dominance criteria. Management Science 50 (10), 1390–1406.

Kusy, M. I., Ziemba, W. T., 1986. A bank asset and liability management model. Operations Research 34 (3), 356–376.

Luedtke, J., 2008. New formulations for optimization under stochastic dominance constraints. SIAM Journal on Optimization 19 (3), 1433–1450.

McKendall, R., Zenios, S. A., Holmer, M., 1994. Stochastic programming models for portfolio optimization with mortgage backed securities: Comprehensive research guide. In: Operations Research Models in Quantitative Finance. Springer, pp. 134–171.

Mulvey, J. M., 1994a. An asset-liability investment system. Interfaces 24 (3), 22–33.

Mulvey, J. M., 1994b. Financial planning via multi-stage stochastic programs. Mathematical Programming: State of the Art.

Mulvey, J. M., Gould, G., Morgan, C., 2000. An asset and liability management system for Towers Perrin-Tillinghast. Interfaces 30 (1), 96–114.

Mulvey, J. M., Simsek, K. D., Zhang, Z., 2006. Improving investment performance for pension plans. Journal of Asset Management 7 (2), 93–108.

Mulvey, J. M., Simsek, K. D., Zhang, Z., Fabozzi, F. J., Pauling, W. R., 2008. Assisting defined-benefit pension plans. Operations research 56 (5), 1066–1078.

Mulvey, J. M., Ural, C., Zhang, Z., 2007. Improving performance for long-term investors: wide diversification, leverage, and overlay strategies. Quantitative Finance 7 (2), 175–187.

Nielsen, S. S., Zenios, S. A., 1996. A stochastic programming model for funding single premium deferred annuities. Mathematical Programming 75 (2), 177–200.

Pflug, G. C., Świetanowski, A., 1999. Dynamic asset allocation under uncertainty for pension fund management. Control and Cybernetics 28, 755–777.

Post, T., 2003. Empirical tests for stochastic dominance efficiency. The Journal of Finance 58 (5), 1905–1931.

Quirk, J. P., Saposnik, R., 1962. Admissibility and measurable utility functions. The Review of Economic Studies (29), 140–146.

Svensson, L. E., 1994. Estimating and interpreting forward interest rates: Sweden 1992-1994. Tech. rep., National Bureau of Economic Research.

Vitali, S., Moriggia, V., Kopa, M., 2017. Optimal pension fund composition for an italian private pension plan sponsor. Computational Management Science 14 (1), 135–160.

Yang, X., Gondzio, J., Grothey, A., 2010. Asset liability management modelling with risk control by stochastic dominance. Journal of Asset Management 11 (2), 73–93.

Zenios, S. A., 1995. Asset/liability management under uncertainty for fixed-income securities. Annals of Operations Research 59 (1), 77–97.

Zenios, S. A., Ziemba, W. T., 2006. Handbook of Asset and Liability Management: Theory and Methodology. Vol. 1. North-Holland Finance Handbook Series, Elsevier.

Zenios, S. A., Ziemba, W. T., 2007. Handbook of Asset and Liability Management: Applications and case studies. Vol. 2. North-Holland Finance Handbook Series, Elsevier.

Ziemba, W. T., 2007. The Russell-Yasuda, InnoALM and related models for pensions, insurance companies and high net worth individuals. In: Zenios, S. A., Ziemba, W. T. (Eds.), Handbook of Asset and Liability Management: Applications and case studies. North-Holland Finance Handbook Series, Elsevier, pp. 861–962.

Ziemba, W. T., Mulvey, J. M. (Eds.), 1998. Worldwide asset and liability modeling. Vol. 10. Cambridge University Press, UK.