

Forecasting Methods and Optimization Models for the Inventory Management of Perishable Products: the Case of “La Centrale del Latte di Vicenza SpA”

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Abstract This paper is a report on a joint project by the Department of Management, Economics and Quantitative Methods of the University of Bergamo (in collaboration with the Department of Economics and Management of the University of Brescia) and the Italian company “La Centrale del Latte di Vicenza SpA” producing different types of milk, yogurt, vegetable drinks, cream, butter, cheese, eggs and vegetables. The aim of the project was to provide forecasting methods and optimization models, to improve the demand forecasts of perishable products and to better manage inventory levels in a *Material Requirements Planning* (MRP) setting.

Key words: Perishable products, Forecasting methods, Material Requirements Planning (MRP), Optimization models.

1 Introduction

Demand forecasting and optimal allocation of resources are critical processes for any company. Managers typically have to take complex decisions in short time. They have to analyze larger and larger amount of data and qualitative information, often in an uncertain setting. Quantitative methods (like statistical methods, Operations Research models and algorithms, ...) can be adopted to support managers in taking these decisions (see [7]).

The purpose of this article is to describe the forecasting methods and the *Material Requirements Planning* (MRP) optimization models we provided to support

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an Italian company, “La Centrale del Latte di Vicenza SpA”, interested in having better demand forecasts and a better inventory management of perishable products. We supported the Company in identifying the best forecasting methods between the *Locally Weighted Regression Estimator* (LOESS) and the *Seasonal Holt-Winters* (HW). Then, we formulated optimization models replicating the MRP used in the Company. An optimal solution of these models allowed us to evaluate the MRP embedded in the information system used by the Company and to reduce the inventory levels of raw materials, packaging materials, components and final products. We applied these methods and models to a large set of real instances provided by the Company.

The structure of the paper is as follows. In Section 2 we give a description of the real problem and of the joint project. In Section 3 we describe the forecasting methods we suggested to the Company. In Section 4 we describe the optimization model we formulated to evaluate and improve the MRP used by the Company. In Section 5 we provide examples of the application of these methods and models to real instances provided by the Company. Conclusions are drawn in Section 6.

2 The real problem and the joint project

“La Centrale del Latte di Vicenza SpA” produces and packages different formats and types of milk, yogurt, vegetable drinks, cream, butter, cheese, eggs and vegetables. These products have different shelf life, ranging for example from 5 days for fresh milk, to 50 days for yogurt and to 120 days for UHT milk. All articles follow a pull production, with exception of UHT milk for which a push system is used. The Company buys from suppliers raw materials - jams and lactic ferments - as well as materials for packaging. Some of these products are perishable and, therefore, if they are not used before their expiry date, additional costs are paid by the Company. Reliability of the suppliers, delivery times, lot sizes have to be taken into account in the production management.

Before this project, the Company used the MRP embedded in the information system as a black box to make requirements planning, without applying any quantitative method to forecast the demand of the final products, but just monitoring the sales trend on a daily basis. For production on behalf of third parties, in the presence of a high demand uncertainty, the Company considered monthly sales budgets and divided it into a weekly time frame.

The aim of the Company in this project was, on one side, to start using quantitative methods to forecast the demand, especially for products on behalf of third parties, identifying possible trends and seasonality. On the other side, the aim was to start using optimization models to have an optimal requirements planning, having in this way a tool to evaluate the MRP used by the Company and to improve it by reducing the inventory levels. Note that these optimization models can use the demand forecasts of the final products, provided in the first part of the project, as an input data.

3 Forecasting Methods

Concerning the demand forecast of the final products, we suggested the following methods to the Company: *Locally Weighted Regression Estimator* (LOESS) and *Seasonal Holt-Winters* (HW). Here we briefly recall them.

LOESS is a modern modeling methods originally proposed in [1], designed to address situations in which the classical procedures do not perform well. It fits simple models to localized subsets of the data to build up a function that describes the deterministic part of the variation in the data, point by point.

Given a dataset consisting of a collection of n (x,y) pairs, one of the purposes of LOESS is to provide a general idea of the dataset's distributional form prior to committing to a specific parametric model. One of the benefits of LOESS is that there is no requirement to specify a global function to fit to the data. Instead, locally weighted regression is performed on a pre-defined number of nearest neighbors around each data point.

In order to carry out the LOESS fitting procedure, we need to specify the number of data points to use for each locally weighted regression, and the degree of the polynomial used for estimation. A user-specified input to the procedure α , called smoothing parameter, determines how much of the data is used to fit each local polynomial. For example, if a dataset contains $n = 20$ observations and $\alpha = 0.5$, then the $q = 20 \cdot \alpha = 10$ most proximate data points will be used for each locally weighted regression. Useful values of the smoothing parameter typically lie in the range 0.25 to 0.5 for most LOESS applications. The local polynomials fit to each subset of the data are almost always of first ($\lambda = 1$) or second degree ($\lambda = 2$). Using a zero degree polynomial turns LOESS into a weighted moving average (MA).

The first step is to define m equally-spaced locations across the range of x -values, identified as v_j , where j ranges from 1 to m . Ultimately, the LOESS curve will be evaluated at each of the v_j 's, and the fitted value at each v_j will be given by $\hat{g}(v_j)$. Generally, if the dataset contains n data points, then $m = n + 1$, i.e., the number of equally-spaced locations will be one more than the total number of data points. For each v_j , it is necessary to determine the q closest points, $d_i(v_j) = |x_i - v_j|$. Once the q -nearest observations have been determined, each distance is rescaled as a proportion of the maximum of the q -nearest distances, $d_i^*(v_j) = d_i(v_j)/d_q(v_j)$, where $d_1^*(v_j)$ represents the nearest data point, and $d_q^*(v_j)$ the furthest. We calculate neighborhood weights for all observations, from $i = 1$ to n . Once the re-scaled distances have been calculated, the weight for observation i is defined using the tri-cube weight function, given by:

$$W(v_j) = \begin{cases} (1 - |d_i^*(v_j)|)^3 & 0 \leq d_i^*(v_j) < 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Because distances are rescaled in terms of the most distant of the q -nearest observations, observations beyond the q^{th} -nearest will be given 0 weight when fitting the locally weighted regression coefficients. For each location v_j , we finally deter-

mine the local weighted regression coefficients using the normal equations adapted to attribute greater weight to more proximate observations.

In matrix form, the regression parameters are given by

$$\hat{\beta} = (X^T W X)^{-1} X^T W y, \quad (2)$$

where

- X is the $n \times (\lambda + 1)$ design matrix, with λ representing the degree of the regression polynomial;
- W is the $n \times n$ diagonal weight matrix;
- y is the $n \times 1$ response vector;
- $\hat{\beta}$ is the $(\lambda + 1) \times 1$ parameter vector estimated for each location.

The first column of X will contain all 1's, representing the intercept term.

$$\text{When } \lambda = 1 \text{ we have: } X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \text{ and } \hat{\beta} = [\beta_0 \ \beta_1].$$

$$\text{When } \lambda = 2 \text{ we have: } X = \begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 \end{pmatrix} \text{ and } \hat{\beta} = [\beta_0 \ \beta_1 \ \beta_2].$$

HW is a method that allows us to take into account trend and seasonality in the data (see [4] and [9]). It comprises the forecast equation and three smoothing equations, one for the level e , one for the trend T , and one for the seasonal component S . Here we provide an additive type model which takes into account both the trend and the seasonality (it is possible to consider also a multiplicative type model). The forecast model for the period $i + \theta$ (with $\theta = 1, 2, 3 \dots$) is:

$$\hat{Y}_{i+\theta} = \hat{e}_i + \hat{T}_i \theta + \hat{S}_{i+\theta-s} \quad (3)$$

where \hat{S} represents the estimate of the seasonality, \hat{e} the estimate of the level and s is the seasonality period. The exponential smoothing models to be used to estimate the average level \hat{e}_i , trend \hat{T}_i and seasonality \hat{S}_i at time i are:

$$\hat{e}_i = \delta_1 (Y_i - \hat{S}_{i-s}) + (1 - \delta_1) (\hat{e}_{i-1} + \hat{T}_{i-1}) \quad (4)$$

$$\hat{T}_i = \delta_2 (\hat{e}_i - \hat{e}_{i-1}) + (1 - \delta_2) \hat{T}_{i-1} \quad (5)$$

$$\hat{S}_i = \delta_3 (Y_i - \hat{e}_i) + (1 - \delta_3) \hat{S}_{i-s} \quad (6)$$

where δ_1 , δ_2 e δ_3 are the corresponding smoothing parameters respectively for the level e_i , the trend T_i and the seasonality S_i .

The three smoothing parameters δ_1 , δ_2 e δ_3 lie between 0 and 1 and their optimal values δ_1^* , δ_2^* and δ_3^* can be found by minimizing the sum of the squares of the prediction errors.

4 An optimized Material Requirements Planning (MRP)

In the second part of the project we provided optimization models replicating the *Material Requirements Planning* (MRP) used by the Company, with the goal to evaluate the MRP embedded in the information system used by the Company and to find better solutions in terms of inventory levels.

The MRP is a requirements planning tool used when a final product is produced by assembling material raws and components (each referred to as *Stock Keeping Unit* - SKU) in successive steps, on the basis of a given *Bill of Materials* (BOM). Starting from the demand forecast of the final product, the MRP allows to the managers to decide when to buy or to produce each SKU and in which quantity. The aim is to buy or to produce each SKU as late as possible, without having stock-out.

We built a user-friendly application in Excel, by using the programming language *Visual Basic for Applications* (VBA). This application, given the data provided by the information system of the Company, is able to compute all advanced data needed for the model, to run an optimization solver (*Solver studio* [11] in our case) and to import the optimal solution in Excel (see Figure 1). The availability of the optimal solution in Excel is useful, for example, to compare the strategy used before by the Company, based on looking on the historical data of the same week of the previous years, with the optimal solution.

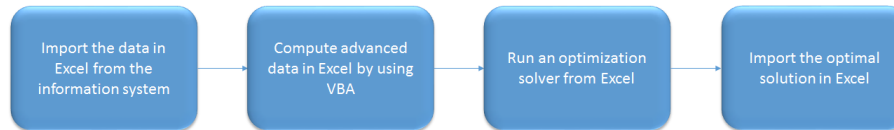


Fig. 1 An optimized MRP

The first step was to define the set of data, available in the information system of the Company, to use in the model. The identified the following data:

1. P : Number of SKU (SKU 1 is the final product)
2. T : Number of time buckets (i.e. the planning horizon)
3. D_t : Demand of the final product at time t
4. C_t : Planned delivery of the final product at time t (from previous planning)
5. S_t : Quantity of the final product to produce at time t from previous planning
6. π_t : Equal to 1 if the production of the final product is feasible at time t and 0 otherwise
7. LS_i : Lot size of the SKU i
8. \mathcal{F} : Set of SKU i for which the quantity produced or purchased is a multiple of the lot size LS_i
9. \mathcal{S} : Set of SKU i for which the quantity produced or purchased is greater or equal than the lot size LS_i
10. LT_i : Lead time of the SKU i

11. \bar{I}_i : Starting inventory level of the SKU i
12. R_{ij} : Number of units of SKU i to produce 1 unit of SKU j (from the *Bill of Materials* - BOM)
13. O_{it} : Number of units of SKU i made available at time t thanks to previously placed orders
14. IA_{it} : Number of units of SKU i used at time t to produce other final products
15. FA_{it} : Number of units of SKU i scheduled at time t to produce other final products
16. F_i : Minimal number of units of SKU i to produce whenever the SKU i is produced
17. M : sufficiently large number.

Concerning the demand of the final product D_t , note that this can be obtained by applying a forecasting technique or it can be the quantity of the final product to produce at time bucket t in the *Master Production Schedule* (MPS). In the latter case, this quantity can be obtained by optimally solving the classical EOQ model (see [3]), in case of constant demand, or the classical Wagner and Whitin model (see [8]), in case of time varying demand. In any case, the aim is to find the best balance between set-up and inventory costs. We refer to [5] for an overview of multiperiod planning in production management.

Concerning the lot sizes, note that the introduction of the two sets \mathcal{F} and \mathcal{S} allows us to handle, as particular cases, both the case in which all products must be ordered by multiple of the lot size and the case in which all products must be ordered for a quantity at least equal to the lot size. Each SKU belongs either to \mathcal{F} or \mathcal{S} depending on the agreements of the Company with the suppliers.

The second step was to formulate an optimization model replicating the MRP. We first introduced the following decision variables:

1. x_{it} : non-negative continuous variable representing the number of units of SKU i produced or purchased at time t
2. \hat{x}_{it} : non-negative continuous variable representing the net number of units of SKU i produced or purchased at time t (for the final product this is equal to $x_{it} - S_t$, while it is equal to x_{it} for the other SKUs)
3. I_{it} : non-negative continuous variable representing the inventory level of SKU i at the end of time bucket t
4. z_{it} : non-negative integer variable representing the number of lot sizes of SKU $i \in \mathcal{F}$ produced or purchased a time t
5. y_{it} : binary variable equal to 1 if $\hat{x}_{it} > 0$ and to 0 otherwise, used to guarantee that $\hat{x}_{it} \geq LS_i$ for SKU $i \in \mathcal{S}$ whenever i is produced or purchased
6. \hat{y}_{it} : binary variable equal to 1 if SKU $i = 2, 3, \dots, P$ is used at time t and to 0 otherwise, used to guarantee that the minimal production quantity F_i is produced whenever i is used.

Then, we formulated the following mixed integer linear programming model:

$$\min \sum_{i=1}^P \sum_{t=1}^T (T-t+1)x_{it} \quad (7)$$

$$I_{1t} = \bar{I}_1 + \sum_{\tau=1}^{t-LT(1)} x_{1\tau} - \sum_{\tau=1}^t (D_{\tau} + C_{\tau}) \quad t = 1, 2, \dots, T \quad (8)$$

$$I_{it} = \bar{I}_i + \sum_{\tau=1}^{t-LT(i)} x_{i\tau} - \sum_{\tau=1}^t \left(\sum_{j=1}^P R_{ij} x_{j\tau} + F_i \hat{y}_{it} + IA_{i\tau} + FA_{i\tau} \right) \quad i = 2, 3, \dots, P \quad t = 1, 2, \dots, T \quad (9)$$

$$x_{1t} = \hat{x}_{1t} + S_t \quad t = 1, 2, \dots, T \quad (10)$$

$$x_{1t} \leq M\pi_t \quad t = 1, 2, \dots, T \quad (11)$$

$$x_{it} = \hat{x}_{it} \quad i = 2, 3, \dots, P \quad t = 1, 2, \dots, T \quad (12)$$

$$\sum_{j=1}^P R_{ij} x_{jt} \leq M\hat{y}_{it} \quad i = 2, 3, \dots, P \quad t = 1, 2, \dots, T \quad (13)$$

$$\hat{x}_{it} = LS_i z_{it} \quad i \in \mathcal{F} \quad t = 1, 2, \dots, T \quad (14)$$

$$\hat{x}_{it} \geq LS_i y_{it} \quad i \in \mathcal{S} \quad t = 1, 2, \dots, T \quad (15)$$

$$\hat{x}_{it} \leq M y_{it} \quad i = 1, 2, \dots, P \quad t = 1, 2, \dots, T \quad (16)$$

$$x_{it} \geq 0 \quad i = 1, 2, \dots, P \quad t = 1, 2, \dots, T \quad (17)$$

$$\hat{x}_{it} \geq 0 \quad i = 1, 2, \dots, P \quad t = 1, 2, \dots, T \quad (18)$$

$$I_{it} \geq 0 \quad i = 1, 2, \dots, P \quad t = 1, 2, \dots, T \quad (19)$$

$$y_{it} \in \{0, 1\} \quad i = 1, 2, \dots, P \quad t = 1, 2, \dots, T \quad (20)$$

$$\hat{y}_{it} \in \{0, 1\} \quad i = 2, \dots, P \quad t = 1, 2, \dots, T \quad (21)$$

$$z_{it} \geq 0 \text{ integer} \quad i = 1, 2, \dots, P \quad t = 1, 2, \dots, T \quad (22)$$

The objective function (7) penalizes the production or the purchase of SKUs performed before the time bucket in which these quantities are needed (delivery for SKU 1 and next steps in production for the other SKUs). Constraints (8) compute the inventory level of the final product at the end of each time bucket as the sum of the starting inventory level (I_1), plus the total quantity produced up to time $t - LT_1$ (and therefore available at time t), minus the sum of the cumulative demand and of the planned deliveries up to time t . Constraints (9) compute the inventory level of each other SKU i at the end of each time bucket, as the sum of the starting inventory level I_i , plus the total quantity produced up to time $t - LT_i$, minus the cumulative demand, given by the sum of the number of units needed to produce other SKUs, the minimum quantity to produce whenever the SKU i is used, the number of units of SKU i used at time bucket t to produce other final products and the number of units of SKU i scheduled at time t to produce other final products. Constraints (10) compute the quantity of the final product produced at each time bucket t as the sum of the net quantity and of the quantity to produce at time t from previous planning. Constraints (11) guarantee that the production of the final product can be performed only when it is allowed. Constraints (12) define the quantity of each SKU different than the final product equal to the net quantity. Constraints (13) guarantee that if

a SKU is used at time bucket t , then the minimal number of units of the SKU is produced. Constraints (14)–(16) guarantee that, for each time bucket at which a SKU is produced or purchased, the produced or purchased quantity at each time bucket is a multiple of the lot size for the SKUs in the set \mathcal{F} and at least equal to the corresponding lot size for the SKUs in the set \mathcal{S} . Constraints (17)–(22) define the decision variables of the problem. Note that Constraints (19) also guarantee that no stock-out can occur for each SKU.

This model is a new variant of the one introduced in [6]. It was implemented by using the modeling language AMPL in the Solver Studio framework embedded in Excel ([11]). We also formulated a second variant of the model able to guarantee that the inventory level of each SKU at the end of each time bucket is enough to satisfy the demand of the SKU for a given number of time buckets.

5 Computational Results

In this section we provide, as an example, some of the computational results obtained, by applying the suggested forecasting methods and optimization models to the instances provided by the Company.

5.1 Forecasting Methods

In this section we describe how to predict sales data for the 200 different products produced by the Company (including third-party brands). We have at our disposal weekly/bi-weekly selling data starting from January 2013.

The analysis of each product was splitted into two parts: We first analyzed the historical time series by the models described in Section 3 (LOESS and HW) with the aim to identify the best method taking into account the seasonality and the presence of trend. The second part focused on the prediction of each product by choosing the best method between LOESS and HW based on the lowest mean squared error of the forecasts obtained by removing the last 26 observations with the 26 observations themselves.

The analysis has been performed in R [10], a free software environment for statistical computing and graphics. The tool takes as input the historical series of all the products and creates as output a worksheet containing for each product the predictive method used (i.e. LOESS or HW) and the demand forecast for the next six future observations. The tool can be used autonomously by the Company, updating the input file week after week with the new observations and recalculating the future forecasts. Notice that the instrument will become more and more reliable over time handling longer historical series.

Let us now look in detail at the analysis carried out for each of the products. For the sake of brevity, we will limit the analysis to a particular type of yogurt identified

as 3073 FPLUS BIANCO 600G CLV. The same methodology has been applied for all the remaining products.

Figure 2 show the decomposition of the series of yogurt 3073 FPLUS BIANCO 600G CLV with the LOESS model.

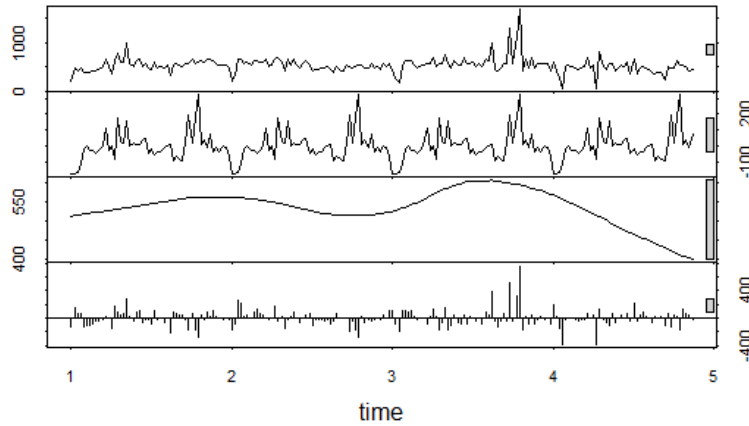


Fig. 2 LOESS decomposition for the product FPLUS BIANCO 600G CLV

The first row shows the historical series of the last five years, the second the seasonality, the third the trend decreasing in the last period and in the fourth row the residuals.

On the other hand, Figure 3 shows a comparison between the original historical series (in black) with the Holt-Winters model (in gray) over the last five years.

In order to identify which one of the two considered models best describes the trend of the series, we have computed the mean square error (MSE) and analyzed the Q-Qnormal Plot, which is constructed with the quantiles of the empirical distribution of the residues and the quantiles of the standard normal distribution.

From the analysis of the Q-Qnormal Plot the best method seems to be LOESS, even if the choice between the two methods turns out to be difficult because, in both cases, the graphical representation of the cumulative distribution of the observed variable seems to follow the cumulative distribution of the normal distribution. In order to determine the best method to describe the series, we therefore calculated the respective mean square errors (MSE): $MSE_{LOESS} = 14848.43$, $MSE_{HW} = 30504.33$. Since the mean square error of the LOESS method is the smallest one, we can therefore conclude that LOESS is the method that best describes the trend of the series of the product analyzed.

We finally decided which of the LOESS and HW methods is the best for a prediction. Given the historical series in which we remove the last 26 observations, we

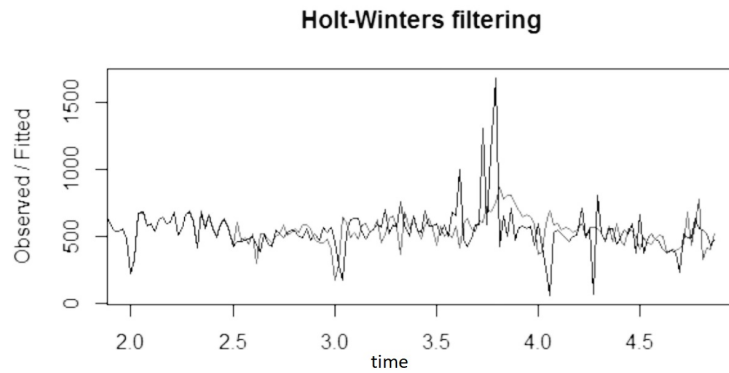


Fig. 3 HW modelling for the product FPLUS BIANCO 600G CLV

generated for these 26 observations a prediction with LOESS and HW and finally we chose the best method for a forecast between the two, by calculating their mean square errors (MSE).

Figure 4 refers to the LOESS prediction: In black the original series is represented, while in black thick line the LOESS forecast up to 2018. The overlapped part is the part we compared (26 observations) and the LOESS mean square error is $MSE = 13898.83$, while the one of HW prediction is $MSE = 59863.83$.

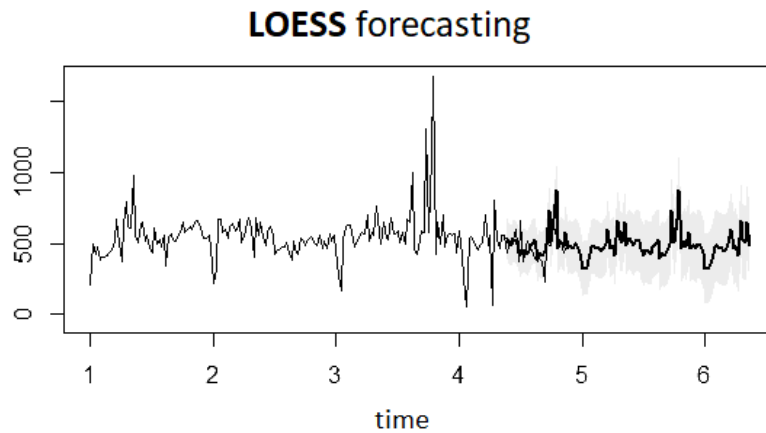


Fig. 4 LOESS forecasting: in black the original series while in black thick line the LOESS forecast up to 2018.

We can therefore conclude that LOESS turns out to be the best forecasting methods for FPLUS BIANCO 600G CLT.

The application of this methodology to all the 200 products has allowed to the Company to reduce the average percentage error on a 6 weeks production schedule from 29.71% to only 7.94%.

5.2 An optimized Material Requirements Planning (MRP)

We now provide an example of the application of the MRP model formulated in Section 4 to a final product (a UHT milk) produced by the Company, having the following data: Number of SKU P : 6 (final product: SKU 1, components: SKU 2, SKU 3, SKU 4, SKU 5, SKU 6); number of time buckets T : 42 days. Figure 5 shows the Bill of Materials (BOM) of this final product: One unit of the final product SKU 1 is obtained by assembling SKU 2, SKU 3, SKU 4, SKU 5, SKU 6 in the quantities R_{ij} reported on the BOM.

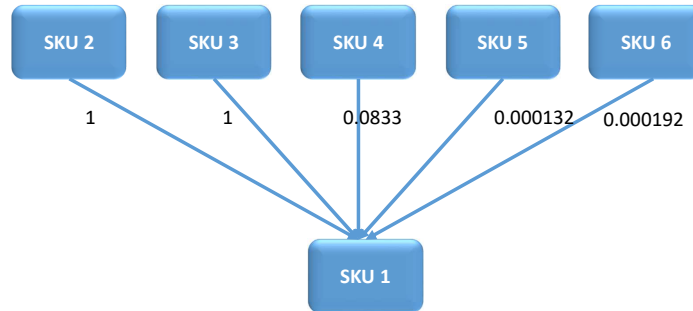


Fig. 5 Bill of Materials (BOM)

The demand of the final product over 6 weeks (from July, 11 to August, 21 in 2016) is the following (the demand is 0 in the days not included in the table):

Day t	1	8	15	22	29	36
Demand D_t	15276	68364	84900	17820	17820	17820

The MRP requires, for each SKU, the following data:

SKU i	Lead time LT_i	Lot size LS_i	Lot size set	Starting inventory level \bar{I}_i
1	0	1000	\mathcal{F} (multiple)	153475.00
2	28	1440000	\mathcal{S} (minimum)	1024928.00
3	28	16200	\mathcal{F} (multiple)	611547.00
4	28	10000	\mathcal{S} (minimum)	9236.00
5	21	675	\mathcal{S} (minimum)	636.33
6	28	384	\mathcal{S} (minimum)	561.61

The application of the model formulated in Section 4 to this case showed that, if the production and purchase quantities applied by the Company are used in the optimization model, the optimal requirements planning is equal to the one provided by the MRP embedded in the information system of the Company. This states that our application is equivalent to the MRP embedded in the information system, when the same solution is adopted. Moreover, when an optimal solution is computed, a significant reduction in the inventory levels are obtained, as shown in Figure 6 for the final product, where we compare the inventory levels in the solution used by the Company and in the optimal solution.

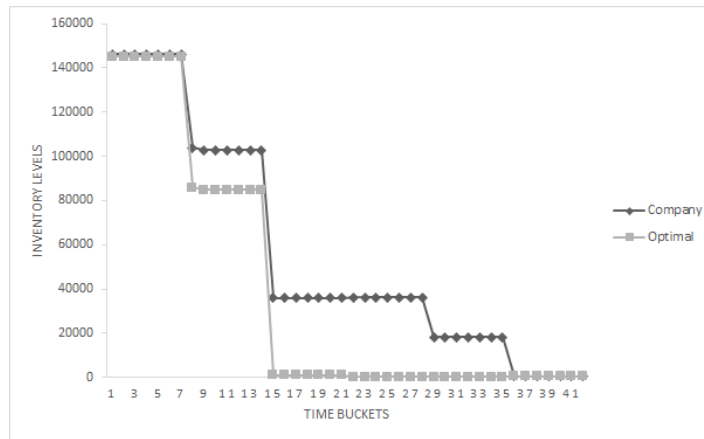


Fig. 6 Inventory levels of the final product

We applied this methodology to the instances provided by the Company, related to the production of two types of final products in 2016. The results showed that the inventory levels can be improved by adopting an optimal solution, similarly to what happened in the example shown before. Better results could be obtained by defining the lot sizes as decision variables.

6 Conclusions

Operations Research was recently defined as “The Science of Better”. This project provides an example of successful application of statistical and Operations Research techniques to a complex real case. However, this project is just a first step. Further efforts could be devoted, for example, in embedding demand forecasts in the MRP optimization models, maybe formulating *Stochastic Programming* models able to compute optimal requirements planning given a set of scenarios of demands.

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