Traders, courts, and the home bias puzzle

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Abstract

Recent evidence shows that the “home bias puzzle” in international trade may be associated with the mere presence of national borders (McCallum (1995)). In this paper we provide a theoretical framework to explain why borders may matter so much for trade. Our argument is that even between perfectly integrated and similar countries the legal system differs, so that legal costs are higher when business is done abroad. Using a matching model of trade, we show that the home bias is associated with both less searching foreign sellers in the home market and a lower probability of cross-border matches being accepted. In industries characterized by high turnover legal costs may reduce trade because reducing the mass of searching foreign sellers and increasing at the same time that of searching domestic sellers.

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1 Introduction

The “home bias puzzle” is one of the most intriguing unsettled issues in international trade. The traditional view is that national borders matter for trade because their existence is associated with discriminatory policies and physical distance. This view, rather common in theory, has been challenged in applied work. Trade costs alone cannot explain fully the extent to which domestically produced tradable goods are over-represented in the consumption of residents. This mismatch has been often solved in applied equilibrium analysis resorting to “Armington preferences”, i.e., by assuming that, for some reason, individuals’ residents are biased towards domestically produced goods.\(^1\) This explanation is probably relevant when we consider trade flows between countries that differ considerably in income per-capita, geography, or history, less when we consider similar countries. In some cases, relying to a bias in tastes is simply an ad-hoc short-cut.

The dissatisfaction with a taste-based explanation for the observed home-bias in consumption finds strong support in recent evidence.\(^2\) McCallum (1995), estimating the volume of trade through a gravity equation across US States and Canada provinces, found that the presence of the national border reduces trade by a factor of twenty. This result is quite surprising. The mere fact that the counterpart is located across the border reduces dramatically the volume of trade even between countries like the US and Canada, that have almost completely liberalized trade and that are quite homogenous culturally. These findings stimulated debate and further research. Subsequent work confirmed that the extent of the ”border effect” is substantial, even though probably not striking as found by McCallum (1995).\(^3\) Overall, consensus is shaping around the idea that much of the observed home bias is simply related to the presence of borders. But then, why do national borders matter so much?

Obstfeld and Rogoff (2000) mention the (border related) home bias in trade as one of the six major puzzles in international economics. They emphasize that what matters to explain this puzzle is not the level of trade costs per-se, but their interaction with the elasticity of substitution between

\(^1\)There are several plausible reasons that may explain a bias in tastes: habit, culture, or consumption externalities.

\(^2\)Trellet (1995), analysing empirically trade volumes for 33 countries in a Hekescher-Ohlin-Vanek framework finds that differences in factor endowment can predict only a fraction of actual trade even allowing for differences in production technologies and tastes biased in favor of domestic goods.

\(^3\)Helliwell (1996) analyses trade between US States and Canada provinces using 1993-1996 data, i.e., relative to the post-Nafta period. In this study, national borders reduce trade by a factor of twelve. As for other countries whose trade data are not available at a subnational level, Wei (1996) and Evans (2000) examine OECD trade, while Head and Mayer (2000), Nitsch (2000), and Chen (2000) look at trade flows in the EU. All these studies provide (indirect) evidence that national borders reduce trade substantially, but less compared with the results obtained by McCallum (1996) and Helliwell (1996).
domestic and foreign goods. They claim that plausible values for trade costs and substitution elasticities can account for much of the observed home-bias. This argument, however, still leaves unexplained why the mere presence of a national border can dramatically reduce trade. In some work the home bias puzzle is approached as a measurement error issue. Anderson and van Wincoop (2000) analyze border effects between Canada and the US by implementing a theoretically founded gravity equation, taking into account relative distance across countries with appropriate functional forms. Their results show that the surprisingly high border effect found in McCallum (1995) can be explained somehow by omitted variables and the small size of the Canadian economy. Also in Anderson and van Wincoop (2000), however, it is found that borders reduce trade across national boundaries by substantial magnitudes. Head and Mayer (2001), analyzing trade among states in the US and among EU countries, also manage to shrink somehow the border effect adopting a micro-founded measure of intra-state trade, but not to eliminate it. Other papers relate border effects to regulatory asymmetries across countries and technical barriers to trade. However, results for the EU indicate that these factors do not seem to be a major source of border effects (Head and Mayer (2000), Brenton and Vancauteren (2001)). Some recently proposed explanations for the home bias do not rely on trade barriers or biased tastes. A series of papers emphasize informational barriers to trade and the developments of commercial networks—often with an ethnic connotation—to reduce search costs in international trade (see Rauch (2001) for a survey). Other work focuses on contract incompleteness and imperfect enforcement. As argued, for instance, in Anderson and Marcouiller (1999) the rule of law is weaker when trade is international, compared with domestic trade. The probability of expropriation, bribery, theft, is higher when transactions occur across the border and contracts are more hardly enforceable in a transnational setting. All this adds costs to international trade. Anderson and Marcouiller (1999) provide empirical support to their argument showing that, ceteris paribus, trade is significantly lower for countries with weak institutions and widespread corruption. This argument, however, leaves unexplained why the presence of national borders can cause a so strong reduction in trade flows even between countries with solid institutions (e.g., between the US and Canada)

In this paper, we develop a model that may help to understand how the mere presence of borders may lead to home bias. Our argument is that national borders matter for international trade because they draw the frontier between different legal systems. This view is not new. Rodrik (2000, page 179), for instance, argues that “...national borders demarcate political

\[ 4 \] For instance, using CES preferences, a value of (iceberg) trade costs of 25% and one of the substitution elasticity of 6 one obtains that the share of expenditure in foreign goods is about 40%.
legal jurisdictions. Such demarcations serve to segment markets in much the way that transport costs or border taxes do”. The main message from our analysis is that differences in jurisdictions associated with national borders may lead to home bias, though in a quite different way compared with tariffs or transport costs. Our point of departure is that, as observed in Rauch (1996, 1999), the international exchange of manufactures does not occur in organized markets like those of basic commodities. Manufactures differ too much in their quality and characteristics for quoted prices to reveal all the information required by traders to finalize their operations. Hence, the connection between sellers and buyers is often the result of a lengthy search process. Since this process is costly, successful matches enjoy a rent and tend to be long lasting. Conversely, unsuccessful matches will be rejected by the parties. This is especially true when trading occurs in sophisticated goods such as machines or capital equipments. In this context, the terms of exchange can only be fixed ex-post, after the realization of a satisfactory match. Thus, the price at which trade occurs reflects the relative bargaining power of buyers and sellers, which is shaped by their outside options. Parties always have the option not to fulfill their obligations. However, by doing that, they will be confronted with legal sanctions. It is easy to understand why borders do matter in this context. When a transaction occurs across the border, it involves different jurisdictions, and the legal costs in case of trial are higher. This may result into a shift of bargaining power in favor of the party that can gain from opportunistic behavior, into a lower incentive to search for business partners abroad, and then into a reduction of cross-border trade flows. It is to note that this change in bargaining power would not realize in case of trade costs associated only with transport costs or border taxes. In our model, it is the possibility of reneging on international contracts that shape the outside options available to traders and that are responsible for the home bias in trade flows.

A basic assumption of our model is that commercial disputes occurring across the border are more costly than disputes occurring within the borders, since they involve heterogenous legal systems.\(^5\) Available evidence for the US shows that litigations initiated by foreigners have a higher probability of success (Clermont and Eisenberg (1996)). This regularity is consistent with the hypothesis that the legal costs to solve commercial disputes are higher for transactions occurring across the border. International law-suits will in fact be pursued more seldom, only when the probability of success is high enough. Whenever these extra costs for dealing with across-the-border disputes are not shared equally between buyers and sellers there will

\(^5\)An alternative is that of international arbitration occurring in a third country (e.g., at the International Chamber of Commerce in Paris). Evidence shows that international arbitration is quite costly, and used only in case of large transactions (see, e.g., Casella (1992, 1996)).
also be a shift of bargaining power between the parties, since one party will have a bigger gain by not respecting the terms of the agreement. We consider the case in which it is the seller (the exporter) that bears the biggest share of this extra cost. There are several reasons that induce to think that this is the most likely case. It happens that in international transactions the delivery of goods comes before their effective payment. It follows that it is the buyer the one that can gain from opportunistic behavior. Even when documentary credit is used, the seller still risk that the buyer claims that the occurrence of some contingency has altered the agreed terms of exchange (e.g., deterioration of quality).\textsuperscript{6} This asymmetric position of sellers and buyers in international trade is reflected in public policies aimed at facilitating international transactions. While it is quite common the public support to export credit and insurance, similar practices targeted to importers are very seldom used.

We build a model of international trade where matching between (ex-ante) heterogenous buyers and sellers occurs randomly. The mechanics of the model are similar to those commonly used to analyze equilibrium unemployment (e.g., Marimon and Zilibotti (1999)). A matching function summarizes the number of random matches realized per unit of time between searching buyers and sellers. Sellers are characterized by the variety of the supplied good, and buyers by the variety they would ideally buy. Matches may therefore be “good” or “bad”, depending on whether they occur between “close” or “distant” parties along the product variety dimension. Since search is costly, only sufficiently satisfactory matches will result in a business relation. Buyers may be matched with domestic or foreign sellers. Exporters have to pay higher legal costs compared with domestic sellers to sue a buyer that behaves opportunistically, refusing to pay the due price. We show that this asymmetry in legal costs translates into a loss of bargaining power for sellers doing business abroad, and into the emergence of home bias. The home bias shows up in both a lower probability for each buyer to be matched with a foreign seller and in a higher probability for a cross-border match to be rejected. Asymmetries in legal costs have a direct negative effect on the entry of foreign sellers and their conditions for accepting matches with buyers, but also an indirect, ambiguous effect on the behavior of domestic sellers, through the bargaining power of buyers. Comparative statics performed numerically show that legal costs will reduce international trade especially in industries characterized by high turnover and where existing business relations are easily destroyed and replaced by new ones. In such industries, as trial costs rise, the home bias may increase disproportionately because of a simultaneous lower entry of foreign sellers and higher entry of domestic sellers.

\textsuperscript{6}See, e.g., del Busto (1994) for a description of documentary credit.
The remainder of the paper is organized as follows. In the next section we develop the model. In section 3 we characterize equilibrium and qualify the emergence of home bias. Section 4 concludes.

2  The Model

We consider a world populated by (ex-ante) heterogenous buyers and sellers. Sellers supply differentiated goods, and buyers are heterogenous in terms of the good variety they like most. Sellers may either be domestic or foreign. Buyers and sellers are matched randomly. Some matches are lucky, and generate a high surplus, others are unlucky. Hence, whenever a match occur, the seller has to decide whether to start business with the matched buyer or to wait for a better match. When a buyer and a seller start a business relation, they agree to exchange at each time one unit of the good, until their relation is randomly destroyed. International trade is free. Moreover, we assume away transport costs or any other cost related to physical distance. Nonetheless, doing business with domestic agents or foreigners matters in our model. The reason is that we consider countries that are characterized by different legal systems. Due to such differences, doing business abroad entails higher legal costs in case of default of one of the parties.

2.1 The Economy

We consider a world with two countries (regions), each populated by a unit continuum of sellers’ and buyers’ types. The two economies are identical in all respects, so that we can concentrate the analysis on one of them only. There are two goods: one differentiated good and one homogenous commodity, that we call henceforth the numeraire. Buyers derive utility both from the differentiated good and the numeraire, while sellers only like the numeraire. Utility of both buyers and sellers is additive in the two goods and linear in the numeraire. At each instant of time, if matched with a partner, each buyer consumes one unit of the differentiated good and each seller sells one unit. Sellers may either sell to home buyers or to foreign buyers. All agents are infinitely-lived and discount the future at rate $r$.

Each seller is characterized by the variety of the good she supplies; each buyer is characterized by the variety of the good which is most preferred to him. Sellers and buyers are uniformly distributed over a unit length circle. At each point on the circle, there is a unit mass of buyers, while the mass of domestic and foreign sellers is determined endogenously. The larger the distance between the ideal variety of a buyer and the variety which is actually purchased, the lower her instant utility. The distance is measured by the length of the shortest arc between the location of the ideal variety and the one which is bought. Let $i, j$ be the length of the arc between a seller’s type $i$ and a buyer’s type $j$. Then, since agents are distinguished by one particular
variety located around the circle, we have that $\tilde{i}, \tilde{j} \in [0, 1/2]$. We define by $\rho : [0, 1/2] \rightarrow [\rho, \overline{\rho}] \subset R^+$ the function mapping the distance between a buyer from her seller into the buyer’s instant utility. This function is such that $\rho(0) = \underline{\rho}$ and $\rho(1/2) = \overline{\rho}$, where $0 < \underline{\rho} < \overline{\rho} < \infty$, and that $\rho' < 0$. Hence, we call $\rho(\tilde{i}, \tilde{j})$ the instant utility function.

In the economy, there is only one production factor: labor. The numeraire is produced 1:1 out of labor. The differentiated manufacture is produced under constant returns to scale with marginal costs equal to $c$ for all varieties. Henceforth, we will assume that some varieties, but not necessarily all, can potentially be sold with a positive profit to a given buyer, i.e., that $\overline{\rho} > c$. Each agent is endowed with a sufficient amount of (indivisible) labor to allow for the purchase of any variety and exclude negative consumption.\(^7\)

In each country, sellers of each type may either be domestic or foreign. The variables referred to, respectively, the home and the foreign country are labelled with superscripts $H$ and $F$. We also denote by $H$ and $F$ the set of domestic and foreign sellers.

At each period in time, some sellers and buyers are randomly matched and some existing matches are randomly destroyed. The rate at which matching occur is not affected by the location of agents on the circle where goods’ varieties are represented.

Sellers choose at each instant about entry. As soon as sellers enter in one location, they have to search for a buyer. Their search costs are represented by a flow of $\gamma$ units of the numeraire per unit of time. A seller, after being matched with a buyer, has to decide whether to accept entering a business relation with the buyer —maintaining this business relation until the match is destroyed— or to wait for a better match. If the seller accepts the match, she posts a price to the buyer.\(^8\) Then, it is the buyer that has to accept or refuse the deal. If the buyer agrees, a business relation is started, and the good is delivered. At this point, the buyer has to decide about her business conduct. Buyers may either be compliant (“honest”) or non-compliant (“dishonest”). A honest buyer pays for the delivered good, while a dishonest buyer refuses to pay. Whenever a buyer refuses payment to the seller, the business relation is terminated at a higher rate ($1$ instead of $d$, $d < 1$). The behavior of the buyer is verifiable by a court, which imposes the due payment to the sued seller in case of dishonest conduct. After dishonest behavior by the buyer, either the parties reach a pre-trial agreement through which the buyer directly

\(^{7}\)As will be clear in the following analysis, this is insured when the labor endowment of each agent is greater than $\max[\gamma, \overline{\rho}]$, where $\gamma$ denotes the instant cost paid by sellers when searching for a buyer.

\(^{8}\)The assumption that sellers make take-it-or-leave-it offers to buyers simplifies the analysis but is not crucial. The main qualitative conclusions are obtained allowing parties to share the surplus from the match according to some given rule.
compensates the seller, or the seller sues the buyer to the court. We assume that trial costs are higher in case of an international lawsuit. Without loss of generality, the legal costs for a domestic lawsuit are assumed to be zero, while those that a seller has to pay for in case of an international lawsuit are a flow equal to $2\Delta$. Figure 1 describes the sequence of actions.

2.2 Matching

Buyers and sellers are randomly matched and separated at each instant of time. Hence, in each moment, both sellers and buyers are either matched or searching. The mass of instantaneous matches between type-$j$ buyers and type-$i$ sellers is an increasing function of the mass of searching type-$j$ buyers and type-$i$ sellers. More formally, let $b(j)$ and $s(i)$ be, respectively, the mass of searching buyers of type $j$ and sellers of type $i$, the matching function $m[s(i), b(j)]$, $m : \mathbb{R}_+ \times [0, 1] \to \mathbb{R}_+$, specifies the mass of instant matches between buyers of type $j$ and sellers of type $i$. We assume $m[s(i), b(j)]$ to be increasing in both its arguments, to exhibit constant return to scale, and to respect Inada conditions. Let $\theta(i, j) \equiv s(i)/b(j)$ and $q[\theta(i, j)] \equiv \frac{m[s(i), b(j)]}{s(i)}$. Then, $q[\theta(i, j)]$ represents average flow of matches (Poisson rate) for a searching seller of type $i$ with a buyer of type $j$, while $\theta(i, j) q[\theta(i, j)]$ are the average instant matches for a buyer of type $j$ with a seller of type $i$. In case of honest behavior, buyer-seller matches are destroyed at each time period at the Poisson rate $d$ assumed to be lower than unity. If a buyer behaves opportunistically, and refuses to pay the delivered good, the match is assumed to be destroyed at a Poisson rate equal to 1.

2.2.1 Sellers

Recall that once entered, sellers have to search for a buyer. We restrict the analysis to an economy in the steady state. The steady-state value function for a searching seller supplying variety $i$ belonging to country $k$, $k = H, F$,
(denoted by $i^k$) is the sum of the instant losses ($-\gamma$) and the option value of searching, i.e., the expected gain once matched with a buyer:

$$rV(i^k) = -\gamma + \int_0^1 q[\theta(i^k, \tau)] \left\{ \max \left[ J^k(i^k, \tau), V(i^k) \right] - V(i^k) \right\} d\tau,$$

where $q[\theta(i^k, \tau)]$ is decreasing with $s(i^k)$.

Note that since there is free entry and $q[\theta(i^k, \tau)]$ is decreasing with $s(i^k)$, entry in the supply of any variety $i$ will occur until $V(i^k) = 0$.

The value function of sellers depends both on whether the match occurs domestically or across the border, and on whether the matched buyer behaves honestly (paying after delivery) or dishonestly (refusing to pay). We will limit the analysis to empirically consistent cases in which buyers always prefer to behave honestly. Appendix A.1 identifies sufficient conditions for honest behavior to occur at equilibrium. The value function of a seller supplying variety $i^k$ matched with a buyer with ideal variety $j$ will thus be given by

$$rJ(i^k, j) = p(i^k, j) - c - d[J(i^k, j) - V(i^k)], \quad k = H, F$$

(2)

where $p(i^k, j)$ is the price charged by a type-$i^k$ seller to a type-$j$ buyer, and $d$ is the separation rate. The price $p(i^k, j)$ is set unilaterally by the seller, who is in the position to make a take-it-or-leave-it offer to the buyer. Each seller has therefore to solve the problem of the buyer to set optimally $p(i^k, j)$.

### Buyers

After the price is posted, the matched buyer has to decide, in sequence, whether to start business with a matched seller and, if yes, whether to behave honestly or dishonestly. As soon as the business relation starts, the good is delivered. A honest buyer pays the price posted by the seller $p(i^k, j)$. A dishonest buyer refuses to pay. Since the behavior of the buyer is verifiable, the seller can obtain the due payment by suing the buyer. However, since the trial is costly and both parties are rational, they will always reach a pretrial agreement. We assume that the surplus from avoiding the trial is shared equally. Denote by $h(i^k, j), k = H, F$, the compensation that the parties agree the buyer should give the seller in order to avoid the trial. In case of a purely domestic match (both parties belong to the same country), we necessarily have $h(i^H, j) = p(i^H, j)$ since the trial is assumed to cost the same to each party (zero). When the match is between two parties belonging to different countries we have instead $h(i^F, j) = p(i^F, j) - \Delta$, i.e., a pre-trial settlement which is lower than the posted price. By reaching a pre-trial agreement, the buyer: $p(i^F, j) - h(i^F, j) - (p(i^F, j) - p(i^F, j)) = h(i^F, j) - c - (p(i^F, j) - c - 2\Delta)$. 

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12This is obtained equalizing the surplus from the pre-trial agreement for the buyer and the seller: $p(i^F, j) - h(i^F, j) - (p(i^F, j) - p(i^F, j)) = h(i^F, j) - c - (p(i^F, j) - c - 2\Delta)$. 

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pre-trial settlement, the seller will spend $\Delta$ instead of $2\Delta$ to obtain the due compensation, while the buyer will save $\Delta$ in terms of a lower expense to obtain the good.

Denoting by $C^h(i^k, j)$ the value function of a honest buyer of type $j$ that is matched with a seller of type $i^k$, we get

$$rC^h(i^k, j) = \rho(\hat{i}^k, j) - p(i^k, j) - d[C^h(i^k, j) - W(j)], \quad k = H, F \quad (3)$$

where $W(j)$ is the value function of buyer $j$ if searching. The welfare of a buyer that behaves opportunistically depends crucially on whether she is matched with a domestic or with a foreign seller. Denoting by $C^d(i^k, j)$ the welfare of a dishonest buyer of type $j$, we have, respectively, in the case of a domestic and a cross-border match

$$rC^d(i^H, j) = \rho(\hat{i}^H, j) - p(i^H, j) - [C^d(i^H, j) - W(j)], \quad k = H, F \quad (4)$$

$$rC^d(i^F, j) = \rho(\hat{i}^F, j) - p(i^F, j) + \Delta - [C^d(i^F, j) - W(j)]. \quad (5)$$

When a buyer matched with a home seller decides not to pay, the settlement is the price $p(i^H, j)$, but the business relation is destroyed at the rate 1. As $d < 1$, a home buyer matched with a home seller will always be honest. Things are different when the match occurs across the border, since the required compensation is lower and the instantaneous surplus for a dishonest buyer is higher. We see that a buyer matched with a foreign seller will be honest provided $C^d \leq C^h$. One checks using equations (3) and (5) that this condition is met if and only if

$$C^h(i^F, j) - W(j) \geq \frac{\Delta}{1 - d}. \quad (6)$$

Note that honest behavior requires that the rate of match destruction is higher with a dishonest buyer.

### 2.3 Pricing

We analyze now the buyers’ decision concerning which match to accept, i.e., which sellers to accept doing business with, and the pricing decisions of sellers. Recall that we assumed that the seller makes a take it or leave it offer to the buyer. Therefore, sellers will set the highest price such that buyers accept the proposed deal. Moreover, we restrict the analysis to cases in which sellers will be better off by inducing buyers to behave honestly in case of an across-the-border match.\(^\text{13}\) In case of, respectively, a domestic

\(^{13}\)Alternative equilibria may exhibit dishonest behavior by some or all of the buyers. These possible equilibria will not be considered in the following analysis.
and a cross-border match we have

\[ C^h(i^H, j) - W(j) = 0, \]

\[ C^h(i^F, j) - W(j) = \frac{\Delta}{1 - d}. \]

Buyers matched with foreign sellers are in the position to extract a positive surplus from the match, in spite of the fact that sellers make take-it-or-leave-it offers. This is required to induce buyers to behave honestly. The rent appropriated by the buyer increases with \( \Delta \) because the higher is \( \Delta \) the lower is the compensation in favor of the seller agreed in a pre-trial settlement in case of dishonest behavior. The rent to the buyer also increases with \( d \): the higher is \( d \) the lower is the loss to the buyer in terms of an increased destruction rate in case of dishonest behavior. We see then why national borders matter in our model. When a transaction is carried out across the border, the bargaining power is shifted towards the party that can benefit from opportunistic behavior: buyers. Note that there is a crucial difference here with respect to other types of trade costs. If we had assumed, for instance, higher marginal costs for doing business abroad, we wouldn’t have obtained such shift of bargaining power, with different results. In the present set-up, transport costs or border taxes would simply increase marginal costs and reduce the rents captured by sellers from their business relations, without affecting the way rents are shared between buyers and sellers.

Since we keep perfect symmetry in the model, there will be an equal mass of sellers of all types. Moreover, as all buyers are matched with sellers of each type at the same rate, we can drop type indices and write \( \theta(i^F, j) = \theta^F \), \( \theta(i^H, j) = \theta^H \). Using symmetry, and noting that the acceptance rule for the seller is to define a maximum distance from a buyer \( n^k \) \((k = H, L)\) above which the asset value of the match, \( J(i^k, i^k + n^k) \), is nil, we can simplify the asset equations for searching agents. For a searching seller we have

\[ rV(i^k) = -\gamma + q(\theta^k) \int_{\tau \in M(i^k, n^k)} J(i^k, \tau) d\tau, \quad k = H, F \quad (7) \]

where \( M(i, x) \) is the set of buyers’ types whose distance from \( i \) is smaller or equal to \( x \). The asset equation for a searching buyer \( j \), \( W \), is given by the option value of being matched with a domestic or a foreign seller:

\[ rW(j) = \theta^H q(\theta^H) \int_{\tau \in M(n^H, j)} [C^h(\tau, j) - W(j)] d\tau, \]

\[ +\theta^F q(\theta^F) \int_{\tau \in M(n^F, j)} [C^h(\tau, j) - W(j)] d\tau, \quad (8) \]

where \( M(x, j) \) is the set of sellers’ types whose distance from buyer \( j \) is smaller or equal to \( x \). Note that since the expression of (8) looks identical
for all \( j \) we can drop the index \( j \) and simply write \( W \) to denote the asset value for searching buyers.

From equations (3) and (8) we obtain

\[
(r + d)(C^h(i^k, j) - W) = \rho(i^k, j) - p(i^k, j) - \theta^H q(\theta^H) \int_{\tau \in M(i^H, j)} [C^h(\tau, j) - W]\,d\tau - \theta^F q(\theta^F) \int_{\tau \in M(i^F, j)} [C^h(\tau, j) - W]\,d\tau,
\]

(9)

where \( k = H, F \) and \( M(x, j) \) denotes the set of buyers’ types whose distance from \( j \) is smaller or equal to \( x \). Since the price set by a domestic seller \( i^H \) is such that \( C^h(i^H, j) = W \), and the one set by a foreign seller is such that \( C^h(i^F, j) - W = \frac{\Delta}{1 - d} \), we have from (9) that the price charged by, respectively, a domestic seller \( i^H \) and a foreign seller \( i^F \) to a buyer at distance \( x \) are given by

\[
p^H(x) = \rho(x) - 2\pi^F \theta^F q(\theta^F) \frac{\Delta}{1 - d}, \quad (10)
\]

\[
p^F(x) = \rho(x) - [r + d + 2\pi^F \theta^F q(\theta^F)] \frac{\Delta}{1 - d}. \quad (11)
\]

The expression of prices set by both domestic (10) and foreign sellers (11) are lower than the instant utility for the buyer \( \rho(x) \). Note that, keeping constant the quality of matches, the price set by exporters is lower compared with that fixed by domestic sellers. We thus obtain a “dumping” result, that is explained by the fact that foreign sellers have to leave some rent to the buyer to induce honest behavior. However, the acceptance rule of cross-border and domestic matches is determined endogenously in the model. The average equilibrium price in cross-border business relations may therefore be on average higher if it is higher the average productivity of equilibrium cross-border transactions. It is finally to note that both in the case of matches within and across the borders an “outside option” term appears in the expression of prices. The seller will set a price that is lower the higher is the term \( \Delta/(1 - d) \) –reflecting the amount of the surplus appropriated by buyers matched with exporters– and the higher the rate at which a buyer enters a business relation with a foreign partner (given by the Poisson rate \( \theta^F q(\theta^F) \) at which matches arrive times the probability \( 2\pi^F \) of having a match which is accepted). This is easily explained. Even if setting a take-it-or-leave-it price, the seller cannot fully appropriate the instant surplus from the buyer, since for the latter there is the option value from waiting for a match with a foreign partner. It is to remark that in case markets are segmented only because of transport costs or border taxes, this outside option term would not materialize in the expression of prices. So, while
changes in legal costs $\Delta$ will in general affect the behavior of both domestic and foreign sellers, in the presence of transport costs or tariffs, only the behavior of exporters would be affected.

The candidate equilibrium in which buyers behave honestly that has been characterized so far can be an actual equilibrium only if no seller has an incentive to deviate from their price behavior. The only deviation can occur in case of cross-border matches. When the match occurs within national borders, in fact, the seller is in the position to fully extract the surplus from the matched buyer. A lower price would leave some surplus. A higher price would induce buyers to reject the deal. In case of cross-border matches, instead, sellers have the alternative option of setting a price higher than (11) that induces dishonest behavior on the part of the matched buyer. In that case, sellers would set the highest price that still make the deal acceptable for the matched buyer. Appendix A.1 shows that such deviations cannot be profitable at equilibrium when production ($c$) and legal ($\Delta$) costs are sufficiently small (condition (23), that is assumed to hold in the whole analysis). The intuition is that buyers’ rents in case of honest behavior are associated with legal costs $\Delta$ (check (6)). When such costs are sufficiently low, sellers would not gain from deviating to prices higher than (11), because this would result in a small increase in the rent from the match, too small to compensate for the increase in the destruction rate of the match.

3 Equilibrium

Henceforth, we restrict attention to cases in which cross-border trade takes place in our economy and in which there is an internal solution for the equilibrium (i.e., in which $\bar{n}^k \in (0, 1/2)$, $k = H, F$). An equilibrium with cross-border trade boils down to a strictly positive solution $(\bar{n}^H, \pi^F, \theta^F, \theta^H)$ of the following system

$$J^H(\bar{n}^H) = 0, \quad (12)$$

$$J^F(\bar{n}^F) = 0, \quad (13)$$

$$V^H = 0, \quad (14)$$

$$V^F = 0, \quad (15)$$

where $J^k(\bar{n}^k)$ is the welfare of a seller matched with a buyer at distance $\bar{n}^k$ and $V^k$ is that of a searching seller, $k = H, F$. The mass of searching buyers,
Proposition 1 For small values of legal costs $\Delta$ an equilibrium with honest behavior and cross-border trade exists and is unique.

Proof: See Appendix A.1.

The equilibrium can be characterized graphically. In Figure 2 we show, separately, in the $(\bar{n}^H, \theta^H)$ and in the $(\bar{\pi}^F, \theta^F)$ space, respectively, the equilibrium for variables relating to domestic and foreign sellers. There, the values for $\bar{n}^H$ and $\bar{\pi}^F$ are implicitly defined by equations (12) and (13) respectively, while those of $\theta^H$ and $\theta^F$ are given by (14) and (15). It is shown in Appendix A.1 that while domestic sellers’ variables depend upon $(\bar{\pi}^F, \theta^F)$, those relating to foreign sellers only are independent of $(\bar{n}^H, \theta^H)$. In fact, while the entry condition and the stopping rule for foreign sellers is not affected by variables relating to domestic sellers $(\bar{n}^H, \theta^H)$, the decisions of domestic sellers are affected by $\bar{\pi}^F$ and $\theta^F$ via the outside option available to buyers. The pricing equations (10) and (11) show that the rent appropriated by buyers increase with the Poisson rate of being matched with foreign sellers $\theta^F q(\theta^F)$ and with the distance below which such matches are accepted $\bar{\pi}^F$. Note that while legal costs $\Delta$ only have an effect on the decisions of domestic sellers through changes in the outside option of buyers, they affect the entry condition and the stopping rule of foreign sellers both directly and through the buyers’ outside option. It is also to note that no outside option effect would materialize when markets are segmented only because of the existence of transport costs or border taxes. In such a case, there would just be a greater marginal costs for sellers when supplying foreign buyers. This would affect directly the entry decisions of foreign sellers and the acceptance rules for foreign matches, but would not alter the outside option of buyers, thus having no effect on $(\bar{n}^H, \theta^H)$.

Appendix A.1 shows that whenever $\bar{\pi}^F$ or $\theta^F$ rise (fall), other things being equal, $\bar{n}^H$ and $\theta^H$ fall (rise). A higher value of $\bar{\pi}^F$ or $\theta^F$ translates into a higher rate of arrival and acceptance of cross-border matching, and then into a higher bargaining power for buyers. This explains the reduced entry rate (lower $\bar{\theta}^H$) and the more selective stopping rule (lower $\bar{n}^H$) for domestic sellers.

In which way legal costs contribute to segment cross-border trade? How do they shape the home bias? Comparative statics are quite involved in this model. While the direct effect of $\Delta$ is unambiguously negative on $\bar{n}^H, \theta^H$ and $\theta^F$, it is ambiguous on $\bar{\pi}^F$ (check Appendix A.1 and Figure 2). Moreover,

$\Delta$ Alternatively, as we do in the next section, from the steady-state conditions one may characterize the mass of matched agents in the economy.
the total effect of $\Delta$ on $\bar{n}^H$ and $\theta^H$ is hard to assess analytically, since it also depends on how the equilibrium values of $\pi^F$ and $\theta^F$ are affected. The next section, after showing that home bias always takes place for $\Delta > 0$, performs some comparative statics exercises numerically.

3.1 The Emergence of the Home Bias and its Determinants

Define by $z^H$ and $z^F$ respectively, the mass of matched home and foreign sellers. At the steady state, $z^H$ and $z^F$ must be constant. Inflows in $z^H$ must be equal to the outflows from it. The same with respect to $z^F$, so that:

$$dz^k = \theta^k q(\theta^k)2\pi^k b,$$

$$k = H, F$$

(16)

where $dz^k$ and $\theta^k q(\theta^k)2\pi^k b$ are, respectively, the number of destroyed and that of created matches per unit of time. Hence, the trade share at the steady state is defined as

$$\frac{z^F}{z^H} = \frac{\theta^F q(\theta^F)\pi^F}{\theta^H q(\theta^H)\pi^H}.$$  

(17)

The above ratio summarizes the extent to which markets are effectively integrated. The closer to one is this ratio, the more we can speak about effective trade integration. Note that the trade share depends both upon differences between $\theta^F$ and $\theta^H$ and between $\pi^F$ and $\pi^H$. So, two factors contribute to shape the degree of trade integration. First, there is the “relative tightness of the market”, reflected in $\theta^F q(\theta^F)/\theta^H q(\theta^H)$. The higher this term is, the easier it is for a buyer to be matched with a foreign seller rather than with a domestic one. The second determinant is the “relative selectivity”, measured by $\pi^F/\pi^H$, which summarizes the extent to which a domestic seller requires a successful match to be “closer”, compared with a foreign seller. It can be shown that the asymmetric pricing behavior we have previously characterized inevitably leads to home bias because of both lower market tightness for foreign sellers and a higher required proximity.

Further, from the steady-state equality $z^H + z^F = 1 - b$ it follows $b = d/(d + \theta^H q(\theta^H)2\pi^H + \theta^F q(\theta^F)2\pi^F)$.

15
Proposition 2 Legal costs $\Delta$ segment markets because of two reasons: i) buyers are more hardly matched with foreign sellers ($\theta^H > \theta^F$); ii) cross-border matches are more easily rejected ($\pi^F < \pi^H$).

Proof: See Appendix A.2.

There are two self-reinforcing reasons that lead to reduced trade in the presence of legal costs. The first is the fact that the matching is more difficult with foreign sellers. Since an equally productive match is less profitable for a seller if realized with a foreign buyer, in the steady state there will be less foreign sellers searching for a partner compared with domestic ones. Second, matches between parties belonging to different countries will be more easily rejected. Again, since an equally productive match is less profitable if realized across the border, it will be accepted only if more productive, or “closer” than a match occurring within domestic boundaries. Since changes in relative tightness and in relative selectivity tend to reinforce each other, there is a legitimate presumption that, as legal costs rise, the extent of the home bias may rise substantially. Comparative statics are not easy performed analytically. However, an insight can be obtained through numerical simulations, once functional forms are specified for the instant utility of buyers and the matching function. Consider then a linear instant utility, such that

$$\frac{1}{2} \left( x \right) = \frac{1}{2} - 2x \frac{1}{2} - \frac{1}{2}, \ x \in [0, 1],$$

and a Cobb-Douglas matching function, such that

$$q(\theta) = \theta^{-1/2}.$$

Table 1: The behavior of the home bias; numerical simulations

<table>
<thead>
<tr>
<th>$\Delta/\rho$</th>
<th>$\Delta/\rho = 0.01$</th>
<th>$\Delta/\rho = 0.03$</th>
<th>$\Delta/\rho = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 0.1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.084</td>
<td>0.108</td>
<td>0.127</td>
</tr>
<tr>
<td>$\theta^H$</td>
<td>0.349</td>
<td>0.260</td>
<td>0.220</td>
</tr>
<tr>
<td>$\theta^F$</td>
<td>0.325</td>
<td>0.206</td>
<td>0.145</td>
</tr>
<tr>
<td>$\pi^H$</td>
<td>0.47</td>
<td>0.437</td>
<td>0.419</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>0.462</td>
<td>0.412</td>
<td>0.378</td>
</tr>
<tr>
<td>$z^F/z^H$</td>
<td>0.947</td>
<td>0.838</td>
<td>0.73</td>
</tr>
<tr>
<td>$d = 0.5$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.634</td>
<td>0.699</td>
<td>0.725</td>
</tr>
<tr>
<td>$\theta^H$</td>
<td>0.03</td>
<td>0.029</td>
<td>0.03</td>
</tr>
<tr>
<td>$\theta^F$</td>
<td>0.018</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>$\pi^H$</td>
<td>0.488</td>
<td>0.485</td>
<td>0.495</td>
</tr>
<tr>
<td>$\pi^F$</td>
<td>0.433</td>
<td>0.32</td>
<td>0.217</td>
</tr>
<tr>
<td>$z^F/z^H$</td>
<td>0.698</td>
<td>0.287</td>
<td>0.08</td>
</tr>
</tbody>
</table>

$\gamma = 0.5, c = 1, \rho = 1, \bar{\rho} = 1.1.$
In Table 1 we provide numerical simulations to assess how the extent of the home bias is shaped by the presence of legal costs associated with cross-border trade.\textsuperscript{16} We consider “small” differential trial costs. The ratio $\frac{\Delta}{\rho}$ is assumed to be between 1 and 5 per cent. Two cases are considered as far as parameter $d$ is concerned. One case ($d = 0.1$) considers small "turnover", i.e. a relatively low value for the rate of match destruction. The other is a "high turnover" case ($d = 0.5$).

The mass of searching buyers $b$ always rises with the magnitude of legal costs $\Delta$. This means that the overall steady state number of business relations (equal to $1 - b$) falls. Note that the steady state value of $b$ is equal to $d/ \left[ d + \theta_H q(\theta_H) 2\pi^H + \theta_F q(\theta_F) 2\pi^F \right]$. So, a fall in $b$ must be associated with a reduction in the number of successful matches for the average searching buyer. As for the behavior of the mass of searching sellers, we note from the changes in $\theta_H$ and $\theta_F$ that it is quite different depending on whether they are domestic or foreign. The same is observed for $\pi^H$ and $\pi^F$. While the entry of foreign sellers drops in all cases (so, $\theta_F$ falls) and also $\pi^F$ unambiguously falls, we see that $\theta_H$ and $\pi^H$ fall in the case of low turnover, while have a non-monotonic behavior in the case of high turnover. In that case, they fall first and then rise. As for the extent of the home bias, (inversely related to $z^F / z^H$) we see that it always rises with $\Delta$, and that can do so quite dramatically in the case of high turnover.

How can we interpret these results? On the one hand, $\Delta$ rises the outside option of buyers directly, thus entailing a reduction in $\pi^H$ and $\theta_H$. On the other hand, rising legal costs $\Delta$ also affect $\pi^F$ and $\theta_F$, thus affecting indirectly the outside option of domestic sellers. Since in the simulations in Table 1 $\pi^F$ and $\theta_F$ always to be lowered by $\Delta$, this translates into reduced bargaining power of buyers, and then into easier entry of domestic sellers ($\theta_H$ tends to rise) and reduced selectivity in accepting matches ($\pi^H$ tends to rise). The indirect effect on domestic sellers’ variables is thus positive. In the case of high turnover this indirect effect of $\Delta$ may prevail over the direct one.\textsuperscript{17} So, when $d$ is relatively high, “small” legal costs are sufficient to choke-off a substantial amount of trade. This is because the role of the outside option for the buyer gets more important as $d$ rises, thus leading to a substantial drop in $\pi^F$ and $\theta_F$. This, in turn, leads to a strong indirect effect on the outside option of buyers, and then into a possible drop in $\pi^H$ and $\theta_H$ as $\Delta$ rise. The fact that $\theta_F$ and $\pi^F$ fall when $\theta_H$ and $\pi^H$ rise shifts the market towards domestic matches further, reinforcing the extent of the steady-state home bias.

\textsuperscript{16} In all cases presented in Table 1 it is checked that condition (23) provided in Appendix A.1. holds. This conditions guarantees that no deviation is profitable for sellers in an equilibrium with buyers behaving honestly.

\textsuperscript{17} It is to note that this occurs provided that the values for $\pi$, $\rho$, and $c$ are sufficiently close, as it is in the simulations in Table 1.
4 Conclusions

National borders matter for trade. In this paper we offer an explanation of the home bias based on the existence of asymmetries in legal systems across countries. The starting point of the analysis is that international trade in many manufacturing sectors does not occur in organized markets like those of basic commodities. In these sectors, the connection between sellers and buyers is the result of a costly search process, so that successful matches enjoy a rent and tend to be long lasting, while unsuccessful matches will be rejected by the parties. Moreover, the price at which trade occurs reflects the relative bargaining power of buyers and sellers, which is shaped by their outside options. In this context, the change in the legal system associated with crossing the border translates into a shift of bargaining power towards the party that has the opportunity to behave opportunistically (buyers, in our example), by refusing to fulfill the agreed obligations. This reduces cross-border trade for two reasons. First, sellers prefer to invest resources to search for domestic partners, rather than for foreign ones. Second, cross-border matches will translate into operating business relations more hardly than domestic ones: only the most productive matches will be accepted. These two effects reinforce each other, and can choke-off a large proportion of trade. This is true especially in the case of sectors characterized by high turnover, where business relations are not long-lasting. In this case, buyers enjoy a high outside option and the bargaining power of foreign sellers is weak. As trial costs rise, the home bias increases disproportionately because of lower entry of foreign sellers and higher entry of domestic sellers.

There are several implications from the analysis. First, without further developments in contractual arrangements such as international arbitration or documentary credit or some degree of effort of sovereign countries to further integrate their economies also from the viewpoint of the settlement of international disputes, the volume of cross-border trade is doomed to remain lower, probably substantially lower, compared with that taking place within national boundaries. Moreover, the relative importance of law asymmetries in explaining the home bias may not necessarily fall over time, without harmonization efforts. The reason is that the share of long-distance trade in search-intensive, tailor-made, sophisticated goods tends to increase as a consequence of technological progress, falling communication costs and manufacturing production reorganization. In particular, there is evidence of a rising fraction of trade in intermediate inputs and capital goods associated with outsourcing and “production fragmentation” (see, for instance, Feenstra (1998) for a survey on the topic). Since it is in this type of goods that trading requires a greater search effort, our analysis suggests that the relative importance of legal asymmetries in shaping cross-border segment-

\[18\] See also Rauch (2001), page 38, on this point.
tation of markets may rise as the process of disintegration of production proceeds. Second, the analysis may help to identify in which sectors trade is more likely to remain internationally segmented due to asymmetries in law systems. Industries where search matters and where firms’ expected life is shorter are those in which border effects may play a stronger in reducing international trade. Available evidence shows that average firms’ size is significantly negatively related to the extent of border effects (Chen (2000)).

Since average firms’ life tends to be shorter in industries with smaller firms (see, e.g., Dunne, Roberts and Samuelson (1988)), this can be an indirect confirmation that higher turnover is positively associated with border effects.

\footnote{Dummies of product differentiation, instead, do not prove significant in explaining border effects across sectors (Chen (2000)).}
A Appendix

A.1 Proof of Proposition 1

In a first step, we take as given the existence of a candidate equilibrium with honest equilibrium and show that for small values of legal costs $\Delta$ and production costs $c$ this candidate equilibrium can be an actual equilibrium because sellers would not have an incentive to deviate from their price decisions. In a second step we show that for small values of $\Delta$ an interior equilibrium with honest behavior and cross-border trade exists and is unique. There, we show that: i) for any pair $(\pi^F, \theta^F)$ only one pair of values $(\hat{n}^H, \theta^H)$ solves (12) and (14); ii) that a unique pair of values $(\pi^F, \theta^F)$ solves equations (13) and (15).

**Step 1**

An equilibrium with honest behavior by all buyers must be such that no seller has an incentive to deviate from (11), assuming that all other sellers are setting (11) and that all buyers are behaving honestly. The price $\tilde{p}$ set by a seller matched with a foreign buyer at distance $x$ as a result of deviation solves $C_{dF}(x) = W$, where the superscript $d$ and $F$ refers, respectively to the conjectured behavior on the part of the matched buyer (dishonest) and to the fact that the match occurs across the border. Since, by refusing to pay the buyer will have to compensate the seller with $\tilde{p} - c$ in a pre-trial settlement, we must have

$$rC_{dF}(x) = \rho(x) - \tilde{p} + \Delta - [C_{dF}(x) - W]. \quad (18)$$

From (8), and since buyers’ surplus is nil in domestic matches, it must be that $rW = 2\pi^{F} \theta^F \frac{q(\theta^F)}{1-d}$, so that, $C_{dF}(x) = W$ yields the following price set in a deviation

$$\tilde{p} = \rho(x) + \Delta[1 - \frac{2\pi^{F} \theta^F q(\theta^F)}{1 - d}]. \quad (19)$$

Denote now by $J_{dF}(x)$ and $J_{hF}(x)$ the asset value of the seller when setting, respectively, the optimal price inducing dishonest and honest behavior in the buyer, i.e., (19) and (11). A deviation from an equilibrium with honest behavior is profitable as long as $J_{dF}(x) > J_{hF}(x)$. From (14) and (15) it must be that

$$J_{hk}(x) = \frac{p^{k}(x) - c}{r + d}, \quad (20)$$

where $k = F, H$. Moreover, recalling that with dishonest behavior the match is destroyed at rate 1 and that the compensation to the seller will be $\tilde{p} - \Delta$ we must have
\[ J^{dF}(x) = \frac{\tilde{p} - \Delta - c}{r + 1}, \quad (21) \]

Substituting, respectively, (11) in (20) and (19) in (21) after some algebra it is obtained that \( J^{dF}(x) > J^{hF}(x) \) if and only if

\[ \rho(x) < c + \frac{\Delta}{(1 - d)^2} \left[ (1 + r)(d + r) + (1 - d) 2\pi^F \theta^F q(\theta^F) \right]. \quad (22) \]

It follows that if the condition

\[ \rho(x) \geq c + \frac{\Delta}{(1 - d)^2} \left[ (1 + r)(d + r) + (1 - d) 2\pi^F \theta^F q(\theta^F) \right]. \quad (23) \]

holds for all \( x \leq \pi^F \) no deviation can occur from an equilibrium with honest behavior. When the value of \( \Delta \) is sufficiently small, sufficiently low values for \( c \in [\tilde{p}, \overline{p}] \) always exist such that condition (23) is satisfied for \( x < 1/2 \).

Step 2.a. The values for \( \pi^H \) and \( \pi^F \) are implicitly defined, respectively, by (12) and (13). It follows from (20) that (12) and (13) can be rewritten as

\[ \rho^H(\pi^H) = c, \quad (24) \]
\[ \rho^F(\pi^F) = c. \quad (25) \]

From equation (10) one can rewrite (12) as follows

\[ \rho(\pi^H) = c + 2\pi^F \theta^F q(\theta^F) \frac{\Delta}{1 - d}. \quad (26) \]

As \( \rho' < 0 \), for each \((\pi^F, \theta^F)\) if there exists a value for \( \pi^H \) (independent of \( \theta^H \)) that solves (26) and that is interior to \((0, 1/2)\), this value must be unique. It is also easily checked that this solution is negatively related to \( \pi^F \), \( \theta^F \) and \( \Delta \).

From (7), (10) and (20), (14) transforms to

\[ \gamma = \frac{2q(\theta^H)}{r + d} \left[ \int_{0}^{\pi^H} (\rho(x) - 2\pi^F \theta^F q(\theta^F) \frac{\Delta}{1 - d} - c)dx \right]. \quad (27) \]

Equation (27) implicitly defines \( \theta^H \) as a continuous function of \( \pi^H \), at given \((\pi^F, \theta^F)\). By the properties of \( q(\theta) \) the right hand side of (27) is monotonically decreasing in \( \theta^H \). Furthermore, the right hand side of (27) goes to infinity when \( \theta^H \to 0 \) and to zero when \( \theta^H \to +\infty \). It follows that, for any
there is always a single value of $\theta^H$ that solves (27). The right hand side of (27) is trivially monotonically increasing in $\pi^H$. Hence, $\theta^H$ is an increasing implicit function of $\pi^H$. It is also checked that $\theta^H$ is negatively related to $\pi^F$, $\theta^F$, and $\Delta$.

Graphically, the implicit value of $\pi^H$ obtained from (26) is a horizontal line in the $(\pi^H, \theta^H)$ space, while $\theta^H$ obtained from (27) is a positively sloped curve starting from 0. So, at given $(\pi^F, \theta^F)$ if there is an internal solution $(\pi^H, \theta^H)$ to (12) and (14) it must be unique.

**Step 2.b.** First note that equations (15) and (13) do not depend on $(\pi^H, \theta^H)$. Again, (13) can be rewritten as follows

$$\rho(\pi^F) = c + [r + d + 2\pi^F \theta^F q(\theta^F)] \frac{\Delta}{1 - d}. \quad (28)$$

If, given $\theta^F$, equation (28) has an internal solution in $\pi^F$, it must be unique, since the left hand side is decreasing and the right hand side is increasing in $\pi^F$. Moreover, one checks that $\frac{d\pi^F}{d\theta^F} = \frac{2\pi^F \Delta (\rho(\theta^F) q(\theta^F)) / d\theta^F}{(1 - d)\rho(\pi^F) - 2\pi^F \theta^F q(\theta^F) \Delta} < 0$ and that

$$\lim_{\theta^F \to 0} \pi^F = \nu,$$

where $\nu$ solves $\rho(\nu) = c + (r + d) \frac{\Delta}{1 - d}$. Note that for sufficiently low values of $\Delta$ it must be that $\nu > 0$. Moreover, since $c \geq \rho$, $\nu \leq 1/2$. As for $\Delta$, its effect on $\pi^F$ is negative.

Substituting (20) and (11) in (15), the following equation is obtained

$$\gamma = \frac{2q(\theta^F)}{r + d} \left[ \int_0^{\pi^F} (\rho(x) - (r + d + 2\pi^F \theta^F q(\theta^F)) \frac{\Delta}{1 - d} - c) dx \right]. \quad (29)$$

Using the properties of the function $q(\theta)$ one checks that the right hand side of (29) is strictly decreasing in $\theta^F$. Moreover, the right hand side of (29) goes to infinity when $\theta^F \to 0$ and to zero, a negative finite value or $-\infty$ when $\theta^F \to +\infty$. Therefore, (29) must have a unique solution $\theta^F \in (0, +\infty)$ at given $\pi^F$. By (25), and since $p^F(x) > c$ for all $x \geq \pi^F$, the right hand side of (29) is monotonically increasing in $\pi^F$. It follows that the value of $\theta^F$ solving (29) is an increasing implicit function of $\pi^F$. Moreover, $\lim_{\pi^F \to 1/2} \theta^F = 0$ whereas $\lim_{\pi^F \to 1/2} \theta^F = \Theta$, where $\Theta$ is a positive constant. As for $\Delta$, after differentiation it is checked that its effect on $\theta^F$ is negative.

Graphically, $\pi^F$ obtained from (28) is a negatively sloped locus in the $(\pi^F, \theta^F)$ space, while $\theta^F$ obtained from (29) is a positively sloped one starting from 0. It follows that if $\pi^F(\theta^F)$ defined by (28) and $\theta^F(\pi^F)$ defined by (29) cross, they cross only once.

From the previous analysis it follows that when $\Delta$ is sufficiently small there is a unique solution to (28) and (29) where $\pi^F \in (0, 1/2)$ and where
\( \theta^F \) has a finite value. Moreover, given the results in step 2.a and \( c \geq \rho_0 \), for sufficiently small values of \( \Delta \) the value of \( n^H \) obtained from (26) is positive and lower than 1/2 for any possible solution \((\pi^F, \theta^F)\). It follows that a range of small values for \( \Delta \) exists such that there exists a unique interior solution to the system given by (12)-(15) (an equilibrium with cross-border trade). Q.E.D.

### A.2 Proof of Proposition 2

We show first that \( \pi^F < \pi^H \). The values for \( \pi^H \) and \( \pi^F \) are implicitly defined, respectively, by (12) and (13). Recalling that (24) and (25) must hold, that both prices are decreasing functions of the distance \( x \), and that \( p^H(x) > p^F(x) \), it must be that \( \pi^H > \pi^F \).

Second, we can show that \( \theta^H > \theta^F \). The values of \( \theta^H \) and \( \theta^F \) are implicitly defined by (12) and (13) which, from (7) can be written as

\[
\gamma = \frac{2q(\theta^H)}{r + \Delta} \int_0^{\bar{n}^H} (p^H(x) - c)dx,
\]

\[
\gamma = \frac{2q(\theta^F)}{r + d} \int_0^{\bar{n}^F} (p^F(x) - c)dx.
\]

Since, by (10) and (11), \( p^F(x) = p^H(x) - (r + d) \frac{\Delta}{1 - \Delta} \), (12) and (13) imply that

\[
\frac{\int_0^{\bar{n}^F} (p^H(x) - c)dx - \bar{n}^F (r + d) \frac{\Delta}{1 - \Delta}}{\int_0^{\bar{n}^H} (p^H(x) - c)dx} = \frac{q(\theta^H)}{q(\theta^F)}.
\]

The left hand side of the above equality is lower than one because \( \pi^F < \pi^H \), and since \( \int_0^{n} (p^H(x) - c)dx \) is increasing in \( n \), as shown in Appendix A.1, step 2.a. Recalling that \( q' < 0 \) it follows that \( \theta^H > \theta^F \). Q.E.D.
References


Figure 1: Sequence of events

- Seller enters
- Match occurs
- Seller posts price
- Buyer
  - Accepts match
  - Refuses match
    - Match expires
      - Refuses match
        - Refuses match
          - The parties reach a pre-trial agreement
          - No pre-trial agreement
            - Good is delivered
              - Business relation destroyed at rate $d < 1$
            - Court enforces payment
              - Pays
                - Business relation destroyed at rate 1

- Seller posts price
- Buyer
  - Accepts match
  - Refuses to pay
    - Refuses to pay
      - The parties reach a pre-trial agreement
        - No pre-trial agreement
          - Good is delivered
            - Business relation destroyed at rate $d < 1$
          - Court enforces payment
            - Pays
              - Business relation destroyed at rate 1

Figure 2: Characterization of equilibrium