Meta-models of the Phillips curve and income distribution

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On June 4, 2022, Piero Ferri passed away, after this paper was completed; Annalisa and Fabio are indebted to him for his generosity, his continuous stimulation to study and learn, and his proactivity. They will miss a mentor and a dear friend.

Abstract

This paper reconsiders the flattening of the Phillips curve that has taken place over the last twenty-five years, together with the recent claim that a wage-price spiral could ignite an inflationary process. The two aspects are interrelated. The time-variation nature of the parameters plays a fundamental role in feeding a process of instability that may affect the reappearance of inflationary episodes. This paper first examines the extent of the U.S. Phillips curve flattening and nonlinearity by comparing the results of different econometric specifications and models. Based on these findings, the use of a regime-switching device, particularly studied by Richard Day, is proposed and applied to a Harrod-Minsky macro model. Three results are worth mentioning. The detection of nonlinearities in the U.S. Phillips curve leading to breaks in the curve; the ability of the model to generate robust bounded fluctuations, in which both Phillips curve loops and distributive loops can arise; the presence of both wage-led and profit-led distributive loops in the same model with different combinations of real and monetary parameters that can generate complex dynamics.

Keywords: Phillips curve, threshold models, time-variation of parameters, regime switching, distributive loops, medium-run growth models, simulations.

JEL Classification: E1, E24, E3, E6, C1, C53.

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1 Introduction

The flattening of the price-Phillips curve that has taken place over the last twenty-five years and peaked during the Great Recession (see e.g. Forbes et al., 2021 and Cristini and Ferri, 2021)[1] has analytical and methodological implications beyond the relationship between inflation and unemployment. In particular, in this paper, we study the dynamic effect of the time variation of the parameters that characterize the changes in the Phillips curve. This theme is developed in four steps that form the structure of the present paper.

First, we follow the strategy used in a companion paper (Cristini and Ferri, 2021) and test econometrically for the presence of nonlinearities in the U.S. wage Phillips curve.[2] While the latter has flattened less than the price Phillips curve, it has still changed significantly and attracted some attention (see Blanchard, 2018 and Galí and Gambetti, 2019). To anticipate the main results, we find evidence confirming nonlinearities also for the wage Phillips curve, with changes at the breakpoint affecting the slope and other coefficients. As will become clear below, these findings are particularly important for the study of dynamics and allow the use of a regime-switching device (Day, 1994).

The second step is to consider a wage-price sub-system to test whether year-to-year feedback between wage growth and inflation can be generated. Although this spiral appears to belong to the years preceding the mid-1990s (Rudd, 2022), there are concerns that it could reappear in the current post-pandemic period. We do not intend to examine this possibility from a historical standpoint. Rather, we want to explore the analytical and methodological problems that a wage-price spiral might trigger. This research step, therefore, implies the need to bring stability analysis into the picture and test whether runaway situations can occur. (See also Flaschel and Krolzig, 2006)

In the third step, the wage-price subsystem is inserted into a macro model. While the wage-price subsystem allows phenomena such as inflation, deflation or stagflation, to be incorporated directly into the analysis, it can also generate strong instability processes. These phenomena are the result of the interaction between price and quantity adjustments, which typically characterize macro disequilibrium processes with feedback

[1] The literature on the U.S. price Phillips curve is extensive. See recent work by Doser et al. (2023) and Hazell et al. (2022) on nonlinearities; Cristini and Ferri (2021) provide new econometric results for the period 1961q1 to 2019q4 across different models. Forbes et al. (2021) offers an empirical analysis of 31 advanced economies for the period 1996q1 to 2017q4.

[2] Nonlinearities typically refer to changes in the slope of the Phillips curve over the cycle, that is, the response of wage or price inflation to a measure of slack differs between positive and negative phases of the cycle or the level of slack in the economy. Note that this leads to a 'kink' at the threshold or break point. One could argue that, if a step also occurs at the threshold, i.e., the constant also changes, we observe a 'discontinuity' in the Phillips curve at the threshold. (See for example Fong et al. (2017).

reactions. These complex interactions depend not only on the specification of the wage-price subsystem, but also on the nature of the overall model in which they are inserted. In this paper, we refer to a macro model of the Harrod-Minsky variety, (Ferri, 2022) which allows us to operate in an environment characterized by bounded and persistent fluctuations and in which loops can be generated in the Phillips curve.

The last step reinforces these dynamic processes by resorting to a regime-switching technique that focuses on time-variation in the parameters. As is well known, this technique has been at the heart of Day (1994) analysis. In this context, two aspects are particularly noteworthy. The first is that the Phillips curve nonlinearities, implying a break at the threshold, are in keeping with the nature of this device. The second is that this helps produce complex dynamics, as Richard Day had taught. One of the authors of the present paper collaborated with Day in studying the effects of switches in the Phillips curve on macro dynamics (Ferri et al., 2001). At that time, the subject was considered mainly from a mathematical point of view, while the dimension of the system was small (see Tramontana et al., 2010 for a more in-depth discussion). In the present work, the switches in the two Phillips curves are considered from a broader perspective and treated in a simulation framework due to the increase in the dimensionality of the system.

On the whole, there are two comparative advantages of the regime-switching methodology that emerge from the present work. The first relates to the robustness of the dynamics, where both loops in the Phillips curve and distributive loops, i.e., the dynamic relationship between the distribution of income and some indicators of economic activity, are systematically present. The second comparative advantage of this technique is that the meta-model, which consists of two regimes separated by a threshold, can deal with different combinations of distributive loops. In other words, in the present model, the dichotomy of wage-led versus profit-led varieties does not characterize different economic systems; on the contrary, both varieties are present in the same system at different states of the economy. This makes it possible to generate complex dynamics and possibly further stimulate the so-called income distribution debate (Blecker and Setterfield, 2019, Chiarella et al., 2021, Palley, 2017 and Ferri, 2022).

We contribute to the existing literature in several ways. From an empirical perspective, we add to the evidence on nonlinearities and breaks in the U.S. wage Phillips by looking at a long period of time (1964q1 to 2021q3) and comparing results from different models. Linear, piecewise linear, log-transformed and threshold models are contrasted, and the magnitude of nonlinearities is assessed across all specifications. We test our results for various measures of inflation (core PCE, headline PCE, CPI), various lags of past inflation, and the inclusion of wage inflation persistence. Moreover, we show some new results on the role of trend productivity growth in the slope, persistence and thresholds of the wage Phillips curve. From a theoretical perspective, by proposing a model in which wage- and profit-led cycles can result from different states of the same economy, we contribute to the recent revival of the debate on distributional cycles and on the role that policy plays in determining such cycles (Palley, 2017). We also provide further insight into the long-standing debate over the Taylor parameters by discussing the stability of the system to changes in the response of monetary policy to the activity rate and inflation. Finally, we contribute to the current debate on how the return of a wage-price spiral could fuel inflation, a topic that has attracted the attention of scholars (Schiller, 2022; Lavoie and Rochon, 2023) and institutions (see, e.g., Alvarez et al., 2022; Boissay et al., 2022 and references therein), to name a few.

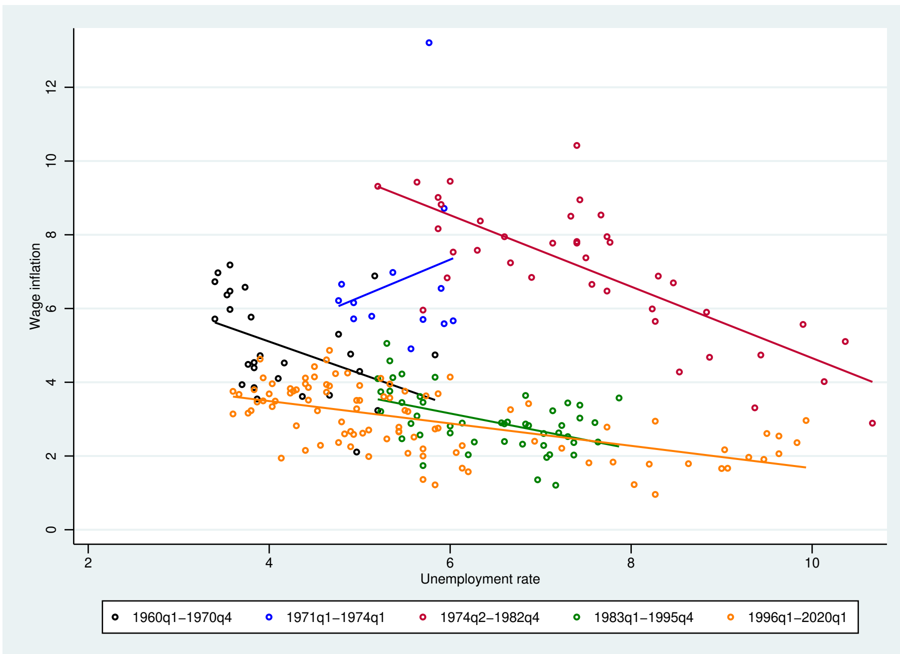
The structure of the paper is the following. Section 2 deals with the econometrics of time-variation in the parameters. Section 3 presents the structural aspects of regime switching. Section 4 discusses switching in a wage-price subsystem. Section 5 illustrates the meta-model covering both regimes. Section 6 shows the overall dynamics. Section 7 introduces the loops that characterize both the Phillips curves and the distributive phenomenon. Section 8 discusses the validity of the parameters used in the simulation and presents robustness tests of the dynamics. Section 9 concludes. The Appendix contains supplementary material.

2 The econometrics of time-variation in the parameters

As Cristini and Ferri (2021) show, the U.S. price Phillips curve is nonlinear and exhibits kinks and jumps at the threshold. The authors tested various nonlinear specifications (log-transformed, stepwise and threshold models) using U.S. core PCE data from 1961 q1 to 2019 q4; they found that: (i) in terms of RMSE, each of the tested nonlinear specifications is preferable to the linear one; (ii) the estimated convexity is model-specific and increases when moving from the log-transformed model to the piecewise model and to the threshold model; (iii) in the latter, a break is detected in correspondence to an unemployment gap of 0.517 that, considering the average non-cyclical unemployment rate of the period, corresponds to an unemployment rate of 5.95%; this estimated slope is six times as steep below the threshold than above it; (iv) the difference between the slopes of the estimated Phillips curve in the different specifications is particularly large below the threshold, suggesting that the choice of the model itself may have policy implications, especially in times of a tight labor market.

In this paper, we repeat the econometric analysis for the canonical Phillips curve. Figure 1 shows the wage Phillips curve for the United States.

Figure 1: The wage Phillips curve 1964 q1- 2020 q1



Note: Data points 2020q2 and 2020q3 are outliers and are excluded for the sake of clarity. Wage inflation is the annualized log change of Hourly Earnings for Production and Nonsupervisory Employees, Total Private (FRED), available from 1964 q1. The rate of unemployment is from BLS. The straight lines are linear interpolations.

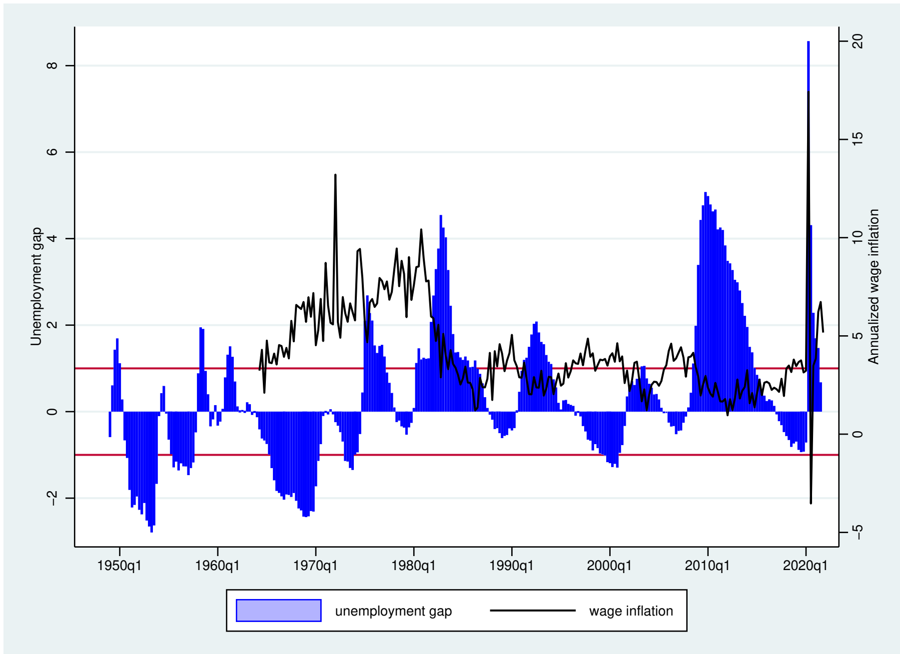
It shows the remarkable outward shift of the 1970s, the successive return to an inward position, and the concomitant moderate flattening of the slope. Both the frequency of negative unemployment gaps and their intensity decrease over time, as can be seen from Figure 2. On the contrary, since the mid-1970s, periods of slack have dominated the scene in terms of their intensity.[3]

2.1 Econometric analysis

We begin by considering the following specification:

[3]On this point see also Hooper et al., 2019 on the role of central banks.

Figure 2: Unemployment gap and wage inflation



Note: the unemployment gap is defined as the difference between the rate of unemployment (BLS) and the (time-variable) CBO non-cyclical (previously called natural) rate of unemployment (long-term) not seasonally adjusted (FRED). The horizontal red lines indicate the ± 1 range of the unemployment gap.

$$w_t = \alpha + \beta_{00}(U - U^*)_t + \beta_{01}(U - U^*)_t^+ + \beta_1 \pi_{t-1}^e + \sum_{i=1}^{7} \beta_{2i}\pi_{t-i} + \beta_3 \tau_t + \epsilon_t \tag{1}$$

where w is wage inflation, defined as the annualized log change of Hourly Earnings for Production and Nonsupervisory Employees, Total Private (FRED),[4] π is the annualized core PCE inflation[5] and enters with 7 lags. In fact, the number of lags varies in the literature; Hooper et al. (2019) also use 7, as well as Cristini and Ferri (2021); Yellen (2017) for example uses 2. Since statistical significance is normally only up to the second lag, this choice could be an alternative. However, in this case, the AIC tests, reported in the last row of Table 1, are always larger than those obtained using seven lags, which supports our

[4]See notes to Table 1 for the precise variable definitions.

[5]Core PCE inflation excludes food and energy prices, thus ruling out supply shocks due to these components (Ball and Mazumder, 2011). In its quarterly economic projections, the FOMC focuses on core PCE inflation but also states its longer-run inflation goal in terms of headline PCE inflation. Core PCE is also widely used empirically and allows us to compare our results with the existing evidence. However, since supply shocks can lead to price increases that can trigger a wage-price spiral, we test the results using PCE headline inflation and CPI. See Tables A1 and A3 and comments in the text below.

choice. π^e are one-year ahead inflation expectations from the Michigan consumers survey.[6] The sum of β_1 and β_{2i} are constrained to unity; this implies that we consider nominal wage growth deflated by a mix of past inflation and inflation expectations. The slack measure is the unemployment gap defined as the difference between the rate of unemployment U and the non-cyclical time-varying rate of unemployment U^*; $(U-U^*)^+ = I_{(U>U^*)} \cdot (U-U^*)$ is obtained by interacting the unemployment gap with an indicator dummy denoted by $I_{(U>U^*)}$ that takes the value 1 when the unemployment gap is positive and 0 otherwise. This allows for a piecewise linear Phillips curve with a slope varying between positive and negative unemployment gaps.[7] In the linear specification we then impose $\beta_{01} = 0$. τ is trend productivity growth defined as the growth in real output per hour in the non-farm business sector smoothed using the HP-filtered trend. ϵ denotes the residuals of the estimated regression. Finally, in one specification, we use a log-transformed Ugap defined as $log(U/U^*)$ to allow for a built-in convex curvature of the estimated Phillips curve.

The Phillips curve described above, which can be viewed as the 'current consensus augmented Phillips curve'(Hooper et al., 2019), largely results from Friedman (1968) restatement of the original Phillips analysis. Friedman's interpretation transformed the original relationship between nominal wage inflation and unemployment into a relationship between expected real wage inflation and unemployment. Depending on how inflation expectations are formed, past inflation or past inflation and expected inflation enter the augmented Phillips curve equation. In addition, as shown by Whelan (1999), productivity growth is also included in the equation if firms are assumed to use the markup price-setting rule, which implies that the average real wage is proportional to average labor productivity.

Interestingly, during the 1990s, the theory of the augmented Phillips curve benefited from the debate over the microeconomic wage curve (Blanchflower and Oswald, 1994) and how the latter could be reconciled with the macroeconomic Phillips curve. Blanchard and Katz (1999) show that the Phillips curve can be recovered from a standard non-competitive model of the labor market in which the lagged real wage enters the equation as a proxy of the reservation wage. They also use this rationale to test the accelerationist hypothesis. For an overview, see also Montuenga-Gómez and Ramos-Parreño (2005).

[6]The Michigan survey of consumers refers to CPI inflation expectations and has been shown to be more effective than professional forecasters in the Phillips curve estimation (Doser et al., 2017 and Coibion and Gorodnichenko, 2015). The Michigan series (MICH) begins in 1978; we extend it back to 1964 using a VAR on PCE inflation and CPI inflation, both calculated as quarter above the corresponding quarter of the previous year. See Figure A3 in the Appendix. Details are available upon request.

[7]We expect positive unemployment gaps to exert a lower pressure on inflation; the estimated coefficient of a positive unemployment gap is, therefore, the algebraic sum of the two coefficients.

Table 1 reports the initial results of estimating equation 1 using the linear specification, the piecewise linear specification and the log-transformed specification.

Two columns are shown for each specification. In the first one, we omit trend productivity growth, to assess its role in the wage inflation regression. Note also that in each specification we need to include two dummies for the last two quarters of 2020 to account for the pandemic shock. The time span is from 1964 q1 to 2021 q3.[8]

The slope of the Phillips curve is always steeper when the trend productivity growth is omitted, and this is true for all three specifications. This omitted variable bias implies that trend productivity growth is positively correlated with wage inflation and negatively correlated with our measure of slack - an expected result. At the same time, the coefficients on expected inflation and inflation lagged by one period are higher when trend productivity growth is controlled for, while further lags on inflation reduce their coefficient magnitudes. By omitting trend productivity growth, wage inflation then appears to be more responsive to slack and less responsive to inflation than in the specification that takes trend productivity growth into account. As we will see in the next subsection, omitting trend productivity growth also leads to a different number of thresholds. Even when productivity is taken into account, inflation expectations dominate past inflation, suggesting an important role for the Central Bank in anchoring inflation expectations.[9]

In the final specification, we estimate a somewhat steeper slope in all the models compared to the existing evidence. For example, with respect to Hooper et al. (2019), who apply a similar empirical analysis (except for the measure of expected inflation, as described in the previous footnote), the difference could be due to the different time period. While Hooper et al. (2019) span the analysis to 2018 q4, we extend it to 2021 q3, capturing COVID and the first POST-COVID quarters in which inflation and the unemployment gap increased. How the COVID experience has affected the Phillips curve

[8] While longer periods are available for some variables, this is not the case for our measure of wage inflation, average hourly earnings of production and nonsupervisory employees, which is available from 1964, and for price expectations - the Michigan series begins in 1978. We use quarterly frequency; therefore, average hourly earnings, the unemployment rate and price expectations are converted from monthly to quarterly.

[9] This result differs markedly from Hooper et al. (2019); they use long-run (10 year) inflation expectations - the same series used in the FED's FRBUS model - and find that inflation expectations are not statistically significant in the wage Phillips curve. On the contrary, the coefficient on inflation persistence is about 0.9, thus supporting a close to an accelerationist conclusion. This discrepancy in the results calls for a deeper analysis of the role of inflation expectations in the wage Phillips curve. Coibion and Gorodnichenko (2015) for example, show that the linear Phillips curve performs better when it controls for short-term consumer expectations. Later in the text, we show that the role of inflation expectations also depends on the inflation measure used and increases when CPI inflation replaces core PCE.

Table 1: Linear, piecewise linear and log-transformed wage Phillips curve models

1964q1 — 2021q3

	Linear		Piecewise		Log-transformed	
	(1)	(2)	(3)	(4)	(5)	(6)
$(U-U*)$	–0.522***	–0.422***	–1.105***	–0.927***		
	(0.05)	(0.06)	(0.15)	(0.16)		
$(U-U*)^+$			0.783***	0.654***		
			(0.19)	(0.19)		
$log(U/U*)$					–3.266***	–2.678***
					(0.32)	(0.36)
π_{t-1}	0.283***	0.307***	0.244**	0.271***	0.266***	0.292***
	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
π_{t-2}	–0.163	–0.159	–0.178*	–0.172	–0.180	–0.173
	(0.11)	(0.11)	(0.11)	(0.10)	(0.11)	(0.11)
π_{t-3}	0.015	0.008	0.011	0.006	0.008	0.003
	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)
π_{t-4}	0.044	0.035	0.033	0.028	0.037	0.030
	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)
π_{t-5}	0.122	0.115	0.121	0.116	0.122	0.116
	(0.12)	(0.11)	(0.11)	(0.11)	(0.12)	(0.11)
π_{t-6}	0.072	0.069	0.087	0.083	0.076	0.073
	(0.12)	(0.11)	(0.11)	(0.11)	(0.12)	(0.11)
π_{t-7}	0.118	0.097	0.123	0.105	0.114	0.096
	(0.10)	(0.09)	(0.09)	(0.09)	(0.10)	(0.09)
π^e_{t-1}	0.510***	0.527***	0.558***	0.564***	0.556***	0.563***
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
dummy 2020q2	18.839***	18.187***	17.499***	17.190***	17.903***	17.455***
	(1.34)	(1.31)	(1.34)	(1.31)	(1.30)	(1.27)
dummy 2020q3	–3.820***	–3.960***	–4.410***	–4.427***	–3.870***	–3.997***
	(1.31)	(1.27)	(1.27)	(1.25)	(1.29)	(1.26)
trend productivity growth		0.502***		0.408***		0.459***
		(0.13)		(0.13)		(0.13)
constant	0.946***	–0.095	0.537***	–0.240	0.856***	–0.081
	(0.09)	(0.28)	(0.13)	(0.28)	(0.09)	(0.28)
N	230	230	230	230	230	230.
RMSE	1.264	1.224	1.220	1.195	1.247	1.216
AIC	771.0	757.5	755.8	747.4	765.1	754.2
BIC	808.9	798.8	797.0	792.0	802.9	795.5
AIC (2 lags)	785.1	767.5	773.3	760.6	779.1	764.3

Notes: *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level. The sum of lagged inflation and expected inflation is constrained to unity. The dependent variable is the annualized log change of Hourly Earnings for Production and Nonsupervisory Employees, Total Private (FRED). π is the annualized rate of growth of the price index of Personal Consumption Expenditures Excluding Food and Energy (FRED). Trend productivity growth is the growth in real output per hour in the non-farm business sector smoothed using the HP-filtered trend, as done for example by Hooper et al., 2019. Expected inflation π^e is one-year ahead inflation expectations from the Michigan consumers survey (MICH). The Michigan series begins in 1978; we extend it back to 1964 using a VAR on PCE inflation and CPI inflation, both calculated as quarter above the corresponding quarter of the previous year. See Figure A3 in Appendix A. Details are available upon request. U is the rate of unemployment (BLS) and U^* is the (time-variable) CBO non-cyclical (previously called natural) rate of unemployment (long term) not seasonally adjusted (FRED). The last row reports the AIC test obtained when running the regressions with only two lags on lagged inflation.

is a topic that deserves further investigation.[10]

The last three rows of Table 1 compare the six specifications presented in the Table based on three quality measures: RMSE and two information criteria, AIC and BIC. Based on all three measures, the piecewise linear model with trend productivity growth (column 4) outperforms the other specifications, providing initial support to the presence of nonlinearities in the U.S. Phillips curve.

We perform some robustness tests on these initial specifications. First, we included two lags of wage inflation to control for wage inflation persistence or pre-trends. The results are reported in Table A2; we find that the first lag of wage inflation is statistically significant and nonlinearities, as well as their magnitudes, are confirmed.[11]

Second, we consider different inflation measures. The results are reported in (Table A1). They show that in all specifications the absolute value of the slope coefficient increases when core PCE inflation is replaced by headline PCE inflation or CPI inflation. The difference is particularly clear in the CPI case. In the linear specification, the slope changes from -0.422 (core PCE) to -0.443 (headline PCE) to -0.508 (CPI) and in the piecewise linear model it changes, respectively, from -0.927 to -0.974 to -1.095, during tight labor markets or from -0.273 to -0.320 and -0. 404 during slack periods. Similarly, convexity also increases in the log-transformed model. Overall, nonlinearities are confirmed, although using CPI inflation leads to a steeper Phillips curve in both tight and slack markets; when using CPI inflation the difference in slope between the two phases is also more pronounced (0.69 versus 0.65). Moreover, the use of headline PCE and, in particular, CPI, reduces the role of inflation persistence in favor of inflation expectations, the coefficient of which rises from 0.5, in the core PCE regression, to 0.8 in all specifications. Thus, an inflation measure that better reflects the shopping basket makes wage inflation to be more responsive to the cycle and inflation expectations and less responsive to past inflation.

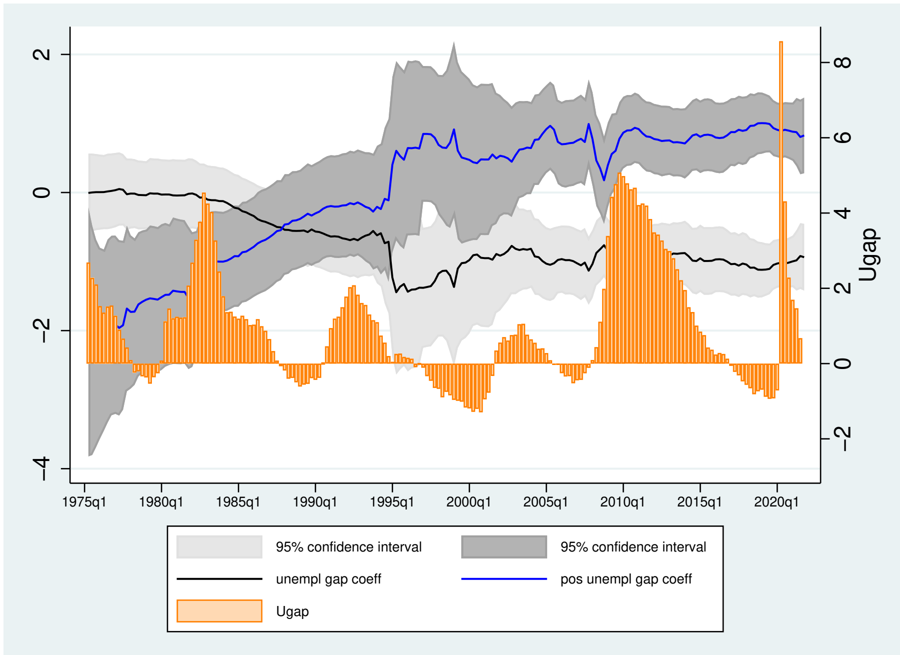
Using the specification of column (4), we estimate a rolling regression using a window of 25 years and a one-year advancing step in order to test for changes in the parameters through time. Figure 3 illustrates the resulting coefficients for β_{00} and β_{01} and their confidence intervals.

While the estimated coefficients of the two slack measures clearly change in time and move in opposite directions, the picture is best read by considering three periods: (i) 1975

[10]See for example Smith et al. (2023) for a recent Phillips curve estimation that includes the COVID years.

[11]However, given the endogeneity of the lagged dependent variable, IV should be considered. We leave this for further investigation.

Figure 3: Rolling estimated coefficients of the piecewise linear model



to 1985 (*Great Inflation*), (ii) 1985 to 2005 (*Great Moderation*) and (iii) 2005 onward (*Great Recession and Pandemic*). In the first period, unemployment gaps are largely positive and the coefficient on this component (β_{01}: blue line) captures most of the (negative) impact of slack on wage inflation. This coefficient increases in magnitude, approaching zero in the mid-1990s and becoming positive thereafter, although it is statistically significant only from 2005 onward and steadily so from 2010 onward. In contrast, the unemployment gap coefficient (β_{00}: black line) is essentially zero and plays no role initially; it then declines and becomes negative and statistically significant in the mid-1980s, a period when the unemployment gap was historically contained. Since 2005, the two coefficients have been fairly stable, although the Great Recession seems to have abruptly changed a possible downward trend of β_{01}.

2.2 The threshold model

A drawback of the previous nonlinear specifications is the arbitrary choice of the threshold, in the piecewise linear model, and the imposed curvature in the log-transformed model. Threshold models allow us to estimate both the optimum number of thresholds and their values. Table 2 reports the results. In order to compare the threshold model results and diagnostics with those of the piecewise linear specification, which scored best in Table

1, we proceed in four successive steps. In the first one (Model A) productivity is not included and the constant is invariant across regions, hence only the coefficient of the unemployment gap is allowed to change across the threshold; next, we add productivity growth, but hold it invariant across regions (Model A1). Model B allows the constant to vary across regions and Model B1 adds productivity growth variant across regions. Note that although the constant assumes significantly different values when allowed to change, then supporting a step at the threshold, the diagnostics are only marginally affected. Instead, it is again the presence of trend productivity growth (Models A1 and B1) that makes the difference: it affects the estimated coefficient of the unemployment gap and improves the diagnostics of the regression. Compared to the piecewise linear specifications (columns 3 and 4 of Table 1), Models A and A1 dominate in terms of diagnostics, thus favoring the threshold specification.

In fact, the estimated coefficients of model A1 are not very different from those of the piecewise linear (column 4). However, when productivity is allowed to change across regions (model B1), results change significantly. In region 2, when the unemployment gap is above the threshold, only lagged inflation and inflation expectations play a role while real variables (labor market slack and trend productivity growth) are not significant in determining wage inflation. For values of the unemployment gap lower than or equal to the threshold, the estimated coefficient of the unemployment gap is significant. The slope of the wage Phillips curve is negative but flatter than in the piecewise linear model. As reported in Appendix A, when trend productivity growth is omitted the optimal model conceives two thresholds (see Donayre and Panovska, 2016 for similar results in this case).[12] Notice also that the estimated threshold slightly declines by moving from a less flexible specification (Model A) to a more flexible one (Model B1); considering the average of the non-cyclical rate of unemployment, the thresholds correspond to a rate of unemployment of 6.2 (Models A and A1), 6.3 (Model B) and 6.8 (Model B1). These values are in between those found by Donayre and Panovska (2016) (5.6% and 7.6%).

Overall, based on RMSE, AIC and BIC the threshold model outperforms the piecewise linear model; this result strengthens the support of nonlinearities in the U.S. Phillips curve and the evidence of kinks and breaks.

Analogously to the previous models, we test the findings of the threshold regressions for different inflation measures. The results are reported in Table A3. Again, the estimated slope is steeper than using core PCE, and the increase in the steepness is particularly

[12]Notice that when trend productivity growth enters the model and is allowed to vary across regions it likely absorbs part of the nonlinearities and breaks otherwise captured by the output gap. This finding deserves further investigation.

Table 2: Wage Phillips curve: threshold models. 1964q1 - 2021q3

	Threshold Model A			Threshold Model A1			Threshold Model B			Threshold Model B1		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	w	Region1	Region2	w	Region1	Region2	w	Region1	Region2	w	Region1	Region2
$(U - U*)$		–0.935***	–0.346***		–0.810***	–0.284***		–0.845***	–0.034		–0.690***	–0.067
		(0.10)	(0.06)		(0.10)	(0.06)		(0.13)	(0.11)		(0.14)	(0.10)
trend productivity growth				0.407***							0.520***	–0.087
				(0.13)							(0.14)	(0.25)
π_{t-1}	0.266***			0.288***			0.258***			0.280***		
	(0.10)			(0.09)			(0.09)			(0.09)		
π_{t-2}	–0.173			–0.168			–0.159			–0.160		
	(0.11)			(0.10)			(0.11)			(0.10)		
π_{t-3}	0.019			0.014			0.027			0.030		
	(0.11)			(0.10)			(0.11)			(0.10)		
π_{t-4}	0.044			0.037			0.061			0.041		
	(0.11)			(0.11)			(0.11)			(0.11)		
π_{t-5}	0.094			0.092			0.113			0.119		
	(0.11)			(0.11)			(0.11)			(0.11)		
π_{t-6}	0.103			0.097			0.093			0.088		
	(0.11)			(0.11)			(0.11)			(0.11)		
π_{t-7}	0.121			0.104			0.116			0.106		
	(0.09)			(0.09)			(0.09)			(0.09)		
π^e_{t-1}	0.525***			0.537***			0.492***			0.496***		
	(0.07)			(0.07)			(0.07)			(0.07)		
dummy 2020q2		0.000	17.482***		0.000	17.097***		0.000	15.909***		0.000	16.085***
		(.)	(1.31)		(.)	(1.29)		(.)	(1.40)		(.)	(1.36)
dummy 2020q3		0.000	–4.414***		0.000	–4.465***		0.000	–4.692***		0.000	–4.621***
		(.)	(1.26)		(.)	(1.23)		(.)	(1.26)		(.)	(1.23)
_cons	0.766***			–0.059				0.879***	–0.304		–0.180	–0.046
	(0.09)			(0.27)				(0.12)	(0.29)		(0.32)	(0.51)
N	230.000			230.000			230.000			230.000		
RMSE	1.209			1.184			1.204			1.176		
SSR	312.994			298.407			308.669			291.781		
AIC	100.9			91.9			99.6			90.7		
BIC	152.4			146.9			154.7			152.6		
Threshold	1.3756			1.3756			0.9013			0.7581		

Notes: *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level. The threshold variable is the unemployment gap $(U - U^{*})$.Region 1 corresponds to the portion of the sample in which the unemployment gap is less than or equal to the threshold. Region 2 corresponds to the portion of the sample in which the unemployment gap is greater than the threshold. Threshold models A and B differ for the inclusion or otherwise of the trend productivity growth. Results from each model are reported in three columns: columns (1) and (4) reported the coefficients of region-invariant variables (inflation expectations and lagged inflation on which the constraint is imposed); the other columns report estimated coefficients of region-varying variables. Variables are defined as described in the Note to Table 1.

large when using CPI. The slope is now statistically significant even above the threshold. Moreover, the coefficient of inflation expectations increases from 0.5 (core PCE) to 0.62 (headline PCE) to 0.84 (CPI), while the role of inflation persistence correspondingly declines.

On the whole, nonlinearities are confirmed and the use of core PCE inflation appears to provide a lower bound to the estimated slope of the wage Phillips curve. In addition, the use of CPI shows the wage inflation as being more an expectational phenomenon than an inertial one suggesting, in turn, a more important role of the monetary authorities to contain wage-price spiral events.

To assess the extent of nonlinearity captured by the models, Table 3 reports the impact on wage inflation of positive and negative unemployment gaps of different magnitudes, using the specifications with trend productivity.

Table 3: The degree of nonlinearity
Percentage change in wage inflation due to various levels of unemployment gaps

	Linear	Piecewise linear	Log-transformed	Threshold
$U - U^* = -0.5$	0.21	0.43	0.12	0.34
$U - U^* = 0.5$	-0.21	-0.13	-0.11	-0.03
$U - U^* = -1.0$	0.42	0.93	0.26	0.69
$U - U^* = 1.0$	-0.42	-0.27	-0.21	-0.07
$U - U^* = -3.0$	1.26	2.79	1.06	2.07
$U - U^* = 3.0$	-1.26	-0.81	-0.55	-0.20

*Source:*Authors computation using Table 1 and 2

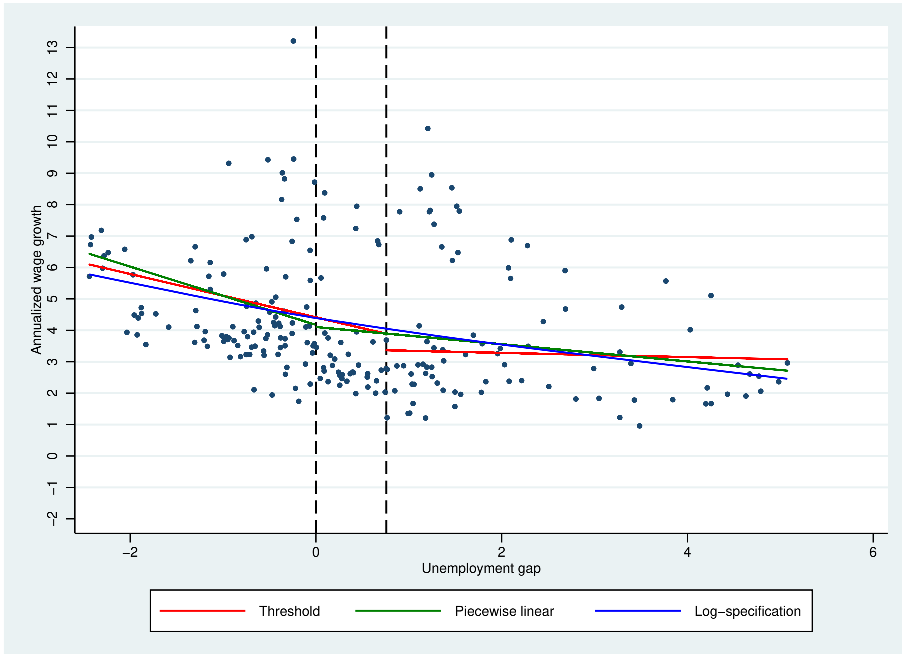
When the unemployment gap is positive, the linear model estimates a stronger decline in wage inflation than the nonlinear specifications; on the contrary, in tight labor markets, the linear model underpredicts the rise in wage inflation estimated by the piecewise linear model. Notice also that the degree of nonlinearity captured by the spline specification is greater than that captured by the log-transformed model.

Figure 4 finally compares the estimated Phillips curve in the three nonlinear models.

Three are the lessons that these econometric investigations deliver for the ensuing analysis. First of all, time-variation in the parameters is a robust finding and needs to be taken into consideration in order to catch the changes affecting the economic system. In the second place, the economics of nonlinearity cannot be neglected and this is going to impact the dynamics of the systems. Finally, the presence of piecewise linearity, steps and breaks favor the introduction of a regime-switching analysis.

In the following sections, we introduce a medium-scale macro-dynamic model and outline its features and functioning. We start by describing the structural properties of the regime-switching technique; next, we consider the wage-price subsystem and the

Figure 4: The estimated Phillips curve



Note: The intercepts are computed using the period averages of the variables included in the estimated regressions.

remaining model equations; finally, we discuss the working of the whole system for which we make use of numerical simulations.

3 The structural properties of the regime switching

The econometric results obtained in the case of the Phillips curve, and in particular the time-variation nature of its parameters, along with the presence of nonlinearities and breaks, stimulate the analysis of dynamics within a regime-switching environment.[13]

We consider a two-regime system. Regime 1 is the "stagnation state" characterized, inter alia, by low growth, high unemployment and low inflation, while Regime 2, the "expansionary state", is characterized by the opposite features. The two regimes also differ in terms of distributive properties and expectations.

The distributive properties strengthen the role of income distribution in generating the dynamics of the system. Specifically, in the following analysis Regime 1 is supposed to be wage-led, i.e., in steady state the output level is a decreasing function of the profit share,

[13] "Some forms of nonlinearities are topologically conjugate to regime switching models", which allows the use of the regime switching mathematical device to analytically underpin the nonlinearity of the Phillips curve (Ferri et al., 2001 page 24).

while Regime 2 is of a profit-led variety i.e., in steady state the output level is an increasing function of the profit share (Nishi and Stockhammer, 2020). The intuition of this is that in stagnation, high unemployment and low wages reduce the wage share; aggregate consumption then declines because capitalists have a lower propensity to consume than workers. In the meantime, investment rises due to higher profit share. The assumption is that the latter effect dominates the former.[14] As it will be clear from the meta-model to be presented shortly, different constellations of real and monetary aspects impact the distribution regimes and the interaction between these factors play an important role in generating the overall dynamics.

As far as price expectations are concerned, they are assumed to be anchored at different levels, reflecting the credibility of monetary policy. This argument moves from the consideration that price expectations depend, inter alia, on the Central Bank credibility and that Central Bank credibility may strengthen or weaken across the cycle; as a result, price expectations are taken to be anchored at different levels in the two regimes.[15]

The two Regimes are separated by a threshold that, in the present case, is represented by the following expression:[16]

$$thresh = g_{01} + \Delta \tag{2}$$

Where g_{01} is the steady state rate of growth of Regime 1, while Δ stands for an increase to be specified below.

It follows that when the (lagged) actual rate of growth (g_{t-1}) is greater than the threshold, Regime 2 is prevailing and vice-versa. Notice that each regime has its own steady state and that when the threshold is crossed new parameters become operative. These are discussed in the section 8 (see also Appendix B.1 for details).

[14]Ferri (2022) shows that $\frac{dg_0}{d\omega_0}$ also depends on monetary aspects. The robustness Section 8.1 uses bifurcation diagrams to show how the growth rate changes with different values of the wage share in the two regimes.

[15]See Cristini and Ferri (2021) for a learning process that replaces this assumption.

[16]Different variables can be considered as thresholds (see Tramontana et al., 2010), and the threshold can be implemented in many ways (see Ferri, 2011). On the economic justification of the discontinuity, see Ferri and Greenberg (1989) who also investigate the role of the cost of living stressed by the institutional approach (see Douty (1975)).

4 Switching in the wage-price subsystem

We introduce the meta-model by first discussing the dynamic wage-price subsystem into which the wage equation is inserted. This subsystem is crucial for at least two reasons.

(i) It leads to a structural definition of the income distribution, via the wage share, thus linking the Phillips curve loops and the so-called distributive loops. In this respect, the wage-price subsystem is reminiscent of the Goodwin tradition, for which wage and price dynamics and the resulting variations in the income distribution are of central importance.[17]

(ii) Stability issues enter the analysis because a wage-price spiral can be generated; although these phenomena seem to belong to the period before the mid-1990s, they have become a source of concern after the Covid-19 pandemic because they could trigger an inflationary process.

The wage-price subsystem is composed of a wage and a price equation, a labor productivity equation and the wage share.

From here onwards the following definitions apply: the subscript j ($j = 1, 2$) defines the two regimes characterizing the model; subscript 0 indicates the steady state and superscript $*$ indicates exogenous variables.

$$g_{w,t} = (1 + \tau_t)\{1 + \beta_{wj}\pi_j^* + (1 - \beta_{wj})\pi_{t-1} - \sigma_{1j}(u_{t-1} - u_{0j})\} - 1 \tag{3}$$

$$\pi_t = \beta_{\pi j}\frac{(1 + g_{w,t})}{(1 + \tau_t)} + (1 - \beta_{\pi j})(1 + \pi_j^*) - \sigma_{2j}(1 - h_{t-1}) - 1 \tag{4}$$

$$\tau_t = \theta_0 + \theta_1\frac{i_{t-1}}{\nu_{t-1}} \tag{5}$$

$$\omega_t = \omega_{t-1}\frac{1 + g_{w,t}}{(1 + \pi_t)(1 + \tau_t)} \tag{6}$$

Equation 3 is the wage growth equation, where g_w is the rate of growth of nominal wages; the equation is in keeping with the specification of the previous section. u is the rate of unemployment, π is price inflation and τ is the rate of growth of labor productivity. The slope of the wage Phillips curve is captured by the coefficient σ_1, which can change across the two regimes, while price inflation affects the nominal wage growth partly through last period inflation (π_{t-1}) and partly through the target (exogenous) inflation rate (π^*), which is regime-dependent. The weight of the two components is governed by the coefficient β_w that can change across regimes. Similarly, the dynamics of price inflation (equation

[17] A similar approach is used, for example in Franke et al. (2006).

4) respond partly to wage inflation, net of productivity growth, and partly to the target exogenous inflation rate; again the coefficient that governs the response to these dynamics (β_π) can change across the two regimes. Note that inflation also depends on capacity utilization (h_t), which plays a role analogous to the unemployment gap in the wage growth equation. Both equations are well known in the literature (see e.g. (Franke et al., 2006; Flaschel and Krolzig, 2006). An equation defining the rate of growth of labor productivity (τ_t) has also been added, coherently with the central role it plays and its strong empirical support (equation 5). It depends on the investment ratio (i_t) and the capital-output ratio (ν_t). This equation is derived from a Leontief production function and it is Harrod neutral, i.e., it implies a labor augmenting technical change. It is in keeping with the Kaldorian-Verdoorn interpretation (see Kemp-Benedict, 2020 and Fazzari et al., 2020).

Given wage and price dynamics and labor productivity growth, all the elements necessary to define the labor share (ω_t) are present, as shown in equation (6). Thus, the sub-system is able to determine the impact of macroeconomic activity on the income distribution, i.e., the so-called income distribution curve. According to the canonical classification, the distribution regime can be either labor market led (LML) or goods market led (GML).[18]

The stability property of this sub-model is an essential part of the research, as both the presence of time-variation in the parameters and the possibility of a wage-price spiral can lead to runaway situations. To thwart these tendencies, two strategies are introduced. On the one hand, regime-switching can deal with changing parameters. On the other, incorporating the wage-price subsystem into a broader model allows for the presence of other feedbacks that can generate complex dynamics.

5 A meta-model in the medium run perspective

In what follows the wage-price subsystem is inserted into a Harrod-Minsky model (see Ferri, 2022). From an analytical point of view, the model refers to a monetary economy of production characterized by corporate debt. This framework captures a core aspect of the Minskyan theory, as the seeds of financial instability, though cast in the financial market, produce important effects because they interplay with real markets and investments in

[18]LML distribution regime refers to the case in which a rise in the output gap leads to a decrease of the profit share. By contrast, GML distribution regime refers to the case in which a rise in the output gap leads to an increase in the profit share (Nishi and Stockhammer, 2020 page 262). LML distribution is found in some Kaleckian-Post-Keynesian approaches, while GML distribution is also called classical case. See Kiefer and Rada (2015) for an empirical analysis.

particular. The model dynamics are driven by aggregate demand, while supply is endogenized. There are many feedbacks at work, while adjustments occur in both prices and quantities. It has been called "Harrod-Minsky" because it conjugates Harrod instability with Minskian financial problems, both of which are made more complex by the presence of a wage-price spiral and an endogenous income distribution.

From the methodological point of view, the model refers to a medium-run perspective (see Minsky, 1982), where the medium run is characterized by several features. It is longer than the canonical business cycle and it is long enough to allow investment to be both a source of aggregate demand and a vehicle for capital growth; moreover, dynamics cannot be assumed to be driven only by exogenous forces, but some endogenous mechanism must be sought.

The meta-model encompassing both regimes is expressed in intensive form, i.e., the variables are deflated by the last period level of output or by its monetary value in the case of nominal variables. The order in which the equations are presented reflects the recursive nature of the simulations. To avoid duplications, the wage-price sub-system (equations 3 to 6 are omitted and should be inserted between the equation 7 and equation 8).

The starting point is growth expectations (equation 7), which are based on a mix of adaptive (backward-looking) and prospective (forward-looking) expectations and focus on the steady state value of growth of the ruling regime.

The Taylor rule (equation 8) is then introduced; note that this is the original Taylor rule without a partial adjustment mechanism. Indeed, it is controversial whether interest rate smoothing should be included. Especially at quarterly frequencies, the observed persistence of the federal funds rate could be the result of serially correlated deviations from the Taylor rule due to special circumstances, such as credit crunches and financial crises. (Rudebusch, 2002). On the contrary, Coibion and Gorodnichenko (2012) conclude in favor of the interest smoothing explanation rather than the shock persistence, and similarly Castelnuovo (2003). In our case, the presence of the regime-switching device and the regime-dependent exogenous target interest rate (R^*) could absorb part of the disturbances that generate serial correlation. However, this is an issue that needs further investigation.

Equation (9) represents the debt ratio; this is fed by the excess of business investment over the profit share, corrected by the parameter μ_1 representing the rate of retention (the last two terms on the right-hand side), and by the past debt ratio including the debt service.

Next, the various components of aggregate demand, defined by equation (14), are introduced, starting with consumption. Consumption is defined by the nondurable component (equation 10) and the durable component (equation 11), which is specified as a semiautonomous force. Nondurable consumption depends on both income distribution, through the labor share ω, and debt. In particular, the specification of equation 10 captures two important stylized facts of the Great Recession period: the centrality of household debt, which increased steadily from the previous period, and income inequality. Therefore, nondurable consumption considers heterogeneous households characterized by different propensities to consume and different incomes. For the share of households earning labor income, consumption is a function of the wage share and loans or credit ($R_t d_t$); for profit income earners, consumption is a fraction of the profit share remaining after investments and lending. Note that the two propensities to consume differ across the two regimes.

Investment is determined à la Minsky (13) and it is supplemented by an adjustment process. In this equation, investment, in addition to its lagged value, depends on the expected rate of growth and on the cash flow in real terms; this is defined as the profit share after consumption ($\mu_1(1-\omega)$) minus debt service, measured by $R_t d_t$, where R_t is the nominal rate of interest (it appears in this expression because interest payments are nominally fixed in a monetary economy). Note that the weights of these three components, captured by the parameters η_1, η_2 and η_3 can change during the phases of the cycle. The capital/output ratio is defined in a standard way in equation 12, where δ is the depreciation rate. Labor market variables are determined by labor demand (equation 18), labor supply (equation 17) and unemployment (equation 19). Capacity utilization (equation 16) is determined by the rate of growth of aggregate demand (equation 14) and the rate of growth of capital (equation 15). The so-called natural rate of growth is determined by equation 20.

$$Eg_t = \alpha g_{0j} + (1-\alpha) g_{t-1} \tag{7}$$

$$R_t = R_j^* + \gamma_1(\pi_t - \pi_j^*) + \gamma_2(Eg_t - g_{t-1}^s) \tag{8}$$

$$d_t = \frac{(1+R_{t-1})}{(1+\pi_t)(1+g_{t-1})} d_{t-1} + \frac{i_{t-1}}{(1+g_{t-1})} - \mu_1(1-\omega_{t-1}) \tag{9}$$

$$c_t = c_{wj}(\omega_{t-1} + R_t d_t) + c_{prj}\Big[(1-\omega_{t-1})(1-\mu_1) - R_t d_t\Big] \tag{10}$$

$$f_t = f_{0j}\Big[(1+\kappa_j(u_{t-1} - u_{0j})\Big] \tag{11}$$

$$\nu_t = \frac{\nu_{t-1}(1-\delta)}{(1+g_{t-1})} + \frac{i_{t-1}}{(1+g_{t-1})} \tag{12}$$

$$i_t = (1-\lambda_1) i_{t-1} + \lambda_1\Big[\eta_{1j}(1+Eg_t) + \eta_{2j}\mu_1(1-\omega_{t-1}) - \eta_{j3} R_t d_t\Big] \tag{13}$$

$$g_t = c_t + f_t + i_t - 1 \tag{14}$$

$$g_{k,t} = \frac{i_{t-1}}{\nu_{t-1}} - \delta \tag{15}$$

$$h_t = h_{t-1}\frac{(1+g_t)}{(1+g_{k,t})} \tag{16}$$

$$g_{Ls,t} = \rho_{0j} - \rho_1 u_{t-1} \tag{17}$$

$$g_{N,t} = \frac{(1+g_t)}{(1+\tau_t)} - 1 \tag{18}$$

$$u_t = 1 - (1-u_{t-1})\frac{(1+g_{N,t})}{(1+g_{Ls,t})} \tag{19}$$

$$g_{s,t} = (1+\tau_t)(1+g_{Ls,t}) - 1 \tag{20}$$

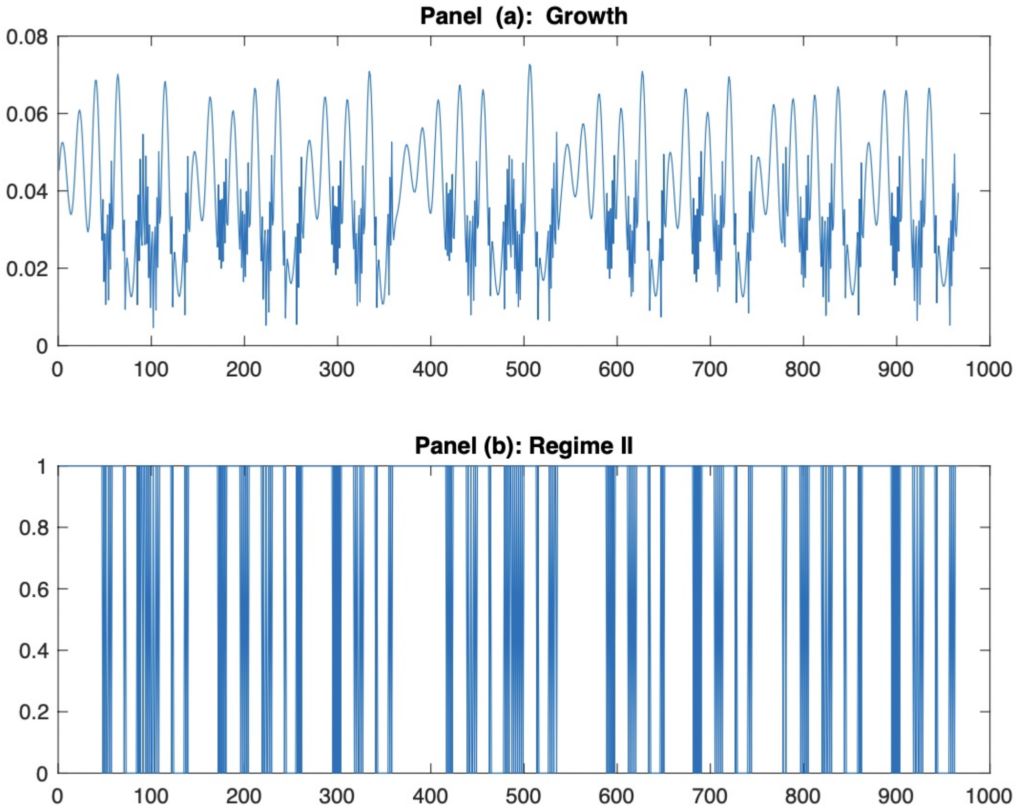
Each system $j = 1, 2$, including the wage-price subsystem, contains 18 equations and 18 unknowns. The parameters determine the values of the steady states, shown in Table A5, and influence the dynamics of the model. The parameters are entered as shown in Table A6. Steady state values and calibration are discussed in Section 8.

6 The overall dynamics

The simulated dynamics are set in motion by a positive shock (0.001) to g_{02} lasting one period.[19] The ensuing dynamics are depicted in Figure 5

[19]As long as the shock leaves the system in the basin of attraction of the invariant set located around the equilibrium, the variable to which we apply the shock and its size do not affect the qualitative results we are going to show. Only by adopting a huge shock, there is the risk to end up in the divergence's basin of attraction. We will avoid this possibility.

Figure 5: The overall dynamics



Note: In panel (a) the dynamics of the growth immediately after the shock. We represent 1000 iterations. In panel (b) the blue stripes characterize iterations where the dynamical system falls in Regime 2. As a consequence, the regions in white characterize iterations corresponding to Regime 1.

Two aspects are worth considering. First, the dynamics remain bounded and persistent (in Figure 5 n=1000). This result is somewhat similar to Takahashi and Okada (2020), although it does not refer to an agent-based model (ABM). In fact, the overall system keeps fluctuating in spite of the presence of a monetary policy. The differences are analytical and relate to their use of a Wicksell-Keynes model. According to this view, the economy changes when full employment is reached, so that a Wicksell economy, based on price adjustment, takes place. In this situation, a profit-led model becomes dominant. In our approach, on the contrary, price and quantity adjustments are always present, even though they are different in the two regimes.

Second, it should be stressed that the two Regimes are characterized not only by different dynamics (ex-hypothesis) but also by different lengths. In the present case, Regime 2 covers about 75% of the cases, i.e., in 75% of the iterations the dynamic system falls in Regime 2. In this environment, the average rate of growth is equal to 0.0378. Two observations are important at this stage of the analysis: (i) the average rate of growth depends on the values of the parameters and the choice of the threshold, which in the present case is equal to 0.0274, i.e., equal to $g_{01} + \Delta$, where the latter increment has been set to 0.005; (ii) the average rate of growth is included in an interval marked by g_{01} and g_{02}. These values, however, cannot measure the actual rate of growth, which is to be considered as a dynamic process that takes place in a disequilibrium environment. This circumstance is bound to interfere with those interpretations that insist on using steady states as benchmarks. This is usually the case in the debate on income distribution.

7 Generating the loops

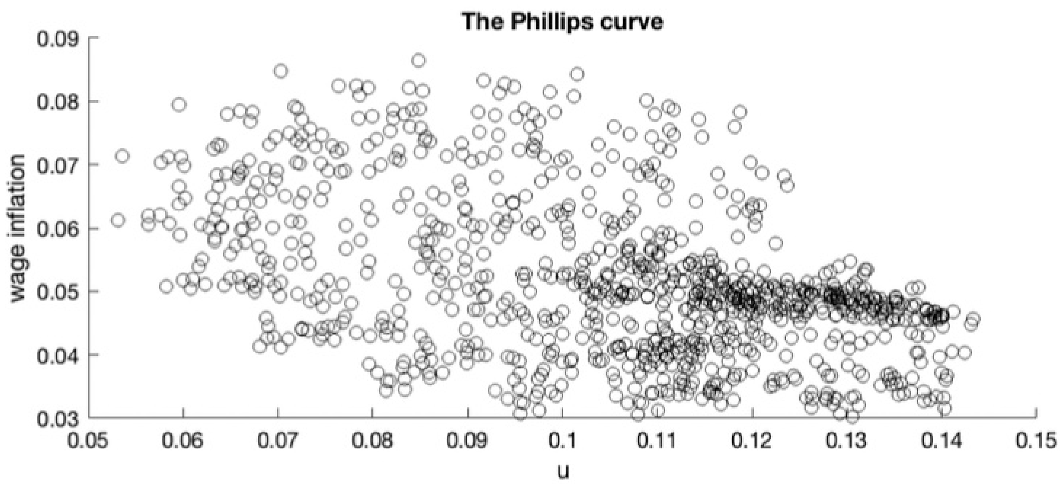
From the bounded and persistent fluctuations generated by the meta-model is relatively straightforward to obtain loops in the Phillips curve, a well-known phenomenon studied in the literature. However, it is also possible to generate distributive loops, which are probably less well-known. They are linked to the previous ones and have strong empirical support (see for instance Barrales-Ruiz et al., 2022 and Chiarella et al., 2021). For this purpose, it is important to refer to Figure 6, which shows three phase diagrams generating loops in the canonical Phillips curve (top panel), in the price Phillips curve (middle panel) and in the distribution sphere (bottom panel).

The presence of loops in the Phillips curve was detected by Phillips himself and in his 1958 paper he implicitly attributed them to the role of import prices.[20] Successively, the loops were mainly justified by the presence of hysteresis in the Phillips curve (see Layard et al., 1991). The loops depicted in Figure 6 are, quite differently, the result of the working of the whole system. Likewise, the presence of two steady state rates of unemployment reflects the working of the whole system and not only that of the labor market; in this sense, they are different from the concept of NAIRU. Finally, the income distribution loop is generated in keeping with empirical analysis (see Barrales-Ruiz et al. (2022)).

The longer-term features of the loops can be represented by the long-term correlations reported in Table 4.

[20]In his 1958 paper, Phillips discusses the devaluation of sterling in September 1949 and the outbreak of the Korean War in 1950 as causes of the increase of retail prices and argues that the rapid rise in import prices can offset the tendency for increasing productivity to reduce the cost of living (Phillips(1958) page 298-299).


u and the wage share
omega
u


The Phillips curve
wage inflation
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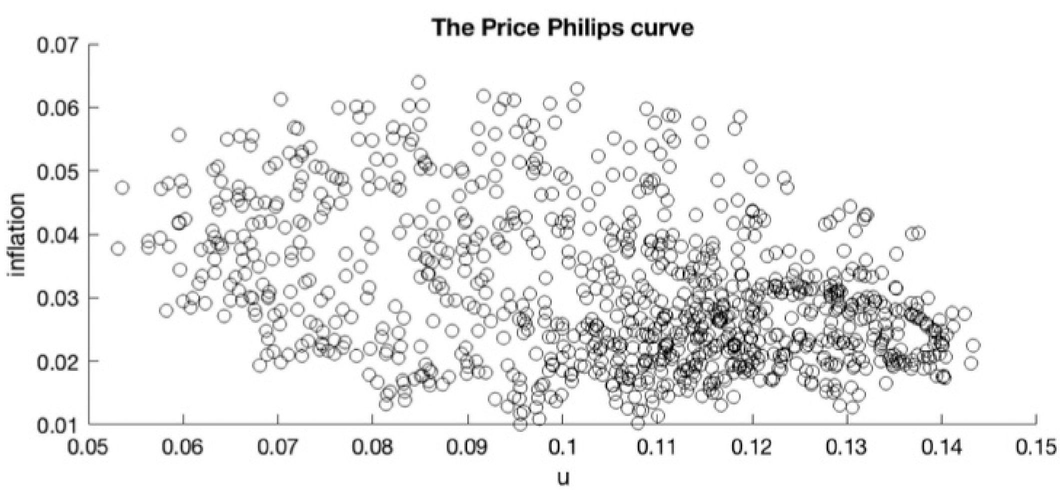
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u

Table 4: The long-term correlations

Variables	Coefficients of correlation
(1) $u - g_w$	-0.4553
(2) $u - \pi$	-0.3710
(3) $u - \omega$	0.1449

Source: Authors computation

The long-term correlations indicate that the Phillips curve is downward-sloping (row 1 in Table 4). The same happens to the price Phillips curve (row 2). More difficult, however, is to find a stylized fact referring to the distributional loop (row 3). In a theoretical model, it depends on both the equations determining the feedback loop and the property of the overall system. In this case, it is compatible with a profit-led situation (Kiefer and Rada, 2015 find a similar result empirically).

8 Validity of the parameter values and robustness of the dynamics

In this section, we evaluate the validity of the calibration and test the robustness of the dynamics to changes in the parameter values. For the latter, we rely on bifurcation diagrams, while for the former we compare the evidence and calibrations in the existing literature with the parameter values used in this paper (Table A7).[21]

We comment here only on the most important results and leave the rest to the Appendix C. Starting from the Taylor rule, the calibration of the parameter on the activity variable (γ_2) is in line with the existing literature, while the value of the inflation parameter (γ_1) is greater. Recently, Schmitt-Grohé and Uribe (2022) propose, e.g., γ_1= 1.5 and γ_2= 0.125, in line with most of the literature. Values of γ_1 close to 2 have been estimated when making use of surveys of professional forecasts (Castelnuovo, 2003) and, similarly, the posterior mode values estimated by Smets and Wouters (2007) also imply a strong policy response to inflation (2.03). On this aspect, the robustness analysis presented in the Appendix C adds an important point. The bifurcation diagram presented in Figure A8 shows that when γ_1 decreases, the system becomes stable, whereas fluctuations enter

[21] We refer to a few papers, each of which considers a part of the macro-model proposed here. In particular, Franke et al. (2006) focus on the wage-price subsystem and on the monetary policy rule, but ignore aggregate demand, aggregate supply and the labor market. In contrast, Fazzari et al. (2020) and Fazzari and González (2023) consider aggregate demand and aggregate supply, but omit the wage-price subsystem and the monetary policy rule. For the latter, we consider the results of an extensive literature cited in particular by Rudebusch (2002), Castelnuovo (2003) and Carvalho et al. (2021).

the picture as the parameter value increases. On the contrary, by increasing γ_2 one passes from chaos to a periodic cycle. These are important results suggesting that the stabilising role of monetary policy depends on how strongly it responds to deviations of inflation from the target. Indeed, a too-strong reaction may trigger increasing fluctuations.

The values of the parameters in the wage and in the price Phillips curves are assessed using the calibration carried out by Franke et al. (2006) to model the wage-price spiral. Although their framework differs somewhat from ours,[22] their calibration of β_w and β_p is in line with our values,[23] while the reactions of price and wage inflation to the economic activity are slightly different.[24] For the values of the remaining equations, we resort to the works of Fazzari et al. (2020) and Fazzari and González (2023). The coefficient on the lagged rate of unemployment in the supply growth equation (ρ_1), which we set to 0.5, is slightly larger than the values, ranging between 0.1 and 0.3, that Fazzari et al. (2020) estimate. In the labor productivity growth, θ_1, the coefficient on the investment rate, which we set to 0.15, is close to the minimum value used by Fazzari and González (2023) (0.19). As far as the values of the two marginal propensities to consume are concerned, the parameters are comprised between the minimum and the maximum values estimated by Fazzari and González (2023) and Fazzari et al. (2020) for the whole economy. Finally, the speed of adjustment of the investment ratio (λ) and the adjustment speed of expected growth $(1-\alpha)$ also fall in between the values estimated in the two papers.

On the whole, we conclude that the values of the parameters are never too far from those in the literature.

To further validate the calibration, Table A8 compares the steady state values derived from the parameters and listed in Table A5, with the actual average values of the variables of interest. The average values and standard deviations are reported for the period 1960 to 2023 and three 20-year subperiods. In addition, Table A9 reports the average values in expansions and recessions by classifying each year as recessionary or expansionary based on the GDP-based recession indicator available since 1967 (source: FRED). Although the 60-year period considered in Table A8 is probably too long and blurs the medium-run

[22] For example, they include the deviation of the wage share from its steady state as an additional factor affecting both price and wage inflation. They also introduce a variable that captures a general inflation climate in the economy, with the central bank's exogenous inflation target playing a role. Another difference is that they measure activity using the employment gap in the wage equation and the output gap in the inflation equation.

[23] They set β_w to 0.225 and β_p to 0.29. In this paper β_w is either 0.2 or 0.6 and β_p 0.1. or 0.9, depending on the regime.

[24] They set σ_1, to 0 versus 0.3 or 0.4 in our case. The lower value can be partly justified by the presence, in their framework, of other variables that capture the labor market tightness. σ_2, on the contrary, is slightly higher than in our case (0.5 versus 0.2 or 0.25, depending on the regime).

changes that are at the core of the theoretical model, especially as far as GDP growth and unemployment are concerned, the steady state values of inflation, interest rates and the private debt ratio are consistent with the data.

8.1 Robustness of the dynamics

The emergence of wage-price spirals results from both the nonlinearity of the system and the regime-switching device. The former generates bounded fluctuations when the equilibrium becomes unstable due to a change in a parameter. A period-doubling or a Neimark-Sacher bifurcation leads to periodic or quasi-periodic trajectories, and can even lead to a chaotic motion with aperiodic and unpredictable fluctuations. The regime-switching mechanism amplifies this feature of the dynamics: when the trajectory crosses the threshold and the regime switches, the new parameter values further accentuate fluctuations, making them even more unpredictable.

To study the role of the different parameters in the stability of the system, we rely on bifurcation diagrams, a typical tool in the qualitative theory of dynamical systems. They are presented and commented on in the Appendix C.

While the robustness tests confirm the overall presence of bounded and persistent fluctuations, they also highlight some specific results. In particular, the monetary policy response to off-target inflation can increase instability if it is too high; labour supply, which reconciles the Harrodian warranted rate of growth with its corresponding natural rate, can also produce increasing fluctuations if its (constant) growth is too low in the bad regime or too high in the good regime. Labor productivity growth can bring the system to a steady state of high growth as long as its constant growth is sufficient and its response to the investment rate is not too high. Moreover, the effects of parameter changes in the wage-price subsystem differ in the two regimes, underscoring the role of regime-switching. Finally, note that all robustness exercises depend very much on the dynamic characterization of the two regimes. Indeed, they differ not only by steady states, but also by the nature of the disequilibrium processes that characterize the wage- or profit-led nature of their distribution loops, in keeping with the theoretical analysis of the present model, where real and monetary aspects interact with each other.

9 Concluding remarks

The present paper has reconsidered the flattening of the Phillips curve along with the recent claim that a wage-price spiral might ignite an inflationary process. The two aspects are related. Indeed, the time-variation of parameters is an essential part of the instability process that can influence inflationary episodes. By using a macro meta-model, the regime-switching methodology, particularly studied by Dick Day, has helped reach two main results. On the one hand, it has generated robust bounded fluctuations where the loops in the Phillips curve are present and where the so-called distributive loops also occur. On the other hand, it has broadened the perspective of the latter since both the wage-led and profit-led varieties may be present in the same model, so that complex dynamics can arise. The analysis can be extended in several directions.

First, even though the model has obtained bounded and persistent fluctuations, more effort should be made ti account for instability, both in the upward and downward directions. Regarding the former, efforts should be made to address the challenges posed by increasing and persistent inflation, which has been subdued in the recent past but is re-emerging today. In this perspective, the feedback from income distribution to inflation should also be considered (Blanchard and Katz, 1999 and Flaschel and Krolzig, 2006). As far as downward instability is concerned, the role of fiscal policy cannot be ignored.

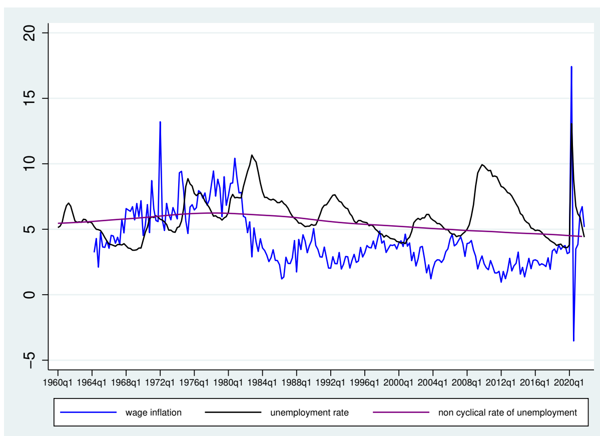
Second, empirical analysis using VAR models should be extended to study the system dimension of the various outcomes, especially the Phillips loops and the distributional loops obtained in the present paper. VAR analyses could also be used to find further support for the calibration and investigate the hypothesis, put forward in this paper, that the wage-led and the profit-led distribution cycles may be different phenomena that do not refer to different economic systems but concern different states of the same economy.

Finally, methodological deepening is necessary. The reference to a regime-switching technique has underlined the disequilibrium nature of the growth process. In this context, steady state values lose their attractiveness, while the average of the actual growth rate becomes a more reliable measure. In this context, the so-called income distribution debate also needs to be reconsidered, not only because it incorporates real as well as monetary aspects, but also because it has to be inserted into a dynamic process in which steady state values might become an insufficient benchmark.

A Econometric Appendix

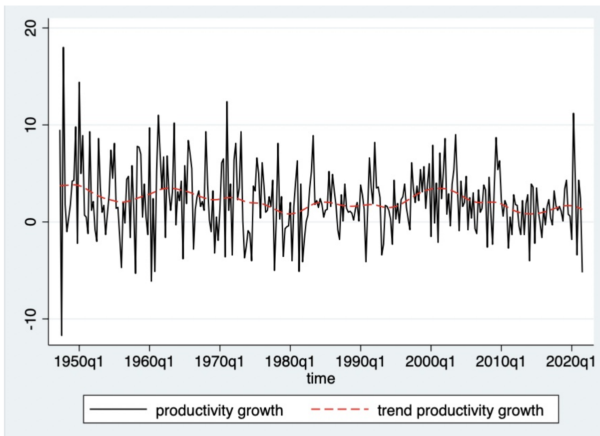
The following figures and tables are presented below. Figure A1 illustrates the main variables of the model, Figure A2 shows productivity growth and Figure A3 illustrates the backward reconstruction of the Michigan consumer survey expectations. Table A1 and Table A3 compare, respectively for the linear, piecewise linear, log-linear models and for the threshold models, the main regression results using various measures of price inflation. Table A2 check the robustness of the linear, the piecewise linear and the log-transformed specifications to the inclusion of two lags of wage inflation. Table A4 presents the overall results of the threshold models and shows that the two-threshold model is optimal, according to the diagnostic test battery, when trend productivity growth is omitted.

Figure A1: The main variables of the model. 1960q1-2021q3



Note: wage inflation is the annualized log change of Hourly Earnings for Production and Nonsupervisory Employees, Total Private (FRED). The rate of unemployment and the non-cyclical (previously called natural) rate of unemployment (long term), not seasonally adjusted are both from FRED.

Figure A2: Productivity growth and estimated trend



Trend productivity growth is the growth in real output per hour in the non-farm business sector smoothed using the HP-filtered trend, as done for example by Hooper et al., 2019.

Figure A3: Backward re-construction of Michigan consumer survey

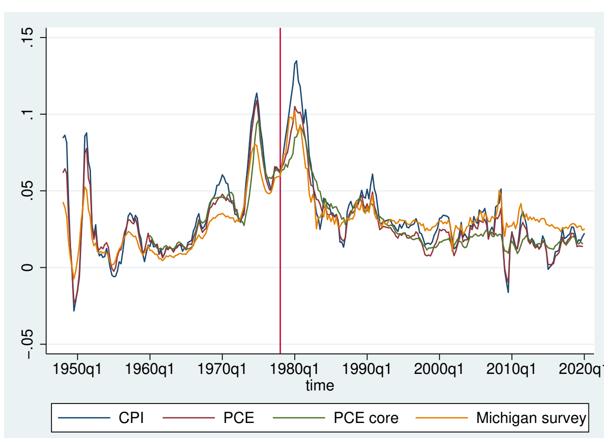


Table A1: Wage Phillips curve: linear, piecewise linear and convex. PCE and CPI

1964q1 - 2021q3

	Linear			Piecewise linear			Convex		
	(1) core PCE	(2) PCE	(3) CPI	(4) core PCE	(5) PCE	(6) CPI	(7) core PCE	(8) PCE	(9) CPI
$(U-U*)$	–0.422***	–0.443***	–0.508***	–0.927***	–0.974***	–1.095***			
	(0.06)	(0.06)	(0.06)	(0.16)	(0.16)	(0.16)			
$(U-U*)^+$				0.654***	0.691***	0.764***			
				(0.19)	(0.20)	(0.20)			
$log(U/U*)$							–2.678***	–2.824***	–3.250***
							(0.36)	(0.38)	(0.39)
π_{t-1}	0.307***			0.271***			0.292***		
	(0.10)			(0.10)			(0.10)		
π_{t-2}	–0.159			–0.172			–0.173		
	(0.11)			(0.10)			(0.11)		
π_{t-3}	0.008			0.006			0.003		
	(0.11)			(0.11)			(0.11)		
π_{t-4}	0.035			0.028			0.030		
	(0.11)			(0.11)			(0.11)		
π_{t-5}	0.115			0.116			0.116		
	(0.11)			(0.11)			(0.11)		
π_{t-6}	0.069			0.083			0.073		
	(0.11)			(0.11)			(0.11)		
π_{t-7}	0.097			0.105			0.096		
	(0.09)			(0.09)			(0.09)		
π_{Ht-1}		0.117*			0.095			0.105	
		(0.06)			(0.06)			(0.06)	
π_{Ht-3}		–0.027			–0.030			–0.033	
		(0.07)			(0.07)			(0.07)	
π_{Ht-3}		0.054			0.054			0.051	
		(0.07)			(0.07)			(0.07)	
π_{Ht-4}		–0.076			–0.079			–0.078	
		(0.07)			(0.07)			(0.07)	
π_{Ht-5}		0.089			0.084			0.085	
		(0.07)			(0.07)			(0.07)	
π_{Ht-6}		0.078			0.080			0.077	
		(0.07)			(0.07)			(0.07)	
π_{Ht-7}		0.146**			0.147**			0.141**	
		(0.06)			(0.06)			(0.06)	
π_{CPIt-1}			–0.016			–0.029			–0.024
			(0.05)			(0.05)			(0.05)
π_{CPIt-2}			–0.056			–0.057			–0.060
			(0.05)			(0.05)			(0.05)
π_{CPIt-3}			0.023			0.021			0.019
			(0.05)			(0.05)			(0.05)
π_{CPIt-4}			–0.063			–0.062			–0.064
			(0.05)			(0.05)			(0.05)
π_{CPIt-5}			0.062			0.057			0.057
			(0.05)			(0.05)			(0.05)
π_{CPIt-6}			0.049			0.051			0.047
			(0.05)			(0.05)			(0.05)
π_{CPIt-7}			0.164***			0.165***			0.159***
			(0.05)			(0.04)			(0.04)
π^e_{t-1}	0.527***	0.619***	0.837***	0.564***	0.648***	0.855***	0.563***	0.653***	0.866***
	(0.07)	(0.08)	(0.07)	(0.07)	(0.08)	(0.06)	(0.07)	(0.08)	(0.06)
dummy 2020q2	18.187***	18.578***	18.988***	17.190***	17.493***	17.788***	17.455***	17.811***	18.118***
	(1.31)	(1.36)	(1.39)	(1.31)	(1.36)	(1.38)	(1.27)	(1.31)	(1.33)
dummy 2020q3	–3.960***	–4.255***	–4.711***	–4.427***	–4.739***	–5.227***	–3.997***	–4.292***	–4.740***
	(1.27)	(1.31)	(1.33)	(1.25)	(1.29)	(1.30)	(1.26)	(1.30)	(1.31)
trend productivity growth	0.502***	0.505***	0.472***	0.408***	0.406***	0.359***	0.459***	0.456***	0.411***
	(0.13)	(0.13)	(0.14)	(0.13)	(0.13)	(0.14)	(0.13)	(0.13)	(0.14)
constant	–0.095	–0.133	–0.155	–0.240	–0.287	–0.306	–0.081	–0.108	–0.099
	(0.28)	(0.29)	(0.30)	(0.28)	(0.29)	(0.29)	(0.28)	(0.29)	(0.30)
N	230.000	230.000	230.000	230.000	230.000	230.000	230.000	230.000	230.000
RMSE	1.224	1.269	1.285	1.195	1.237	1.245	1.216	1.257	1.267
BIC	798.8	815.3	820.8	792.0	807.9	810.7	795.5	811.0	814.6
AIC	757.5	774.1	779.6	747.4	763.2	766.0	754.2	769.8	773.3

Notes: *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level. All variables are defined as described in the Note to Table 1, except for π_H, which defines annualized (headline) PCE inflation and π_{CPI}, which defines annualized CPI inflation.

Table A2: Linear, Piecewise linear and convex. Adding wage inflation lags

	linear	piecewise linear	log-transformed
$(U - U*)$	–0.376***	–0.890***	
	(0.06)	(0.16)	
$(U - U*)^+$		0.663***	
		(0.19)	
$log(U/U*)$			–2.406***
			(0.36)
π_{t-1}	0.234**	0.199**	0.223**
	(0.10)	(0.10)	(0.10)
π_{t-2}	–0.216**	–0.232**	–0.227**
	(0.11)	(0.11)	(0.11)
π_{t-3}	–0.001	–0.001	–0.006
	(0.11)	(0.11)	(0.11)
π_{t-4}	0.026	0.017	0.022
	(0.11)	(0.11)	(0.11)
π_{t-5}	0.115	0.115	0.116
	(0.11)	(0.11)	(0.11)
π_{t-6}	0.067	0.083	0.071
	(0.11)	(0.11)	(0.11)
π_{t-7}	0.127	0.134	0.125
	(0.09)	(0.09)	(0.09)
π^e_{t-1}	0.648***	0.684***	0.677***
	(0.08)	(0.08)	(0.08)
dummy 2020q2	17.916***	16.910***	17.276***
	(1.29)	(1.29)	(1.25)
dummy 2020q3	–5.900***	–6.392***	–5.850***
	(1.49)	(1.46)	(1.48)
L.wage infl	0.116**	0.118**	0.111**
	(0.05)	(0.05)	(0.05)
L2.wage infl	0.034	0.027	0.032
	(0.05)	(0.05)	(0.05)
trend productivity growth	0.560***	0.456***	0.515***
	(0.13)	(0.13)	(0.13)
constant	–0.888**	–1.003***	–0.836**
	(0.39)	(0.38)	(0.38)
N	228.000	230.000	228.000
RMSE	1.204	1.1729	1.196
AIC	745.2102	734.257	742.2911
BIC	793.2211	785.6972	790.3019

Notes: *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level. All variables are defined as described in the Note to Table 1, except for L.winf and L2.winfl, which define the lagged dependent variable (from t-1 to t-2).

Table A3: Wage Phillips curve: Threshold model. Different prices

1964q1 — 2021q3

	core PCE			PCE			CPI		
	(1) w	(2) Region 1	(3) Region 2	(4) w	(5) Region 1	(6) Region 2	(7) w	(8) Region 1	(9) Region 2
$(U - U*)$		–0.690***	–0.067		–0.662***	–0.179*		–0.960***	–0.295**
		(0.14)	(0.10)		(0.18)	(0.09)		(0.12)	(0.14)
π_{t-1}	0.280***								
	(0.09)								
π_{t-2}	–0.160								
	(0.10)								
π_{t-3}	0.030								
	(0.10)								
π_{t-4}	0.041								
	(0.11)								
π_{t-5}	0.119								
	(0.11)								
π_{t-6}	0.088								
	(0.11)								
π_{t-7}	0.106								
	(0.09)								
π_{Ht-1}				0.107*					
				(0.06)					
π_{Ht-2}				–0.027					
				(0.07)					
π_{Ht-3}				0.062					
				(0.07)					
π_{Ht-4}				–0.076					
				(0.07)					
π_{Ht-5}				0.086					
				(0.07)					
π_{Ht-6}				0.078					
				(0.07)					
π_{Ht-7}				0.150***					
				(0.06)					
π_{CPIt-1}							–0.032		
							(0.05)		
π_{CPIt-2}							–0.056		
							(0.05)		
π_{CPIt-3}							0.028		
							(0.05)		
π_{CPIt-4}							–0.053		
							(0.05)		
π_{CPIt-5}							0.058		
							(0.05)		
π_{CPIt-6}							0.058		
							(0.05)		
π_{CPIt-7}							0.157***		
							(0.04)		
π^e_{t-1}	0.496***			0.620***			0.841***		
	(0.07)			(0.08)			(0.07)		
trend productivity growth		0.520***	–0.087		0.659***	0.135		0.346**	0.477
		(0.14)	(0.25)		(0.18)	(0.19)		(0.14)	(0.43)
dummy 2020q2		0.000	16.085***		0.000	16.917***		0.000	17.483***
		(.)	(1.36)		(.)	(1.39)		(.)	(1.50)
dummy 2020q3		0.000	–4.621***		0.000	–4.842***		0.000	–5.374***
		(.)	(1.23)		(.)	(1.27)		(.)	(1.30)
constant		–0.180	–0.046		–0.489	–0.128		–0.082	–0.507
		(0.32)	(0.51)		(0.38)	(0.42)		(0.31)	(0.75)
Threshold	0.7581			0.2676			1.2516		
N	230.000			230.000			230.000		
SSR	291.781			317.845			325.871		
RMSE	1.176			1.227			1.243		
BIC	152.6			172.3			178.0		
AIC	90.7			110.4			116.1		

Notes: *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level. Variables are defined as described in the Note to Table 1 , except for π_H, which defines annualized (headline) PCE inflation and π_{CPI}, which defines annualized CPI inflation. The threshold variable is the unemployment gap $(U - U^*)$. Region 1 corresponds to the portion of the sample in which the unemployment gap is less than or equal to the threshold. Region 2 corresponds to the portion of the sample in which the unemployment gap is greater than the threshold. For each price measure used (core PCE in columns 1-3, headline PCE in columns 4-6 and CPI in columns 7-9), the estimated coefficients of the region-invariant variables (inflation expectations and lagged inflation on which the constraint is imposed) are reported in the initial column; the other two columns report the estimated coefficients of region-varying variables.

Table A4: Threshold models: comparison of two vs. three regions

Variable	Two regions	Three regions	Two regions with productivity growth	Three regions with productivity growth
	(1)	(2)	(3)	(4)
thresholds	0.9013	0.9013	0.7581	-0.6549
		1.5436		-0.0144
lagged inflation				
$\pi t-1$	0.25770639	0.22668938	0.28049145	0.29071615
$\pi t-2$	-0.15919777	-0.1416952	-0.15951841	-0.18704303
$\pi t-3$	0.02697468	0.02064223	0.02958949	0.00287497
$\pi t-4$	0.06118654	0.11617179	0.04116006	0.03125193
$\pi t-5$	0.11295463	0.09940257	0.11904962	0.11865156
$\pi t-6$	0.09258146	0.09929484	0.08817429	0.07267901
$\pi t-7$	0.11620247	0.07170121	0.10553224	0.10531969
Expected inflation	0.4915916	0.50779319	0.49552126	0.56554972
Region1				
$(U-U^*)$	-0.84466177	-0.84251063	-0.68988435	-0.82473585
dummy 2020q2	0	0	0	0
dummy 2020q3	0	0	0	0
trend productivity growth			0.51978242	0.16070608
constant	0.87908897	0.87496524	-0.18031954	0.52636114
Region2				
$(U-U^*)$	-0.03393043	2.6148322	-0.06700931	0.66276502
dummy 2020q2	15.908986	0	16.084825	0
dummy 2020q3	-4.691514	0	-4.6209312	0
trend productivity growth			-0.08653702	1.2169406
constant	-0.30441796	-3.5025102	-0.0460338	-1.0028018
Region3				
$(U-U^*)$		0.04846857		-0.19308667
dummy 2020q2		15.476718	.	16.715391
dummy 2020q3		-4.789107		-4.5261139
trend productivity growth				0.13241535
constant		-0.64170814		-0.01103455
Statistics				
SSR	308.66913	301.78229	291.78093	335.45618
AIC	99.663846	102.4741.	90.722517	132.80476
BIC	154.67311	171.23569	152.60794	211.88059
HQIC	121.85348	130.21114	115.68585	164.70236

Notes: *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level. The sum of lagged inflation and expected inflation is constrained to unity. Dependent variable is the annualized log change of Hourly Earnings for Production and Nonsupervisory Employees, Total Private (FRED). π is the annualized rate of growth of the price index of Personal Consumption Expenditures Excluding Food and Energy(FRED). Trend productivity growth is the growth in real output per hour in the non-farm business sector smoothed using the HP-filtered trend, as done for example by Hooper et al., 2019. Expected inflation π^e is one-year ahead inflation expectations from the Michigan consumers survey (MICH). U is the rate of unemployment (BLS) and U^* is the (time-variable) CBO non-cyclical (previously called natural) rate of unemployment (long term) not seasonally adjusted (FRED).

B Mathematical Appendix

There are two constraints in the meta-model. The first assumes that the nominal interest rate R_t never becomes negative, while the second assumes that the steady state rate of growth is positive. The latter assumption makes it easy to calculate the steady states for all variables. Multiple equilibria cannot occur. The steady state numerical values are shown in Table A5, and the parameters are shown in Table A6. A discussion of the steady state values and the validity of the parameter values is presented in Section 8 in the text and supported by Tables A8, A9 and A7 in this Appendix.

B.1 Steady states

We show the empirical steady state values underlying the simulations of Section 6.

Table A5: The steady state values in the two regimes

Description	Regime 1	Regime 2
Rate of growth	$g_{01} = 0.0224$	$g_{02} = 0.0522$
Rate of unemployment	$u_{01} = 0.1241$	$u_{02} = 0.0846$
Inflation	$\pi^*_{01} = 0.03$	$\pi^*_{02} = 0.04$
Interest rate	$R_{01} = 0.03$	$R_{02} = 0.04$
Debt ratio	$d_{01} = 0.8881$	$d_{02} = 1.7576$
Durable consumption ratio	$f_{01} = 0.235$	$f_{02} = 0.2380$
Investment ratio	$i_{01} = 0.2588$	$i_{02} = 0.2974$
Consumption ratio	$c_{01} = 0.5285$	$c_{02} = 0.5268$
Labor share	$\omega_{01} = 0.7250$	$\omega_{02} = 0.77$

Source: Authors' computation.

Table A6: The parameters of the model

Symbol	Equation in which they appear	Regime 1	Regime 2	Invariant
β_{wj}	Wage inflation: equation 3	0.9	0.1	
σ_{1j}	Wage inflation: equation 3	0.2	0.25	
$\beta_{\pi j}$	Price inflation: equation 4	0.2	0.6	
σ_{2j}	Price inflation: equation 4	0.4	0.3	
θ_0	Labor productivity: equation 5			0.001
θ_1	Labor productivity: equation 5			0.15
α	Expected rate of growth: equation 8			0.2
γ_1	Taylor rule: equation 9			2.5
γ_2	Taylor rule: equation 9			0.8
μ_1	Debt: equation 10			0.85
c_{wj}	Consumption: equation 11	0.695	0.63	
c_{prj}	Consumption: equation 11	0.432	0.35	
κ_j	Durable consumption: equation 12	0.2	0.3	
η_{1j}	Investment: equation 14	0.195	0.2	
η_{2j}	Investment: equation 14	0.3	0.75	
η_{3j}	Investment: equation 14	0.4	0.85	
λ_1	Investment: equation 14			0.2
ρ_{0j}	Labor supply: equation 18	0.065	0.07	
ρ_1	Labor supply: equation 18			0.5

Source: Authors' computation

Table A7: Validity of the calibration

	Fazzari and Gonzales (2023) Table 1	Fazzari et al. (2020) Table 1	Franke et al. (2006) p.459 and eq. 6, 7	Carvalho et al. (2021) Table 1 OLS	Rudebusch (2002)	Castelnuovo (2003) Table 3	Castelnuovo (2003) Table 7	This paper Table 7
Taylor rule								
γ_1			1.24	0.90 - 2.00	1.24	1.508	2.157	2.5
γ_2			0.33	0.75 - 1.01	0.33	0.864	0.867	0.8
Inflation equation								
β_p			(1-0.55)*0.5 = 0.225					0.2 - 0.6
σ_1			0					0.3 - 0.4
Phillips curve								
β_w			(1-0.42)*0.5 = 0.29					0.1 - 0.9
σ_2			0.51					0.2 - 0.25
Labor supply growth								
ρ_1	0.073 - 0.113	0.10 - 0.30	0.065 - 0.07					0.5
ρ_0	0.008 - 0.01							0.065 - 0.07
Labor productivity growth								
θ_0	0.015 - 0.02							0.002
θ_1	0.19 - 0.29	0.30 - 0.70						0.15
Consumption								
$1 - s$	0.559	0.4 - 0.8						
c_w								0.63 - 0.695
$c_p r$								0.35 - 0.432
Investment ratio								
λ (speed of adjustment)	0.003 - 0.008	0.05 - 0.20						0.2
Aggregate demand growth								
α	0.06	0.9						0.2

Source: Authors' analysis

Table A8: Average values of variables of interest

	1960-1980	1980-2000	2000-2023	1960-2023	Steady state Regime 1 (recessionary)	Steady state Regime 2 (expansionary)
Nr of years of recessions	5	3	4	12		
MEANS						
GDP growth	8.688	6.616	4.258	6.448	2.24	5.22
Unemployment Rate	5.59	6.36	5.95	5.96	12.41	8.46
Inflation, consumer prices for the United States	5.134	3.77	2.453	3.765	3	4
Market Yield on U.S. Treasury Securities (10-Year Constant Maturity)	6.587	8.36	3.086	5.905	3	4
Household debt/GDP	0.443	0.591	0.847	0.631	-	-
Non-financial corporation debt/GDP	0.301	0.389	0.44	0.377	-	-
Private debt/GDP (sum of two above)	0.744	0.98	1.287	1.008	0.8881	1.7576
Nondurable consumption/GDP	0.515	0.553	0.599	0.556	0.528	0.5268
Durable consumption/GDP	0.087	0.084	0.078	0.083	0.235	0.238
Investment/GDP	0.23	0.225	0.211	0.222	0.2588	0.2974
Share of Labour Compensation in GDP at Current National Prices for United States	0.631	0.614	0.603	0.616	0.725	0.77
STANDARD DEVIATIONS						
GDP growth	2.383	2.135	2.805	3.046		
Unemployment Rate	1.35	1.579	1.824	1.605		
Inflation, consumer prices for the United States	3.692	1.93	1.692	2.785		
Market Yield on U.S. Treasury Securities (10-Year Constant Maturity)	1.988	2.574	1.186	2.974		
Household debt/GDP	0.019	0.076	0.085	0.182		
Non-financial corporation debt/GDP	0.028	0.038	0.037	0.068		
Nondurable consumption/GDP	0.009	0.014	0.011	0.037		
Durable consumption/GDP	0.004	0.005	0.008	0.007		
Investment/GDP	0.011	0.014	0.014	0.015		
Share of Labour Compensation in GDP at Current National Prices for United States	0.009	0.009	0.014	0.016		

Source: Authors' computation on FRED data

Table A9: Average values of variables of interest by recessionary and expansionary years

	Expansionary years	Recessionary years
GDP growth	6.639	5.545
Unemployment Rate	5.972	6.5
Inflation, consumer prices for the United States	3.419	6.381
Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity	5.653	7.563
Household debt/GDP	0.672	0.613
Non-financial corporation debt/GDP	0.397	0.378
Private debt/GDP (sum of two above)	1.069	0.991
Nondurable consumption/GDP	0.566	0.551
Durable consumption/GDP	0.083	0.081
Investment/GDP	0.22	0.221
Share of Labour Compensation in GDP at Current National Prices for United States	0.612	0.624

Source: Authors' computation on FRED data

Table A10: The variables in the model

Symbol	Description	Equation
g_w	Rate of growth of nominal wages	Equation 3
π	Price inflation	Equation 4
τ	Rate of growth of labor productivity	Equation 5
ω	Labor share	Equation 6
Eg	Expected rate of growth	Equation 7
R	Interest rate	Equation 8
d	Debt ratio	Equation 9
c	Consumption ratio	Equation 10
f	Durable consumption ratio	Equation 11
ν	Capital output ratio ratio	Equation 12
i	Investment ratio	Equation 13
g	Rate of growth	Equation 14
g_k	Rate of growth of capital	Equation 15
h	Capacity utilization	Equation 16
g_{Ls}	Rate of growth of labor supply	Equation 17
g_N	Rate of growth of labor demand	Equation 18
u	Rate of unemployment	Equation 19
g_s	Natural rate of growth	Equation 20

Source: Authors computation

C Bifurcation diagrams

Each bifurcation diagram is obtained by skipping the first 1000 iterations, to avoid the effects of the chosen initial conditions, and plotting the next 1000 iterations. The range of variations of the parameters around their benchmark value is chosen in a way that is possible to see if the variation leads to a qualitative change in the dynamics. The bifurcation diagrams have the growth (g) on the vertical axis, where its values are represented by a red line, while the values of g_{01} are blue and the threshold between the two regimes ($g_{01} + \Delta$) is green. The value of g on the vertical axis permits us to show the effects of the parameters on the equilibrium values of the growth. As it happens for the general model, dynamics are set in motion by a loss of stability of g_{02}.

Below we report and comment on the bifurcation diagrams regarding the following parameters:

- the steady states values of the labor share in the two regimes, ω_{01} and ω_{02};
- the parameters of the investment equation, η_1, η_2, η_3, which are varying across regimes;
- the parameters of the Taylor rule;
- the parameters of the labor supply;
- the parameters of the wage and price inflation;
- the parameters of the labor productivity growth.

Note that, in all simulations, we changed the value of a single parameter, while leaving all others unchanged as in the benchmark shown in the Appendix B, except for λ_1.[25] We begin by examining the role of the steady state values of ω_{01} and ω_{02} in Figure A4 (left and right panels, respectively). When ω_{01} is increased, the amplitude of the fluctuations decreases. This is a consequence of the fact that the higher ω_{01} is, the smaller the distance between the two steady state values of g. It follows that the fluctuations have a smaller amplitude, but they still affect both regimes. In fact, the black simulated points are on both sides of the threshold (in green).

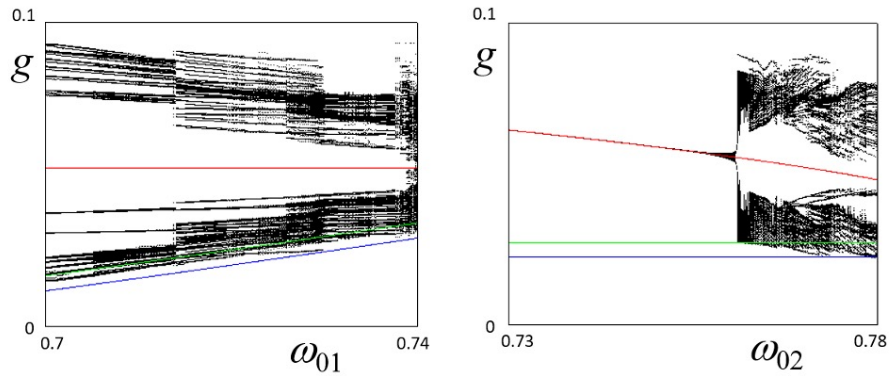
A more important role is played by ω_{02} which, if reduced, can stabilize the equilibrium with a higher growth rate, while the fluctuation dynamic is replaced by a process of convergence to this value. However, if the value is not sufficiently low, the fluctuations between the two regimes will remain.

Figures A5 - A6 - A7 show the role of parameters η_{ij}.[26] When η_{11} and η_{12} are increased, they have the effect of decreasing the amplitude of fluctuations due to the smaller dif-

[25] We set $\lambda_1 = 1$ because this parameter only affects the frequency of oscillations.

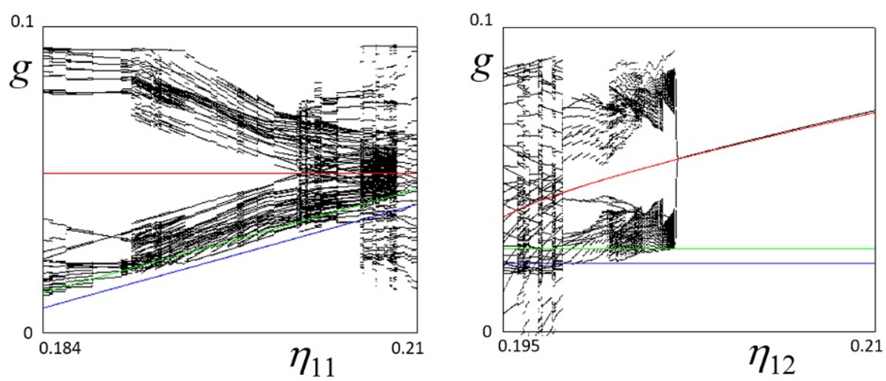
[26] They appear in the equation of investment ratio, equation 13.

Figure A4: Bifurcation diagrams for ω_{01} and ω_{02}



ference between the two equilibrium values (the blue and red curves converge more and more). On the other hand, η_{31} plays an opposite role. As for the parameters η_{21} and η_{22}, they lead to a stable equilibrium with high growth when they are sufficiently increased, while η_{32} has the same effect when it is sufficiently decreased. However, it should be emphasized that they also generate a non-negligible interval of fluctuations.

Figure A5: Bifurcation diagrams for η_{11} and η_{12}



The role of the parameters γ_1 and γ_2 [27] is examined in Figure A8. A decrease of γ_1 in Figure A8 stabilizes the system, while fluctuations come into play as it increases. By increasing γ_2 one goes from chaos to a periodic cycle. Even if fluctuations are of greater amplitude (and still affect both regimes), we lose the unpredictability of chaos in favor of the predictability of a periodic cycle.

[27] They appear in the Taylor rule equation, see equation 8.

Figure A6: Bifurcation diagrams for η_{21} and η_{22}

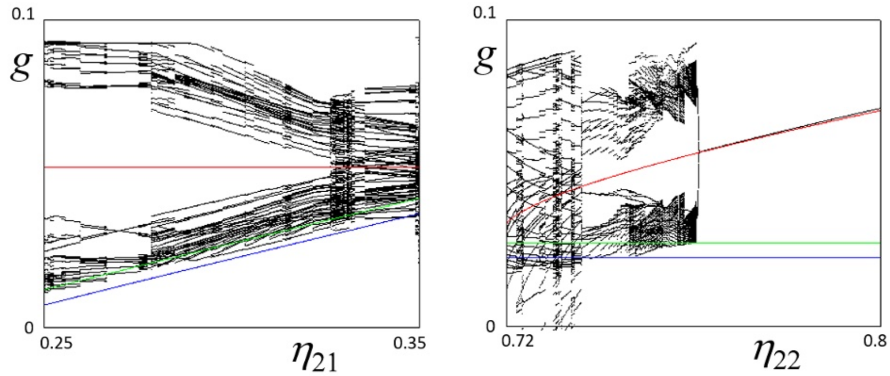


Figure A7: Bifurcation diagrams for η_{31} and η_{32}

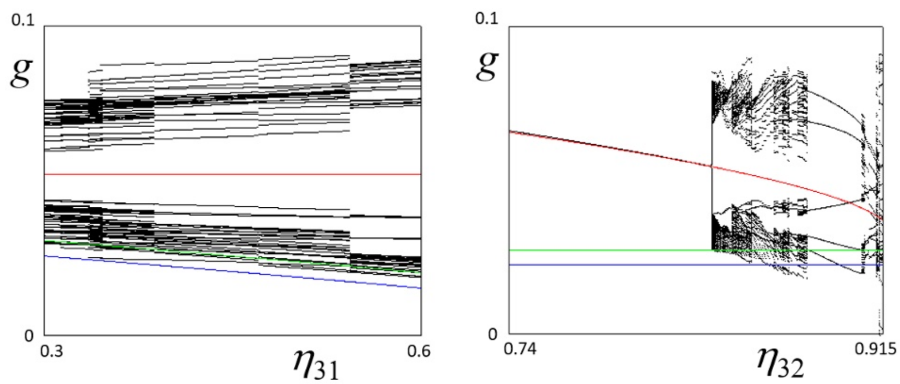
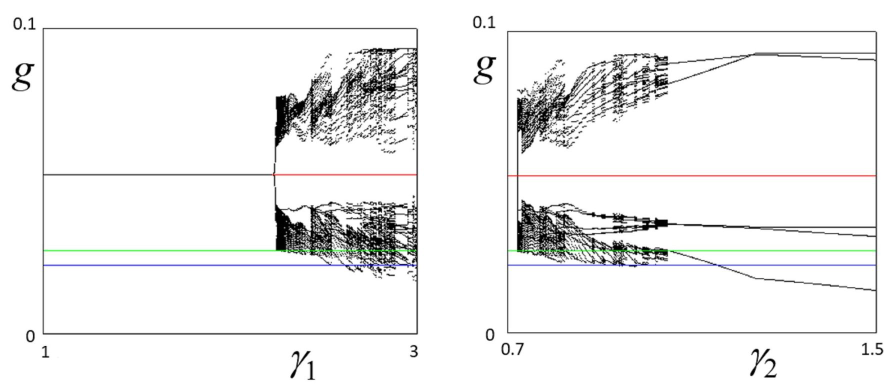
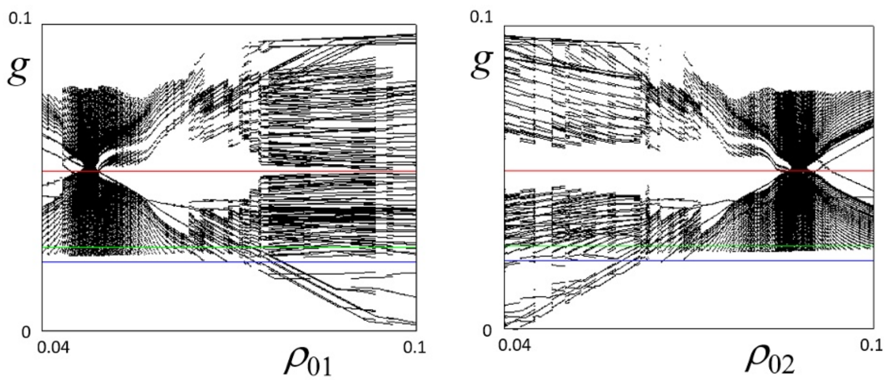


Figure A8: Bifurcation diagrams for γ_1 and γ_2



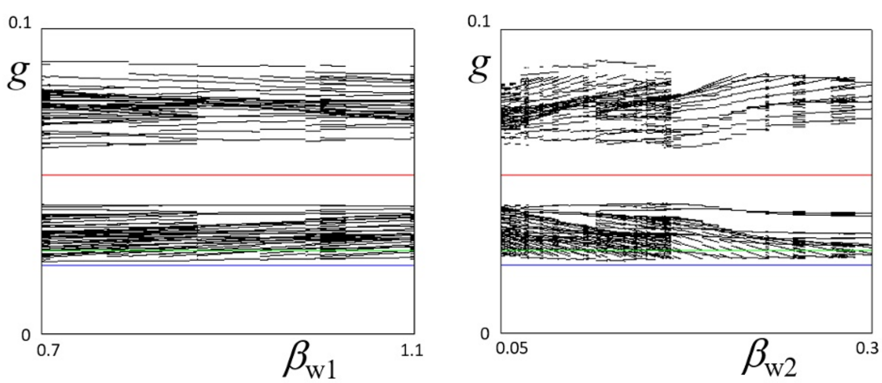
Let us now consider the labor supply that (indirectly) links the general model to the wage-price sub-model via the unemployment rate. In particular, consider the value of the constant (ρ_0) in the two regimes. The fluctuations become larger when ρ_{01} is sufficiently increased (left panel of Figure A9), or when ρ_{02} is sufficiently decreased (right panel of Figure A9). In any case, a very complex picture emerges.

Figure A9: Bifurcation diagrams for ρ_{01} and ρ_{02}



The parameters related to the wage-price subsystem are analyzed using Figures A10 and A11. The two panels of Figure A10 show that β_{w1} and β_{w2}[28] play a fairly limited role in determining the amplitude of the fluctuations. Similarly, the role played by $\beta_{\pi1}$ and $\beta_{\pi2}$. (Figure A11).[29] However, it should be emphasized that the right-hand side panels exhibit greater variability.

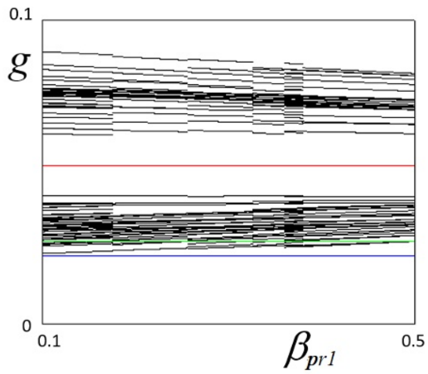
Figure A10: Bifurcation diagrams for β_{w1} and β_{w2}



On the contrary, the parameters θ_0 and θ_1, which refer to the productivity dynamics,[30] can stabilize or destabilize the dynamics. In particular, the high-growth equilibrium

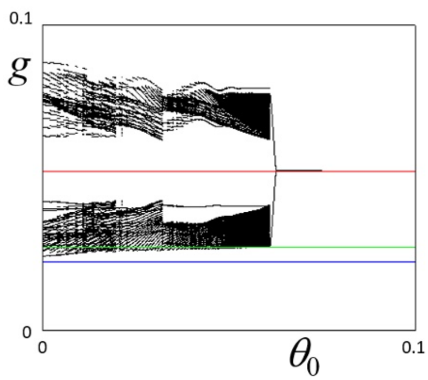
[28]They appear in the equation of the rate of growth of nominal wages 3.
[29]They appear in the inflation equation 4.
[30]They appear in the equation of productivity 5.

Figure A11: Bifurcation diagrams for $\beta_{\pi 1}$ and $\beta_{\pi 2}$



(Regime 2) can be stable for sufficiently high values of θ_0 and sufficiently low values of θ_1 (Figure A12).

Figure A12: Bifurcation diagrams for θ_0 and θ_1



In FiguresA13 and A14 show the bifurcation diagrams of σ_{1j} and σ_{2j}[31] While σ_{11} and σ_{21} play basically no role in the dynamics, σ_{12} and σ_{22}, i.e., the same parameters in the 'good state', can lead to stability if their values are low enough. However, for sufficiently high values they lead to fluctuations.

[31]They appear in the equation of nominal wage inflation 3 and in the price inflation equation 4, respectively.

Figure A13: Bifurcation diagrams for σ_{11} and σ_{12}

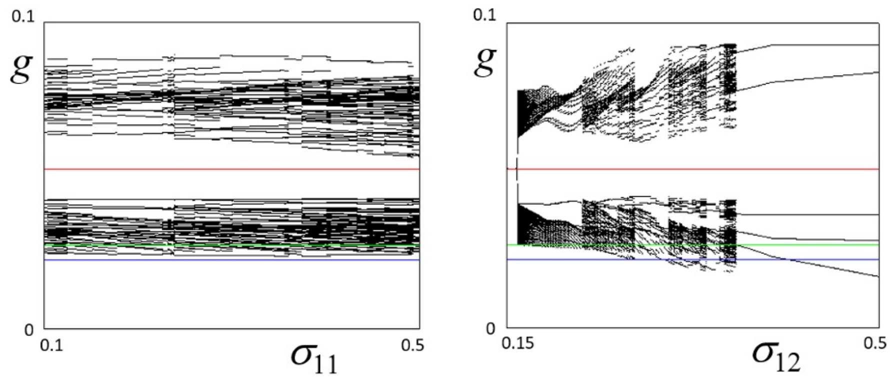
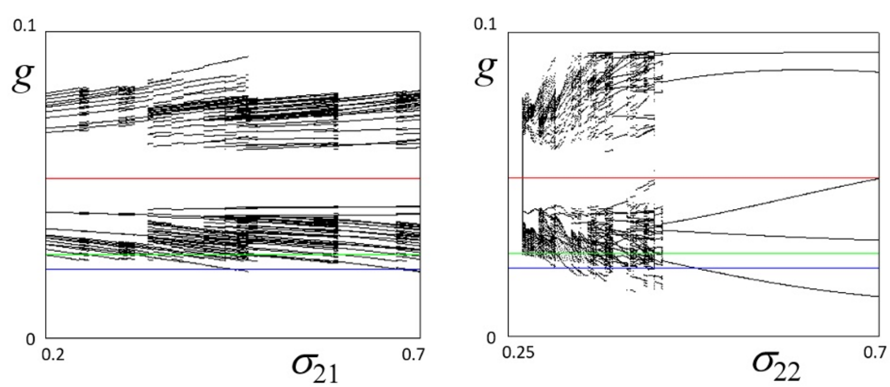


Figure A14: Bifurcation diagrams for σ_{21} and σ_{22}



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