Spatio-temporal modelling for avalanche risk assessment in the North of Italy

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Abstract: The main objective of this work is to evaluate the avalanche activity in a given location and at a given time taking into account a number of variables including the stratigraphy of snow cover, temperature, direction and wind speed, altititude, etc. To this end we propose a space-time point model where the intensity function indicates the limiting expected rate of occurrence of snow avalanches of a given size occurring on a certain day at location (x, y), conditioned on the historical information available prior to time t. Some meteorological and environmental data may be considered as the covariates of the model. To show the ability of the model in assessing the risk avalanche, data from digitalized Avalanche Database of the Trentino Region (North of Italy) is considered. Since not all locations in the Alpine zone are equally likely subject to snow avalanche, the model will be flexible enough for including a spatially-varying background rate of avalanche which may be estimate by kernel smoothing the observed avalanches .

Keywords: Snow avalanches, intensity function, spatio-temporal modelling.

1 Introduction

In recent years the study of avalanche phenomena has attracted growing interest especially for the increase of accidents and deaths, now comparable with those related to natural disasters. This is mainly due to a wide anthropization of mountain areas which has often brought a rapid growth of recreational activities, transportation, and constructions in high-altitude areas without an adequate assessment of avalanche hazard. Hence, the analysis of avalanche activity is extremely important to prevent damage and for activities aimed at land use planning in mountain areas. Many scientists have been studying avalanches to try to map the risk and improve predictions. To that end several statistical methods have been proposed based on different approaches. In this work we propose an approach based on space-time point processes for modeling the avalanche risk. In particular, the intensity function of the process indicates the limiting expected rate of occurrence of snow avalanches occurring on day t at location (x, y), conditioned on the historical information available prior to time t. Also, we use a self-exciting model to deal with unobserved random space-time effects. The location (x, y) represents the baricenter of the polygon which draws the shape of avalanche. For showing the effect of some covariates (such as elevation, slope, temperature, etc.) different models are proposed. Application to the digitalized Avalanche Dataset of Trentino region (Italy) illustrates the ability of the models to forecast the risk avalanche. Although this approach has not been previously applied to avalanche events, it has been used for analysis spatio-temporal analysis of earthquakes occurrences (Ogata, 1998) and wildfire risk (Peng *et al.* 2005; Schoenberg *et al.* 2007).

2 Spatio-temporal models for avalanches

Any spatial temporal point process is uniquely characterized by its conditional intensity function $\lambda(t, x, y|\mathcal{H}_t)$ given by the limiting conditional expectation

$$\lambda(x, y, t|\mathcal{H}_t) \lim_{\Delta t, \Delta x, \Delta y \downarrow 0} \frac{E[N\{(t, t + \Delta t) \times (x, x + \Delta x) \times (y, y + \Delta y)\}|\mathcal{H}_t]}{\Delta t, \Delta x, \Delta y}$$

provided the limit exists. This is a random function that depends on the prior history, $\mathcal{H}_t$, of the point process up to time t. In this preliminary analysis, we considered a small number of models that should capture the main aspects of the avalanche dataset. One first class of models is nonparametric and has separable spatial and temporal effects. This is given by

$$\lambda_{1a}(x, y, t|\mathcal{H}_t) = \lambda(x, y, t) = \beta_0 + \beta_1 S(x, y) + \beta_2 T(t) \tag{1}$$

or by

$$\lambda_{1m}(x, y, t|\mathcal{H}_t) = \exp\left(\beta_0 + \beta_1 S(x, y) + \beta_2 T(t)\right) \tag{2}$$

where β is the parameter vector to be estimated. So, one is an additive model while the other is a multiplicative model. In these models, $S(x, y)$ is a deterministic function of the location (x, y) and it is estimated by a two-dimensional kernel smoother

$$S(x, y) = \frac{1}{n_0} \sum_{j=1}^{n_0} K\left(\frac{x - x_{0j}}{\phi_x}\right) K\left(\frac{y - y_{0j}}{\phi_y}\right)$$

where K is a suitable kernel function, taken as the quartic kernel in this paper. The function $T(t)$ is a periodic with trend deterministic function, also estimated by kernel methods using the events' times. The determinist aspect of these functions make the conditional intensity independent of the past, justifying the first equality in (1). To have an identifiable model and to avoid numerical instabilities, we centered all covariates at zero. It is likely that this model has less predictive power than other models as it does not incorporate important additional information. However the model can be improved using covariates. At this moment, we have the elevation $E(\mathbf{x})$ and slope $S(\mathbf{x})$. In particular, for the slope we created a binary map with areas with slopes angles within this (25, 50) degrees. Hence, another class of models has an intensity varying only with the exogenous covariates and the temporal components. We again have $\lambda(x, y, t|\mathcal{H}_t) = \lambda(x, y, t)$ for these models, a deterministic intensity function. It is given by

$$\lambda_{2a}(x, y, t) = \lambda_{1a}(x, y, t) + \beta_3 E(\mathbf{x}) + \beta_4 S(\mathbf{x}) \tag{3}$$

Another version of this model is the multiplicative form where

$$\lambda_{2m}(x, y, t) = \lambda_{1m}(x, y, t) \exp\left(\beta_3 E(\mathbf{x}) + \beta_4 S(\mathbf{x})\right) \tag{4}$$

Other covariates can be added in the model such as precipitation, temperature and the level of new snow. Additional improvements of these models respect to the first class of models can be tested by means of the difference between the log-likelihood maximum values of each model. The final class of models we are going to consider are those that include the history of previous avalanches events in the area near each point. The conditional intensity is a truly random function that depends on the previous occurrences. Let

$$H(\mathbf{x}, t) = \int\int\int I_{B_{\mathbf{x}}(r)\times[t-\epsilon,t)}(x, y, t) N(dx, dy, dt)$$

where $I_A(\cdot)$ is the indicator function of the set A and $B_{\mathbf{x}}(r)$ is a small disc centered at $\mathbf{x}$ and with radius r. That is, $H(\mathbf{x}, t)$ is the number of events from the point process N that are inside the three-dimensional cylinder $B_{\mathbf{x}}(r) \times [t - \epsilon, t)$. Clearly, $H(\mathbf{x}, t)$ is $\mathcal{H}_t$-measurable. Then, the models incorporating this previous history are of two types, an additive model,

$$\lambda_{3a}(x, y, t|\mathcal{H}_t) = \lambda_{2a}(x, y, t) + \beta_6 H(\mathbf{x}, t) \ , \tag{5}$$

and its multiplicative version,

$$\lambda_{3m}(x, y, t|\mathcal{H}_t) = \lambda_{2m}(x, y, t) \exp(\beta_6 H(\mathbf{x}, t)) \ . \tag{6}$$

3 Applications and results

The data used in this work have been provided by the province of Trento through the availability of digitalized Avalanche Database (based on a permanent survey on avalanches). In this application we consider 3350 avalanche events at 970 sites for the period January 1980 – December 1989. In this preliminary report, we did not fit the models (5) and (6). They require a much heavier numerical work as each time unit (day, in our case) has an associated map with the covariate $H(\mathbf{x}, t)$ that

Models	Intercept	$S(\mathbf{x})$	$T(t)$	Elevation	Slope	Log-Lik
Model 1	0.16017	0.00027	0.01559	NA	NA	-1803.152
Model 2	0.05780	0.00027	0.15635	0.23963	0.00035	-1138.234

Table 1: Estimates from models 2 and 4.

enters the likelihood maximization in each iterative step. We are working on this model and should have final results soon. The results for the models 2 and 4 are in Table 1. Figs. 1 (middle and right) show an example of the estimated intensity functions (risk maps) by the two models on February 1, 1986. As expected, model

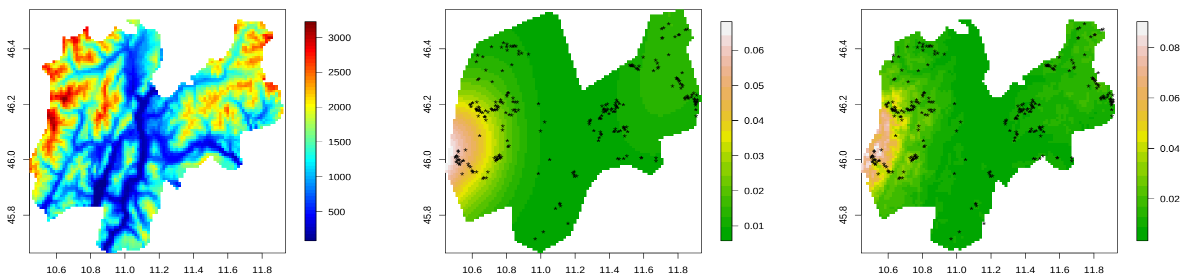

Figure 1: Elevation map of Trentino (left); estimated intensity function at February 1, 1986 by model (2) (middle) and model (4) (right). Asterisks represent avalanche events at the same day.

(4) performs better than model (2). We are going to include other covariates such as temperature and the amount of snow accumulated in the soil. Both are time varying and should be useful in terms of prediction of avalanche events. We are in the process of collecting these covariates and we expect to have an extended version of this paper incorporating these additional information in the near future.

References

Ogata, Y. (1998). Space-time point-process models for earthquake occurrences. *An- nals of the Institute for Statistical Mathematics* (50), 379-402.

Peng, R. D., Schoenberg, F. P., Woods, J. (2005). A space-time conditional intensity model for evaluating a wildfire hazard index. *Journal of the American Statistical Association*, 100 (469), 26–35.

Schoenberg, F., Chang, C., Keeley, J., Pompa, J., Woods, J., and Xu, H. (2007). A critical assessment of the Burning Index in Los Angeles County, California. *International Journal of Wildland Fire*, 16, 473-483.