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# A VULNERABILITY INDEX EVALUATION FOR MASONRY BELL TOWERS IN VENICE

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## ABSTRACT

The skyline of Venice is still nowadays characterized by a high density of slender bell-towers, corresponding to a huge number of churches that marked the built environment of the ancient town, constituting both religious and administrative references.
It is the character itself of such structures that enforces a particular attention to the stability and safety the time during, also if the astonishing event of the sudden collapse of St. Marco bell-tower in 1904 is by now nearly forgotten. More than 90 bell-tower has been classified, on the basis of a research conducted by the "Cultural Heritage and Landscape" Office of Venice, collecting a lot of historical, geometrical and qualitative information about these buildings.
The problem was then to find a reasonable method to compare each other these structures, in order to fix a vulnerability scale helpful also for establishing the priority choice of intervention on a territorial scale, in the spirit of the "Guide-Lines for seismic risk evaluation and reduction on the cultural heritage", according to Italian Building Code.
With the help of a GIS based on the survey previously made, some global limit states have been defined on the basis of a limited number of geometrical or mechanical parameters (qualitatively obtained by visual inspection or building character survey), in order to highlight possible critical situations of the whole tower.
The evaluation of different "vulnerability indexes", homogeneous and comparable among them, allowed for each tower to outline the higher risk factor and to set up an overall rating of bell-tower structures examined, comparing in this way also objects that are heterogeneous by shape, materials and building history.

*Keywords:* *Safety index, Masonry bell-towers, Quality assessment*

**Fig. 1** The roofs of Venice and some (well-known and not) bell-towers

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# 1. INTRODUCTION

## 1.1. The skyline of Venice

The skyline of Venice is still nowadays characterized by a high density of slender bell-towers, many of which are frighteningly leaning, corresponding to some of the huge number of churches that marked the built environment of the ancient town, constituting both religious and administrative references.
It is the character itself of such structures that enforces a particular attention to the stability and safety the time during, also if the astonishing event of the sudden collapse of St. Marco bell-tower in 1904 (but quickly rebuilt in 1912) is by now nearly forgotten. More recently, the sudden collapse of the Pavia Civic Tower in 1989, without any apparent warning signs, awake scientists and public opinion to face the problems of safety of ancient towers [1], and also some bell-towers in Venice gained new attention from authorities, like that of St. Stephan or that of the "*Frari*" church [2].
But starting in 2005, the Cultural Heritage and Landscape Office (SBAP) for Venice and Lagoon committed an extensive survey on the nearly 90 bell-towers of the town, in order to collect some basic information on the state of health of such buildings. Following this survey, a new *Research for the Evaluation of the Structural Stability of Venice's Bell-Towers* was developed in the framework of an agreement between the IUAV University of Venice and the cited Cultural Heritage Office for Venice[4].

## 1.2. A preliminary survey of the bell-towers

The first task of the research was to analyze and organize the data collected in the previous survey realized by the Cultural Heritage Office[5], by transferring them on a geo-referenced support (GIS) in order to make the data inspection easier. The data collected in the *Survey and Control of bell towers of Venice* constituted a very important source for the study of these buildings: the survey was addressed to the whole complex of bell towers in Venice and surroundings, covering around 90 buildings, and focuses on different aspects:

- dimensional data, from geometrical surveys;
- constructive and morphological characterization;
- material deterioration;
- structural damage;
- historical information from archive documents.

**Fig. 2** A GIS screenshot with a general map of Venice and the location of the classified bell-towers

## 1.3. The GIS database support

In order to understand and explain the different damages observed on the bell-towers, beside the geometrical data (height, side size, thickness of the walls, openings,…) is very useful to cross morphological information (like belfry shape, roof, internal floors, but also masonry quality) with data

[4] The research was coordinated by F. Doglioni (IUAV) and A. Lionello (SBAP-Ve), with the participation of G. Mirabella-Roberti, M. Bondanelli, L. Franco, I. Cavaggioni, A. Squassina, F. Trovò and the contribution also of L. Sartorelli, G. Ottaviani and D. Cianciarulo. The results were published in [3]

[5] This study was promoted and realized by I. Cavaggioni and A. Lionello (SBAP-Ve), with the collaboration of M.Furlanetto, I.Olivieri, M.Roncuzzi, A.Vendrame and of the firm RiArte s.r.l. for topographic serveys.

related to damage (cracks, deformations) taking into account constructive aspects (modifications, superstructures) and previous repairs.
The largest part of available information was selected, and adapted to the structure of a relational data-base; other data were integrated, like load and slenderness, attaining nearly 200 items for each of the 90 records: the data-base was then published on-line[6]. This tool made easier the information analysis, from time to time allowing data extraction for qualitative, numerical or synthetic readings, making possible, in subsequent elaborations, to obtain more complex information starting from the same collected data.

## 2. THE VULNERABILITY INDEX EVALUATION

After this preliminary extensive survey, a further step was requested: to establish a first, rough hierarchy in the risk of collapse of this vast collection of monuments (different by age, height, position, subsoil…) allowing to focus a second step of the research only on some of them, those presenting more critical conditions.
In any case, in order to formulate some mechanical estimation of a "vulnerability index", in the spirit of level 1 assessment from "Guide-Lines for seismic risk evaluation and reduction on the cultural heritage" [4], some mechanical parameters had to be evaluate only on the basis of the qualitative observations already collected, integrated by direct observation of some selected typical cases. On the other hand, some typical failure mechanisms have to be selected, in order to fix an overall scale of risk, finding some appropriate, non-dimensional, indexes that can be compared each other, first for the single tower, determining the most probable mechanism, and then for the global population of towers, allowing to fix a priority scale for a deeper research and possibly for strengthening interventions.
The idea was to define an "index of risk", instead of the traditional "safety factor", with the aim of underlining the "estimate" character of the result, avoiding unrealistic feelings of safety vs. danger. So the different indexes **I** are obtained calculating the ratio between the quantity **A**, representing the *action* – static or geometrical – that lead to the failure, and the quantity **R**, representing the *resistance* that the structure oppose to the failure, obtaining $\mathbf{I} = \mathbf{A}/\mathbf{R}$ (that usually has to be < 1: none of the bell towers is falling, actually).

### 2.1 Mechanical parameters

The more critical aspect was at this point to "translate" the qualitative observations collected by the survey in numerical parameters, characterizing the resistance term in the formulas. Here some arbitrary hypothesis have to be made; the more complete are the available observations, the more reliable will be the results of the analysis. On the contrary, the survey gave a very precise information of geometrical parameters, like height, base dimensions, thickness, and also out-of-plumb of the tower, allowing for a reasonable evaluation of the global mechanisms, to be preferred for this reason to local ones.

#### *2.1.1. Masonry Quality definition*

The preliminary evaluation process is based on a first inspection, aimed to outline the qualitative parameters that will be considered the more meaningful for describing the consistency and efficiency of the masonry: the choice of a specific weight for each parameter is conventional, and must be carefully checked against the reference population so that the obtained score can be easily motivated and decoded in its formation process [5].
Among others, the parameter related to masonry quality appears to be crucial for the risk evaluation. The estimate was based on the assumption that a masonry made by homogeneous brick of first utilization, with regular and accurate bonding, is more efficient and less vulnerable; and the more vulnerable is a masonry made by reused heterogeneous bricks, with irregular bonding; also the amount of substitutions and re-bonding executed in the past are considered, as they are nearly absent or limited in the more efficient masonry and prevailing in the lower quality masonry. The observed cases combined those aspects, and the proposed score allowed to determinate 5 increasing classes of quality.

#### *2.1.2. Choice of the mechanical parameters*

The chosen approach make a strict correspondence among the five *classes* of masonry quality obtained from the survey and some arbitrary-selected numerical values, assigned in the range allowed by the

[6] The Information System was designed and realized by F. Trovò and D. Cianciarulo using File Maker Server 10.

recent Italian Building Code [6] par. 11.10.3.1.2 for the characteristic strength $f_{Mk}$ and par. 11.10.3.4 for elastic modulus $E_M$ of masonry. Then the design strength $f_{Md}$, used in calculations is obtained adopting a material factor $\gamma_M$=3,00 for ordinary bond masonry, using the coefficient $\gamma_M$=2,50 only for very well bonded and homogeneous masonry, corresponding respectively to the "execution classes" 2 and 1 of the Building Code, art. 4.5.6.1.

**Table 1**

| **class** | $\mathbf{E_M}$ [N/mm$^2$] | $\mathbf{f_{M,k}}$ [N/mm$^2$] | $\mathbf{f_{M,d}}$ [N/mm$^2$] | |
|---|---|---|---|---|
| | | | $\gamma_M$=3,00 | $\gamma_M$=2,50 |
| 1 | 1500 | 1,5 | 0,500 | 0,600 |
| 2 | 2000 | 2,0 | 0,666 | 0,800 |
| 3 | 2500 | 2,5 | 0,833 | 1,000 |
| 4 | 3000 | 3,0 | 1,000 | 1,200 |
| 5 | 3500 | 3,5 | 1,166 | 1,400 |

## 2.2 Choice of some "limit states"

In the definition of the critical situations the term of limit states was preferred, referring both to some states of stress or deformation of the structure in the elastic domain, and to some ultimate failure mechanisms (equilibrium losses) that can involve the tower as a whole. From the "action" side, static action depending from the self weight were preferred, that strongly characterize the global behaviour of the towers on the long term: all occasional action (like wind effect or seismic action) are to be related to this fundamental component.

In the following the limit states here reported are described:

- Elastic Limit State of maximum compression/decompression,
- Instability Limit State of flexural deformability of the tower or of soil comprimibilty,
- Ultimate Limit State of overturning by rigid rotation on the base.

### *2.2.1. Elastic Limit State of maximum compression/decompression*

The maximum and minimum stresses are calculated on the base section, considering that the tower behave as a St.Venant solid fully bonded at the base subjected to combined compression/bending stresses on both principal axes, with the normal sections that keep herself plane after deformation and stresses proportional to the deformations.

From the Navier formula, assuming positive the compression stresses and being $M_x = N \cdot e_x$, and $M_y = N \cdot e_y$ respectively the bending moments along principal $x$ and $y$ axes of the section due to the eccentricity $e_x$ , $e_y$ of the normal force $N$, the maximum and minimum stresses, $\sigma_{max}$ and $\sigma_{min}$, are calculated as:

$$\sigma_{\max,\min} = \frac{N}{A}\left[1 \pm \left(\frac{d_x e_x A}{I_x} + \frac{d_y e_y A}{I_y}\right)\right] \tag{1}$$

being $A$ the area of the base hollow cross section, $I_x$ and $I_y$ the moments of inertia along principal axes $x$ and $y$, $d_x$ and $d_y$ the maximum distances from $x$ and $y$ axes of the external borders of the section.

For practical reasons, assuming a square hollow base section of side $a_{ext}$ external and $a_{int}$ internal, being $I_x = I_y = I$ , $I = \frac{1}{12}(a_{ext}^4 - a_{int}^4)$, $A = a_{ext}^2 - a_{int}^2$, $\frac{I}{A} = i^2 = \frac{a_{ext}^2 + a_{int}^2}{12}$, and $d_x = d_y = \frac{a_{ext}}{2}$, putting $\sigma_{min}$= 0 in (eq. 1), we obtain the maximum (limit) value for the eccentricity of the normal force at the base $(e_x + e_y)_{lim}$ ; so, whatever the normal force $N$, the term in square brackets vanish when:

$$(e_x + e_y)_{lim} = \frac{2I}{A a_{ext}}, \text{ that is } (e_x + e_y)_{lim} = \frac{2i^2}{a_{ext}} = \frac{a_{ext}^2 + a_{int}^2}{6 a_{ext}}$$

Notice that the kernel limit for the hollow section calculated above is grater than that for the solid section, $a/6$.

The no-tension elastic limit state can be represented by the *no-tension* or *eccentricity index* $\mathbf{i}_e$ comparing the calculated limit value of the eccentricity with the actual (measured) value $(e_x + e_y)_{act}$ :

$$\mathbf{i}_e = \frac{(e_x + e_y)_{\text{act}}}{(e_x + e_y)_{\text{lim}}} = \frac{(e_x + e_y)_{\text{act}}}{2i^2/a_{\text{ext}}} < 1 \tag{2}$$

The corresponding maximum value of compression can be calculated as $\sigma_{\max}\big|_{\sigma_{\min}=0} = 2\dfrac{N}{A} = 2\ \sigma_{\text{med}}$ .

If it happens that for $\sigma_{\min} = 0$ we find $\sigma_{\max} = 2\dfrac{N}{A} > \sigma_{\text{amm}}$ , then the elastic stress limit state is better referred to the maximum compression stress. So the *stress index* $\mathbf{i}_\sigma$ can be evaluated, defining also a *mean stress index* $\mathbf{i}_{\bar{\sigma}} = \dfrac{\sigma_{\text{med}}}{\sigma_{\text{adm}}}$ , as:

$$\mathbf{i}_\sigma = \frac{\sigma_{\max}}{\sigma_{\text{adm}}} = \frac{\sigma_{\text{med}}}{\sigma_{\text{adm}}}\left[1 + \frac{(e_x + e_y)_{\text{act}}}{2I/Aa_{\text{ext}}}\right] = \mathbf{i}_{\bar{\sigma}}\left(1 + \mathbf{i}_e\right) < 1 \tag{3}$$

*2.2.2. Limit State of Eulerian Instability – Critical Flexural Height*

The tower is considered as a heavy deformable bar, of length $h$, standing vertical and fully clamped at the base, with an uniform bending stiffness of $EI$, subjected to an uniform vertical load $q$ (corresponding to the dead load per unit height). The critical Eulerian load $Q_{cr}$ is obtained as:

$$Q_{cr} = q_{cr}h = 7.837\frac{EI}{h^2}, \quad \text{or better} \quad q_{cr} = 7.837\frac{EI}{h^3}$$

The value of $Q_{cr}$ for distributed load is 3.176 higher than the critical value $P_{cr}$.for concentrated load at the top. But for the stability of the tower, better than referring to the *critical load* $q_{cr}$, that represent a limit value of the self weight (that actually can not be variable), it would be better to find a *critical height* $h_{cr}$ of a tower, having the same bending stiffness $EI$ and same weight per unit length $q = \gamma A$ (where $\gamma$ is the specific weight of masonry), beyond which the vertical straight configuration is no more stable, as proposed by [7]. This critical height can be obtained as:

$$h_{\text{cr,E}} = 1.986 \times \sqrt[3]{\frac{EI}{q}} = 1.986 \times \sqrt[3]{\frac{EI}{\gamma A}} \tag{4}$$

So it is easy to set a geometrical *elastic stability index* $\mathbf{i}_{\text{E}}$ as:

$$\mathbf{i}_{\text{E}} = h / h_{\text{cr,E}} < 1 \tag{5}$$

*2.2.3. Limit State of Elastic Instability – rigid-body rotation at the base*

This is also a geometrical problem: the stability of the tower is verified against a possible rotation as a rigid body of the whole tower around the foundation, assuming an elastically deformable soil (of Winkler type) of constant $k_{\text{W}}$, the rotational constant at the base is $m = k_{\text{W}}I_{\text{found}}$, being $I_{\text{found}}$ the second order inertia moment of the foundation base; the critical height of the tower can be calculated as:

$$h_{cr,\text{W}} = \sqrt{\frac{2m}{q}} = 1.4142\sqrt{\frac{k_{\text{W}}I_{\text{found}}}{\gamma A}} \tag{6}$$

being the critical load: $\quad Q_{cr} = \dfrac{2m}{h}, \quad$ or better $\quad q_{cr} = \dfrac{2m}{h^2}$ .

So it is easy to set a geometrical *foundation stability index* $\mathbf{i}_{\text{W}}$ as:

$$\mathbf{i}_{\text{W}} = h / h_{\text{cr,W}} < 1 \tag{7}$$

A difficulty of this stability evaluation is that the dimension of the foundation, as well as the subsoil Winkler constant $k_{\text{W}}$, are very often unknown or uncertain. For the foundation we assumed a small

increase of the base side of 1.2–1.4, giving an increase in the moment of inertia of 2.07–3.86. The subsoil reaction constant $k_W$ for Venice was assumed equal to 2500 kN/m$^3$.
Moreover, we observe that it is possible to calculate the stability limit for a deformable heavy bar elastically pinned at the base, combining in this way the two results above, but the critical height can not be obtained in a closed form [8]; so one possibility is for example to combine the two indexes already obtained (eq. 5) and (eq. 7), resulting $\mathbf{i}_W > \mathbf{i}_E$ , as:

$$\mathbf{i}_{inst} = \mathbf{i}_W \sqrt{(1+\mathbf{i}_E)} \tag{8}$$

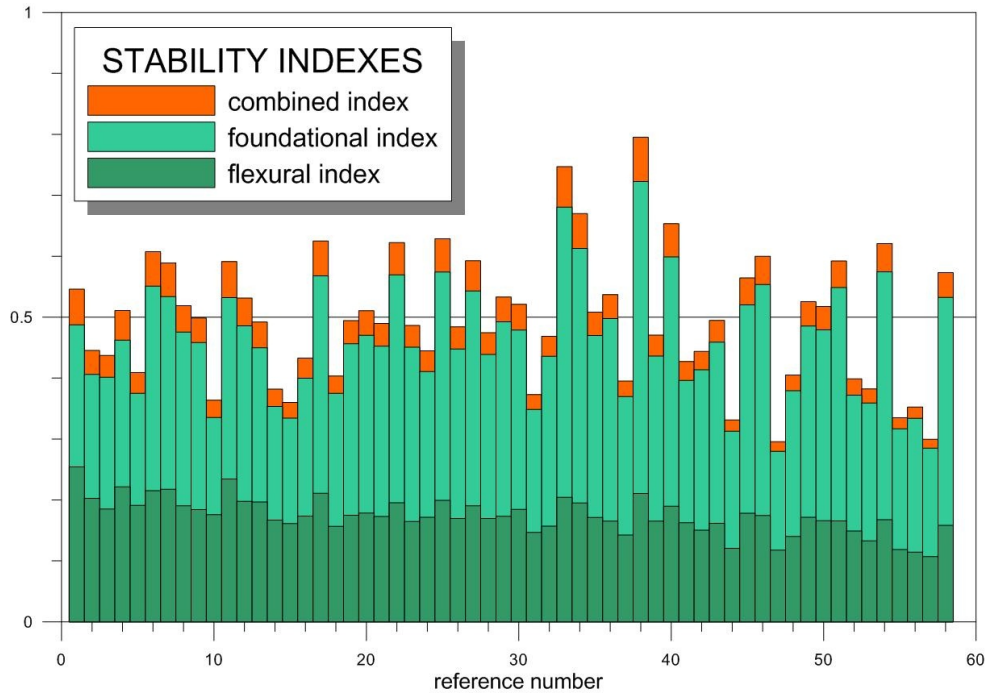


**Fig. 3** Comparison of calculated stability indexes

*2.2.4. Ultimate Limit State of overturning by rigid rotation on the base.*
Limit analysis of leaning masonry towers was proposed first by J. Heyman [9] on 1992, under the hypothesis of material with zero tension and infinite compression strength, avoiding sliding mechanisms for overcoming friction limits; the tower is considered subjected to uniformly distributed horizontal forces, proportional to the self weight by a factor $\lambda$, that could represent in a first approximation a seismic load. Heyman solution can be slightly modified (see [10]) introducing a limited compression strength $f_{mu}$ for the masonry, so that the ultimate limit multiplier $\lambda_u$ for a tower with hollow section of external side $b$ constant along the height $h$ is obtained by the formula:

$$\lambda_u = (b/h)\cdot(1 - \gamma\, h / f_{mu}) \tag{9}$$

here appears the "geometric slenderness" $\lambda_g = b/h$, and the actual slope $\lambda_a$ equals to the ratio between maximum measured out-of-plumb $e_{max}$ and the height of measurement $h_e$. An index of risk for the overturning mechanism can be calculated as:

$$\mathbf{i}_{ov} = \lambda_a / \lambda_u \tag{10}$$

## 2.3. A comprehensive "vulnerability index"

Once the various indexes are set-up, a decision has to be taken on what index has to be considered the more reliable or on how to combine, if it makes sense, the indexes already obtained in order to be able to set up a sort of global risk scale: an example of combination is (eq. 8) for the two stability indexes. As a first gross analysis, the stress index $\mathbf{i}_\sigma$ and the stability indexes $\mathbf{i}_E$ and $\mathbf{i}_W$ are all referred to elastic limit states, but the overturning index $\mathbf{i}_{ov}$ is instead referred to an ultimate limit state, so is better not to mix it with the others; our proposal was then to obtain a *modified stress index* $\mathbf{i}_{\sigma,mod}$ in order to take in account the instability risk, that we can obtain as:

$$\mathbf{i}_{\sigma,\text{mod}} = \frac{\mathbf{i}_{\sigma}}{(1-\alpha\mathbf{i}_{\text{inst}})} = \mathbf{i}_{\bar{\sigma}} \frac{(1+\mathbf{i}_{e})}{(1-\alpha\mathbf{i}_{\text{inst}})} \tag{11}$$

where the stress index $\mathbf{i}_{\sigma}$ is defined in (eq. 3) and the combined stability index $\mathbf{i}_{\text{inst}}$ is defined in (eq. 8); we introduced a correction factor $\alpha = 0.4$.
The control stress index of (eq. 11) was then adopted as the starting value for a more detailed analysis of the real state of risk of each bell-tower, with the aim of taking in account some different specific factor of risk, typical of construction arrangements (like for example the alignment of windows on one side, or a door opening at the base) or caused by previous damage (like passing cracks near the corners or in the middle of one side, or diffused cracking near the base); or else also some factors reducing the risk, like the presence of efficient ties disposed along the height of the tower. All these factors were separately analyzed and classified, assuming detailed scores for each factor, the weighted combination of which conduced to the final estimate of risk. The methodology is well illustrated in [11].

## 3. ANALYSIS OF THE RESULTS

As we can observe in Fig. 5, the vulnerability index defined above (eq. 11) can result greater than 1, due also to the growing effect of the instability index at the denominator; when this happens, it has to be interpreted as an alarming signal, that requires a more detailed analysis.
Anyway it's to be remembered, in particular, that in the analysis made the geometrical dimensions come from a survey, but material properties come from an estimate, so that the results are strongly dependent from this initial approximation.
In our simulation, 9 bell-towers over 58 attain a risk index greater or equal to 1, and 5 have a value greater than 1.25; one in particular attains a value of nearly 1.8, that certainly means a very high vulnerability.
Often, but not always, this is due to a mean stress index > 1, that means a static stress very high compared to the material characteristics observed (and to the assumed allowable stress), increased by the sensitivity to risk of instability: this seems anyway to give a meaningful parameter.

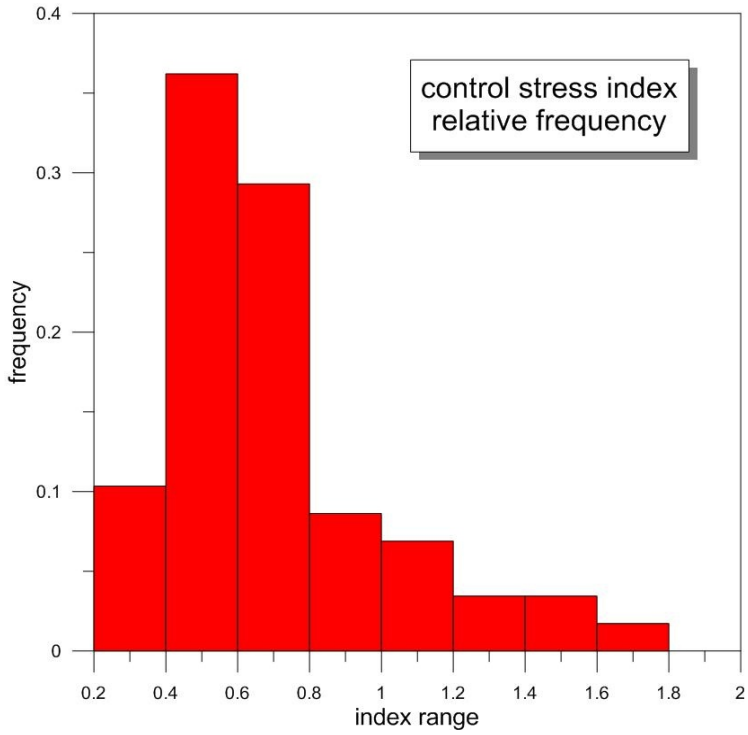


**Fig. 4** Frequency distribution of control stress index

The statistical distribution of the values is highly asymmetric (the mean value is 0,69 and the median is 0.61), with a peak value around 0.5 and the tail on the upper side, suggesting a particular attention to be addressed to the more vulnerable elements.

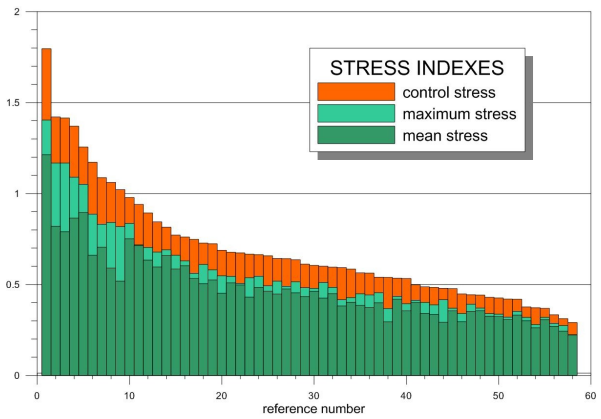


**Fig. 5** Comparison of calculated stress indexes shown in decreasing order

## 4. CONCLUSIONS

Despite some arbitrary assumption, the proposed methodology appears to give an appropriate answer to the basic question of finding a reliable criterion for assessing the state of risk of the population of bell-towers in Venice, in the spirit suggested by some recent code documents; moreover, it is always possible to adjust the initial material parameter scale by means of direct masonry tests, if allowable, obtaining more affordable results. These rough estimations, based on simplified mechanical models, are anyway to be corrected by further examination of the conditions that in practice can increase or decrease the risk index, and these aspect were developed in a subsequent step of the research; in any case the proposed approach remains the basis of the final methodology.

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