

Amplitude Analysis of $B^\pm \rightarrow \pi^\pm K^+ K^-$ DecaysR. Aaij *et al.**
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The first amplitude analysis of the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decay is reported based on a data sample corresponding to an integrated luminosity of 3.0 fb^{-1} of pp collisions recorded in 2011 and 2012 with the LHCb detector. The data are found to be best described by a coherent sum of five resonant structures plus a nonresonant component and a contribution from $\pi\pi \leftrightarrow KK$ S -wave rescattering. The dominant contributions in the $\pi^\pm K^\mp$ and $K^+ K^-$ systems are the nonresonant and the $B^\pm \rightarrow \rho(1450)^0 \pi^\pm$ amplitudes, respectively, with fit fractions around 30%. For the rescattering contribution, a sizable fit fraction is observed. This component has the largest CP asymmetry reported to date for a single amplitude of $(-66 \pm 4 \pm 2)\%$, where the first uncertainty is statistical and the second systematic. No significant CP violation is observed in the other contributions.

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Charge-parity (CP) symmetry is known to be broken in weak interactions. In two-body charged B -meson decays, the only CP -violating observable is the difference of the partial decay widths for particle and antiparticle over their sum. For three- and multibody processes, the decay dynamics is very rich, thanks to possible interfering intermediate resonant and nonresonant amplitudes, and therefore CP violation (CPV) can be manifested as charge asymmetries that may vary and even change sign throughout the different regions of the observed phase space.

Several experiments have reported sizable localized CP asymmetries in the phase space of charmless three-body $B^\pm$ decays [1–7]. The $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ and $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays, having the same flavor quantum numbers, are coupled by final-state strong interactions, in particular through the rescattering process $\pi^+ \pi^- \leftrightarrow K^+ K^-$. The $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ decay, with 3 times larger branching fraction, may proceed through resonances from $b \rightarrow u$ ($\bar{b} \rightarrow \bar{u}$) tree transitions as well as from the $b \rightarrow d$ ($\bar{b} \rightarrow \bar{d}$) loop-induced penguin processes. On the other hand, the production of resonances in the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decay is limited: $\pi^\pm K^\mp$ resonances can only be obtained from penguin transitions; $K^+ K^-$ resonances can come from tree-level transitions, but with the $s\bar{s}$ contribution highly suppressed by the OZI rule [8–10]. In the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decay, no significant $\phi(1020) \rightarrow K^+ K^-$ contribution has been seen [11]. However, a concentration of events is

observed just above the $\phi(1020)$ region in the $K^+ K^-$ invariant-mass spectrum. This corresponds to the region where the S -wave $\pi^+ \pi^- \leftrightarrow K^+ K^-$ rescattering effect is seen, as shown by elastic scattering experiments [12,13]. Intriguingly, in this same region, large CP asymmetries have been observed [1,14]. As proposed in Refs. [15,16], this could be a manifestation of CPV arising from amplitudes with different rescattering strong phases as well as different weak phases.

A better understanding of the CPV mechanisms occurring in three-body hadronic B decays can be achieved through full amplitude analyses. In this Letter, the first amplitude analysis of the decay $B^\pm \rightarrow \pi^\pm K^+ K^-$ is performed based on a data sample corresponding to an integrated luminosity of 3.0 fb^{-1} collected in 2011 and 2012. The isobar model formalism [17,18], which assumes that the total decay amplitude is a coherent sum of intermediate two-body states, is applied. A rescattering amplitude is also included. The magnitudes and phases of the coupling to intermediate states are determined independently for $B^+ \rightarrow \pi^+ K^- K^+$ and $B^- \rightarrow \pi^- K^+ K^-$ decays, allowing for CP violation.

The LHCb detector is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$ equipped with charged-hadron identification detectors, calorimeters, and muon detectors; and it is designed for the study of particles containing b or c quarks [19,20].

Simulated samples, needed to determine the signal efficiency as well as for background studies, are generated using PYTHIA [21] with a specific LHCb configuration [22]. Decays of hadronic particles are described by EVTGEN [23], in which final-state radiation is generated using PHOTOS [24]. The interaction of the generated particles with the detector and its response are implemented using the GEANT4 toolkit [25] as described in Ref. [26].

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In a preselection stage, $B^\pm$ candidates are reconstructed by requiring three charged tracks forming a good-quality secondary vertex, with loose requirements imposed on their momentum, transverse momentum, and impact parameter with respect to any primary vertex. The momentum vector of the B candidate should point back to a primary vertex, from which the $B^\pm$ vertex has to be significantly separated. To remove contributions from charm decays, candidates for which the two-body invariant masses $m(K^\pm\pi^\mp)$ and $m(K^+K^-)$ are within 30 MeV/ c^2 of the known value of the D^0 mass [27] are excluded.

A multivariate selection based on a boosted decision tree (BDT) algorithm [28,29] is applied to reduce the combinatorial background (random combination of tracks). The BDT is described in Ref. [1]; it is trained using a combination of $B^\pm \rightarrow h^\pm h^+ h^-$ samples of simulated events (where h can be either a pion or a kaon) as signal, and data in the high-mass region $5.40 < m(\pi^\pm\pi^+\pi^-) < 5.58$ GeV/ c^2 of a $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$ sample as background. The $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$ sample is used as a proxy for the combinatorial background because, among the various $B^\pm \rightarrow h^\pm h^+ h^-$ channels, it is the only one whose high mass region is populated just by combinatorial background. The selection requirement on the BDT response is chosen to maximize the ratio $N_S/\sqrt{N_S+N_B}$, where N_S and N_B represent the expected number of signal and background candidates in data, respectively, within an invariant mass window of approximately 40 MeV/ c^2 around the $B^\pm$ mass in the data [1].

Particle identification criteria are used to reduce the crossfeed from other b -hadron decays, in particular $K \leftrightarrow \pi$ misidentification. Muons are rejected by a veto applied to each track [30]. Events with more than one candidate are discarded.

An unbinned extended maximum-likelihood fit is applied simultaneously to the $\pi^+K^-K^+$ and $\pi^-K^+K^-$ mass spectra in order to obtain the total signal yields and the raw asymmetry, defined as the difference of B^- and B^+ signal yields divided by their sum. Three types of background sources are identified: the residual combinatorial

background, partially reconstructed decays (mostly from four-body decays) and cross feed from other B -meson decays. The parametrization of cross feed and partially reconstructed backgrounds is performed using simulated samples that satisfy the same selection criteria as the data. From the result of the fit, yields for signal and background sources are obtained [1].

Candidates within the mass region $5.266 < m(\pi^\pm K^+ K^-) < 5.300$ GeV/ c^2 , referred to as the signal region, are used for the amplitude analysis. This region contains 2052 ± 102 (1566 ± 84) of B^+ (B^-) signal candidates. The relative contribution from the combinatorial background is 23%, with a charge asymmetry compatible with zero within 1 standard deviation. The main cross feed contamination comes from $B^\pm \rightarrow K^\pm\pi^+\pi^-$ decays which contribute in 2.7% with a charge asymmetry of 2.5% [1]. Another 0.6% comes from $\phi(1020)$ mesons randomly associated with a pion, with negligible charge asymmetry.

The distributions of the selected $B^\pm$ candidates, represented by the Dalitz plot [31] constructed by the squared mass combinations $m_{\pi^+K^\mp}^2$ and $m_{K^+K^-}^2$, are shown in Fig. 1. The clear differences between the B^+ and the B^- distributions are due to CPV effects [1].

The total $B^+ \rightarrow \pi^+K^-K^+$ decay amplitude $\mathcal{A}$ can be expressed as a function of $m_{\pi^+K^-}^2$ and $m_{K^+K^-}^2$ as

$$\mathcal{A}(m_{\pi^+K^-}^2, m_{K^+K^-}^2) = \sum_{i=1}^N c_i \mathcal{M}_i(m_{\pi^+K^-}^2, m_{K^+K^-}^2), \quad (1)$$

where $\mathcal{M}_i(m_{\pi^+K^-}^2, m_{K^+K^-}^2)$ is the decay amplitude for an intermediate state i . The analogous amplitude for the B^- meson $\bar{\mathcal{A}}$ is written in terms of $\bar{c}_i$ and $\bar{\mathcal{M}}_i(m_{\pi^-K^+}^2, m_{K^+K^-}^2)$. This description for the total decay amplitude is known as the isobar model. In the amplitude fit, the complex coefficients $c_i = (x_i + \Delta x_i) + i(y_i + \Delta y_i)$ and $\bar{c}_i = (x_i - \Delta x_i) + i(y_i - \Delta y_i)$ measure the relative contribution of each intermediate state i for B^+ and B^- , respectively, with Δx_i and Δy_i being the parameters that allow for CPV . The individual amplitudes are described by

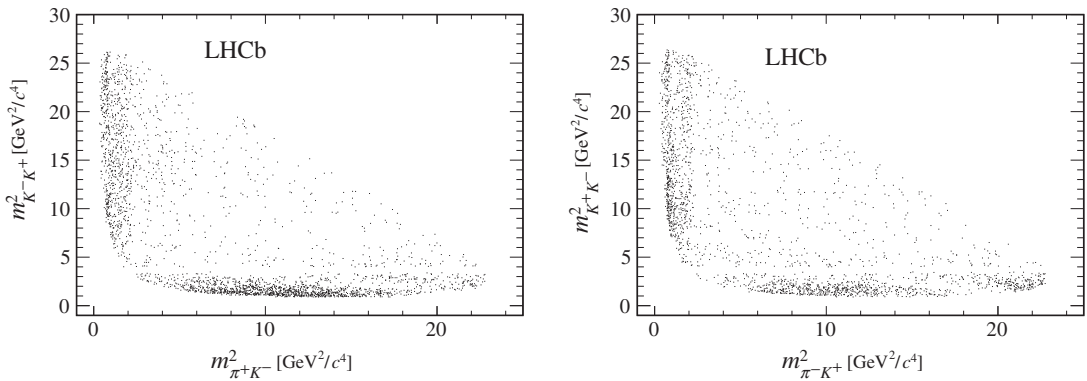


FIG. 1. Dalitz plot for (left) $B^+ \rightarrow \pi^+K^-K^+$ and (right) $B^- \rightarrow \pi^-K^+K^-$ candidates in the selected signal region.

$$\mathcal{M}_i(m_{\pi^+K^-}^2, m_{K^+K^-}^2) = P_i(J, \vec{p}, \vec{q}) F_B(|\vec{p}|) F_i(|\vec{q}|) T_i. \quad (2)$$

The factor P_i represents the angular part, which depends on the spin J of the resonance. It is equal to 1, $-2\vec{p} \cdot \vec{q}$, and $\frac{4}{3}[3(\vec{p} \cdot \vec{q})^2 - (|\vec{p}||\vec{q}|)^2]$, for $J = 0, 1$, and 2 , respectively; $\vec{q}$ is the momentum of one of the resonance decay products and $\vec{p}$ is the momentum of the particle not forming the resonance, both measured in the resonance rest frame. The Blatt-Weisskopf barrier factors [32,33], F_B for the B meson and F_i for the resonance i , account for penetration effects due to the finite extent of the particles involved in the reaction. They are given by 1, $\sqrt{(1+z_0^2)/(1+z^2)}$, and $\sqrt{(z_0^4+3z_0^2+9)/(z^4+3z^2+9)}$ for $J = 0, 1$, and 2 , respectively, with $z = |\vec{q}|d$ or $z = |\vec{p}|d$ and d the penetration radius, taken to be $4.0 \text{ (GeV/c)}^{-1} \approx 0.8 \text{ fm}$ [34,35]. The value of z is z_0 when the invariant mass is equal to the nominal mass of the resonance. Finally, T_i is a function representing the propagator of the intermediate state i . By default a relativistic Breit-Wigner function [36] is used, which provides a good description for narrow resonances such as $K^*(892)^0$. More specific line shapes are also used, as discussed further below.

To determine the intermediate state contributions, a maximum-likelihood fit to the distribution of the $B^\pm \rightarrow \pi^\pm K^+ K^-$ candidates in the Dalitz plot is performed using the LAURA⁺⁺ package [37,38]. The total probability density function (PDF) is a sum of signal and background components, with relative contributions fixed from the result of the $B^\pm \rightarrow \pi^\pm K^+ K^-$ mass fit. The background PDF is modeled according to its observed structures in the higher $m(\pi^\pm K^+ K^-)$ sideband, the contribution from $B^\pm \rightarrow K^\pm \pi^+ \pi^-$ cross feed decays, using the model introduced by the BABAR Collaboration [6], and an additional 0.6% relative contribution from $\phi(1020)$ mesons randomly associated with a pion. The signal PDF for B^+ (B^-) decays is given by $|\mathcal{A}|^2$ ($|\bar{\mathcal{A}}|^2$) multiplied by a function describing the variation of efficiency across the Dalitz plot. A histogram representing this efficiency map is obtained from simulated samples with corrections to account for known differences between data and simulation. The B^+ and B^- candidates are simultaneously fitted, allowing for CP violation. The CP asymmetry A_{CP_i} and fit fraction FF_i for each component are given by

$$A_{CP_i} = \frac{|\bar{c}_i|^2 - |c_i|^2}{|\bar{c}_i|^2 + |c_i|^2} = \frac{-2(x_i \Delta x_i + y_i \Delta y_i)}{x_i^2 + (\Delta x_i)^2 + y_i^2 + (\Delta y_i)^2}, \quad (3)$$

$$FF_i = \frac{\int (|c_i \mathcal{M}_i|^2 + |\bar{c}_i \bar{\mathcal{M}}_i|^2) dm_{\pi^+K^\mp}^2 dm_{K^+K^-}^2}{\int (|\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2) dm_{\pi^+K^\mp}^2 dm_{K^+K^-}^2}. \quad (4)$$

The contribution of the possible intermediate states in the total decay amplitude is tested through a procedure in which each component is taken in and out of the model, and that which provides the best likelihood is then maintained,

and the process is repeated. In some regions of the phase space the observed signal yields could not be well described with only known resonance states and line shapes, and thus alternative parametrizations were also tested.

In the $\pi^\pm K^\mp$ system, a nonresonant amplitude involving a single-pole form factor of the type $[1 + m^2(\pi^\pm K^\mp)/\Lambda^2]^{-1}$, as proposed in Ref. [14], is included. This component, hereafter called single-pole amplitude, is a phenomenological description of the partonic interaction. The parameter Λ sets the scale for the energy dependence and the proposed value of $1 \text{ GeV}/c^2$ is used.

In the $K^+ K^-$ system, a dedicated amplitude accounting for the $\pi\pi \leftrightarrow KK$ rescattering is used. It is expressed as the product of the nonresonant single-pole form factor described above and a scattering term that accounts for the S -wave $\pi\pi \leftrightarrow KK$ transition amplitude, with isospin equal to 0 and $J = 0$, given by the off-diagonal term in the S matrix for the $\pi\pi$ and KK coupled channel. The scattering term is expressed as $\sqrt{1 - \nu^2} e^{2i\delta}$, where the inelasticity (ν) and phase shift (δ) parametrizations are taken from Ref. [39]. For the mass range 0.95 to $1.42 \text{ GeV}/c^2$, where the coupling $\pi\pi \rightarrow KK$ is known to be important, these parameters are given by

$$\nu = 1 - \left(\epsilon_1 \frac{k_2}{s^{1/2}} + \epsilon_2 \frac{k_2^2}{s} \right) \frac{M^2 - s}{s} \quad (5)$$

and

$$\cot \delta = C_0 \frac{(s - M_s^2)(M_f^2 - s)}{M_f^2 s^{1/2}} \frac{|k_2|}{k_2^2}, \quad (6)$$

with parameters set as given in Ref. [39].

For all models tested in the analysis, the channel $B^\mp \rightarrow K^*(892)^0 K^\mp$ is used as reference, with its real part x fixed to one, y and Δy fixed to zero, while Δx is free to vary. The values of x , y , Δx , and Δy for all other contributions are free parameters. The masses and widths of all resonances are fixed [27].

The fit results are summarized in Table I. Seven components are required to provide an overall good description of data; three of them correspond to the structure in the $\pi^\pm K^\mp$ system, and four for the $K^+ K^-$ system. Statistical uncertainties are derived from the fitted values of x , y , Δx , Δy , with correlations and error propagation taken into account; sources of systematic uncertainty are also evaluated as described later.

The $\pi^\pm K^\mp$ system is well described by the contributions from the $K^*(892)^0$ and $K_0^*(1430)^0$ resonances plus the single-pole amplitude. The inclusion of the latter provides a better description of the data than that obtained from the $K_0^*(700)$, $K_2^*(1430)^0$, $K^*(1410)^0$, and $K^*(1680)^0$ resonances. The largest contribution is from the single-pole

TABLE I. Results of the Dalitz plot fit, where the first uncertainty is statistical and the second systematic. The fitted values of c_i ($\bar{c}_i$) are expressed in terms of magnitudes $|c_i|$ ($|\bar{c}_i|$) and phases $\arg(c_i)$ [$\arg(\bar{c}_i)$] for each B^+ (B^-) contribution. The top row corresponds to B^+ and the bottom to B^- mesons.

Contribution	Fit fraction (%)	A_{CP} (%)	Magnitude (B^+/B^-)	Phase [$^\circ$] (B^+/B^-)
$K^*(892)^0$	$7.5 \pm 0.6 \pm 0.5$	$+12.3 \pm 8.7 \pm 4.5$	$0.94 \pm 0.04 \pm 0.02$	0 (fixed)
$K_0^*(1430)^0$	$4.5 \pm 0.7 \pm 1.2$	$+10.4 \pm 14.9 \pm 8.8$	$1.06 \pm 0.04 \pm 0.02$	0 (fixed)
Single pole	$32.3 \pm 1.5 \pm 4.1$	$-10.7 \pm 5.3 \pm 3.5$	$0.74 \pm 0.09 \pm 0.09$	$-176 \pm 10 \pm 16$
$\rho(1450)^0$	$30.7 \pm 1.2 \pm 0.9$	$-10.9 \pm 4.4 \pm 2.4$	$0.82 \pm 0.09 \pm 0.10$	$136 \pm 11 \pm 21$
$f_2(1270)$	$7.5 \pm 0.8 \pm 0.7$	$+26.7 \pm 10.2 \pm 4.8$	$2.19 \pm 0.13 \pm 0.17$	$-138 \pm 7 \pm 5$
Rescattering	$16.4 \pm 0.8 \pm 1.0$	$-66.4 \pm 3.8 \pm 1.9$	$1.97 \pm 0.12 \pm 0.20$	$166 \pm 6 \pm 5$
$\phi(1020)$	$0.3 \pm 0.1 \pm 0.1$	$+9.8 \pm 43.6 \pm 26.6$	$2.14 \pm 0.11 \pm 0.07$	$-175 \pm 10 \pm 15$
			$1.92 \pm 0.10 \pm 0.07$	$140 \pm 13 \pm 20$
			$0.86 \pm 0.09 \pm 0.07$	$-106 \pm 11 \pm 10$
			$1.13 \pm 0.08 \pm 0.05$	$-128 \pm 11 \pm 14$
			$1.91 \pm 0.09 \pm 0.06$	$-56 \pm 12 \pm 18$
			$0.86 \pm 0.07 \pm 0.04$	$-81 \pm 14 \pm 15$
			$0.20 \pm 0.07 \pm 0.02$	$-52 \pm 23 \pm 32$
			$0.22 \pm 0.06 \pm 0.04$	$107 \pm 33 \pm 41$

amplitude with a total fit fraction of about 32%. The $K^*(892)^0$ and the $K_0^*(1430)^0$ amplitudes contribute to 7.5% and 4.5%, respectively. Given that they originate from penguin-diagram processes, their contributions to the total rate are expected to be small. The projection of the data onto $m_{\pi^\pm K^\mp}^2$ with the fit model overlaid, is shown in Fig. 2.

In the K^+K^- system, two main signatures can be highlighted: a strong destructive interference localized between 0.8 and 3.3 GeV^2/c^4 in $m_{K^+K^-}^2$ and projected

between 12 and 20 GeV^2/c^4 in $m_{\pi^\pm K^\mp}^2$, as shown in Fig. 1; and the large CP asymmetry for $m_{K^+K^-}^2$ corresponding to the $\pi\pi \leftrightarrow KK$ rescattering region, as shown in Fig. 3. For the former, a good description of the data is achieved only when a high-mass vector amplitude is included in the Dalitz plot fit, producing the observed pattern through the interference with the $f_2(1270)$ amplitude. The data are well described by assuming this contribution to be the $\rho(1450)^0$ resonance, included in the fit with mass and width

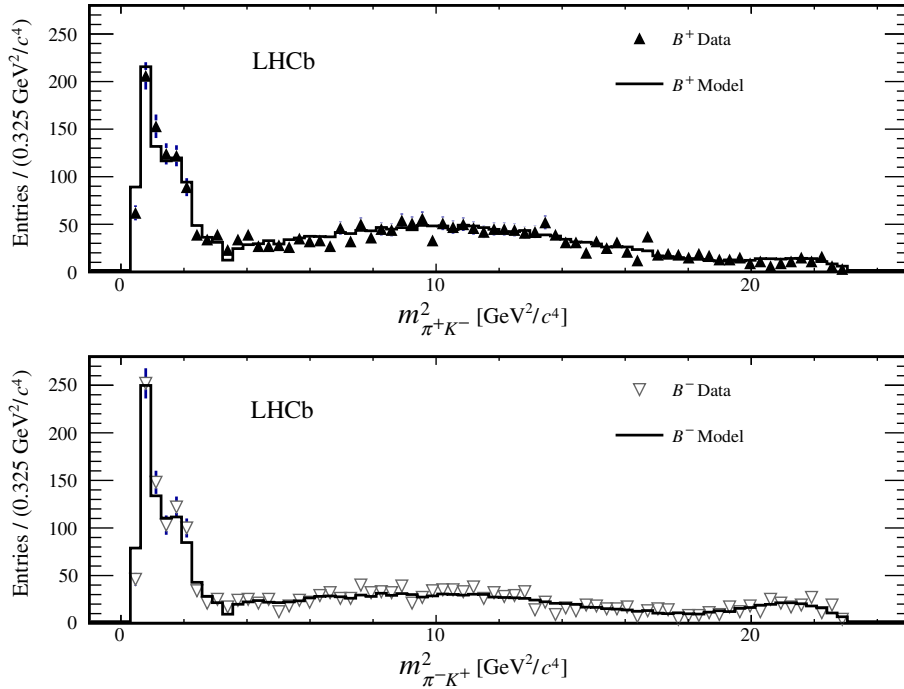


FIG. 2. Distribution of $m_{\pi^\pm K^\mp}^2$. Data are represented by points for B^+ and B^- candidates separately, with the result of the fit overlaid.

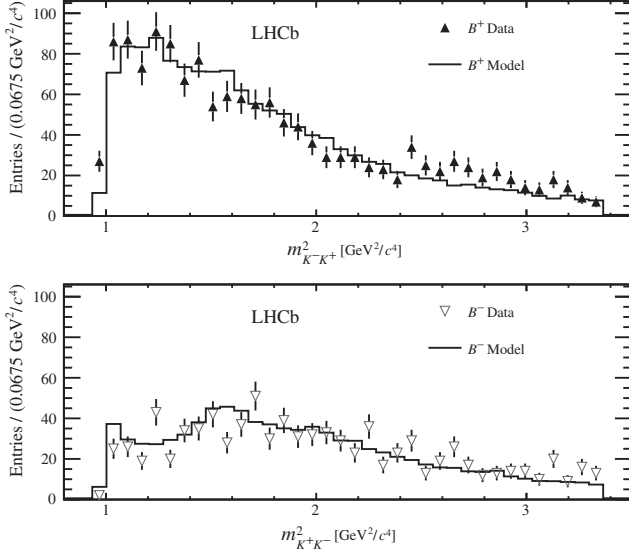


FIG. 3. Distribution of $m^2_{K^+K^-}$ up to $3.5 \text{ GeV}^2/c^4$. Data are represented by points for B^+ and B^- separately, with the result of the fit overlaid.

fixed to their known values [27]. The corresponding $B^\pm \rightarrow \rho(1450)^0 \pi^\pm$ fit fraction is approximately 30%, a rather large contribution not expected for the K^+K^- pair as the dominant decay mode is $\pi\pi$ and the $\rho(1450)^0$ contribution in $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ is observed to be much lower [40,41]. A future analysis with the addition of the run 2 data recorded with the LHCb detector should be able to better pinpoint this effect.

With respect to the low $m^2_{K^+K^-}$ region, shown in Fig. 3, a significant contribution with a fit fraction of 16% from the $\pi\pi \leftrightarrow KK$ S -wave rescattering amplitude is found. This contribution alone produces a CP asymmetry of $(-66 \pm 4 \pm 2)\%$, which is the largest CPV manifestation ever observed for a single amplitude. This must be directly related to the total inclusive CP asymmetry observed in this channel, which was previously reported to be $(-12.3 \pm 2.1)\%$. For the coupled channel $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$, with a branching fraction 3 times larger than that of $B^\pm \rightarrow \pi^\pm K^+ K^-$, a positive CP asymmetry has been measured [1]. This gives consistency for the interpretation of the large CPV observed here originates from rescattering effects. Finally, the inclusion of the $\phi(1020)$ resonance in the amplitude model also improves the data description near the $K^+ K^-$ threshold, however, with a statistically marginal contribution. The model is also not perfect in other regions in $m^2_{K^+K^-}$, for instance, for B^+ decays in a few bins above $2.5 \text{ GeV}^2/c^4$.

A second solution is found in the fit, presenting a large CP asymmetry of 76% in the $K_0^*(1430)^0$ component, compensated by a similarly large negative asymmetry in the interference term between the $K_0^*(1430)^0$ and the single-pole amplitudes. The net effect is a negligible CP

asymmetry near the $K_0^*(1430)^0$ region, matching what is seen in the data. This solution presents a large sum of fit fractions for the B^- decay, of about 130%, indicating this is probably a fake effect created by the fit. As such, this solution is interpreted as unphysical. More data are necessary to understand this feature.

Several sources of systematic uncertainty are considered. These include possible mismodelings in the mass fit, the efficiency variation and background description across the Dalitz plot, the uncertainty associated to the fixed parameters in the Dalitz plot fit and possible biases in the fitting procedure.

The impact of the systematic studies affect differently each of the amplitudes. The main contribution comes from the variation of the masses and widths of the resonances; their central values and uncertainties are taken from Ref. [27] and are randomized according to a Gaussian distribution. This effect is particularly important for the $K_0^*(1430)^0$ and single-pole components, the two broad scalar contributions in the $\pi^\pm K^\mp$ system. The absolute uncertainties on their fractions are found to be 0.8% and 3.0%, respectively. The second main contribution comes from the $\pi^\pm K^+ K^-$ mass fit, impacting most on the $K^*(892)^0$, $K_0^*(1430)^0$ and single-pole fractions with uncertainties of 0.4%, 0.8%, and 2.0%, respectively. The systematic uncertainty associated with efficiency variation across the Dalitz plot is studied by performing several fits to data with efficiency maps obtained by varying the bin contents of the original efficiency histogram according to their uncertainty; this results in uncertainties in the fit fractions that range from 0.01% to 0.1%. The systematic uncertainty due to the background models is evaluated with a similar procedure, also resulting in small uncertainties. The $B^\pm$ production and kaon detection asymmetry effects are taken into account following Ref. [42], with associated uncertainties less than 0.1%. All systematic uncertainties are added in quadrature and represent the second uncertainty in Table I.

In summary, the resonant substructure of the charmless three-body $B^\pm \rightarrow \pi^\pm K^+ K^-$ decay is determined using the isobar model formalism, providing an overall good description of the observed data. Three components are obtained for the $\pi^\pm K^\mp$ system: two resonant states [$K^*(892)^0$, $K_0^*(1430)^0$] with a CP asymmetry consistent with zero, and a nonresonant single-pole form factor contribution with a fit fraction of about 30%. Two other components are found, $\rho(1450)^0$ and $f_2(1270)$, which provide a destructive interference pattern in the Dalitz plot. The rescattering amplitude, acting in the region $0.95 < m(K^+ K^-) < 1.42 \text{ GeV}/c^2$, produces a negative CP asymmetry of $(-66 \pm 4 \pm 2)\%$, which is the largest CP violation effect observed from a single amplitude.

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