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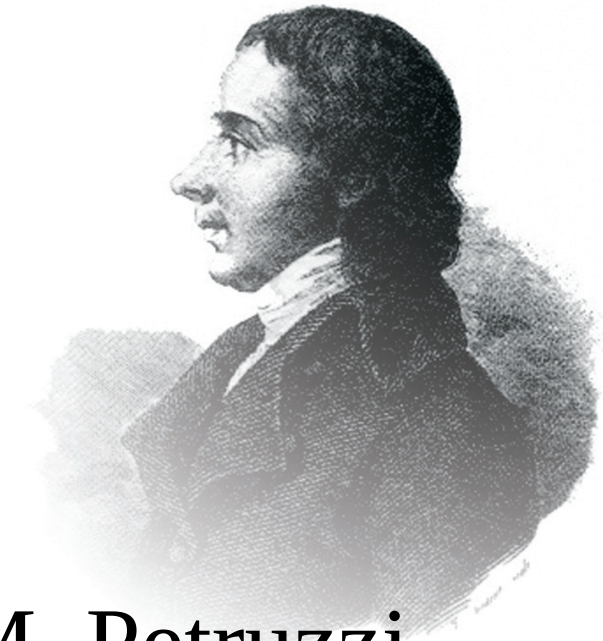

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Serie Ricerca

Dipartimento
di Matematica, Statistica,
Informatica e Applicazioni
"Lorenzo Mascheroni"

UNIVERSITÀ DEGLI STUDI DI BERGAMO

ON THE ORIGIN OF THE MOON: A REVIEW OF CURRENT THEORIES AND A FOUR-BODY SCENARIO FOR A RECENT CAPTURE EVENT

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January 2008

Abstract

The problem of Moon origin has been studied since centuries. Till man brought back lunar rocks, a commonly accepted theory (among over thirty presented) considered that Moon arose together with Earth from the same cloud of gas and dust whose condensation, via complex processes, is thought to explain the formation of planets. However analysis of the rocks showed a different isotopic structure than the terrestrial rocks, a fact that invalidates the common origin theory. Here we review some of the new theories, the dominant one being that Moon formed from material ejected in space, but remaining gravitationally tied to Earth, after a giant tangential impact with a body of size similar to Mars. Then we give arguments from ancient traditions and religions that Moon's origin may be quite recent, probably to be dated at circa 9450 BC, when Earth captured it from a large body passing by. This passage was catastrophic to our planet, ending rapidly the

last Ice Age and the associated Atlantis civilization. We also argue that Shinto's divinity was originally not Sun, but Moon.

0.1 Some theories on the origin of the Moon

The problem of the origin of the Moon is still one of the open problems in astronomy. Indeed up to the first collection of lunar rocks, while several theories had been presented in the literature, it was widely believed that the Moon originated coevally with Earth from the giant gas and dust cloud, consisting of material emitted by the explosion of an earlier star (so that our Sun is a second generation star), whose condensation led to the formation of Sun, of the terrestrial type planets, and of the external gaseous planets. When the Kuiper belt existence passed from a hypothesis to a fact, it too was considered to have been formed by this process. The more distant and spherically distributed Oort cloud, whose existence is doubted by some, is considered, see e.g. Clube and Napier (1) to be formed to a large extent by material captured in different parts of the galaxy when the solar system crosses a molecular cloud (where part of the existing Oort cloud is also lost). The above scenario was due to a large extent to the work of Whetherill (2), who concluded that terrestrial type planets could form only in the vicinity of Sun, say within half a billion km, while the large gas planets could form only beyond. This main point of Whetherill scenario crashed with the discovery in the last 10 years or so of several (more than 200 presently) planetary systems around other stars. Here large gas planets were found very close to the star, even only about 1 million km. Even considering that the observed systems are biased towards those with large planets and that only a handful of small planets have been found, it is clear that Whetherill model must have unacceptable simplifications and hypotheses. The first mechanism able to explain why the large planets are so close to the star has been proposed by Del Popolo (3,4) Moreover additional mathematical computations (5) have shown that large gas planets can form not in times order 10 million years, as it was before estimated, but just in about one century! And quite strong arguments by Ackerman (6,7) even indicate that the so called gas planets may have a solid inner part, consisting of clathrates and other materials.

The problem of satellite formation is similar but possibly more rich of solutions. Indeed satellites can form coevally with the planet, but they can also arise from later impacts or capture of bodies being originally free or gravitationally tied to another body. As said before for the case of our satellite, the Moon, the coeval formation, that was preferred till not long ago, has now been dropped since analysis of the lunar rocks has shown that they have a different isotopic composition than Earth. A mechanism that would separate particles by their isotopic constitution seems not to be available. Hence one has looked at other possible origins, in particular to the three body capture and to a large impact with debris condensation in a circum Earth orbit. The last mechanism is the one presently accepted by most astronomers. It assumes that a body of mass estimated order of that of Mars impacted Earth at the beginning of its life by a tangential hit. The material of the body essentially vaporized with part of Earth material. Part of the product fell later over Earth, part escaped to the space, part condensed around Earth forming the Moon. The process has been simulated with different parameters and by assigning a suitable composition to the impacting body one can explain the mineral composition of the Moon.

In the following section we give a partial presentation of additional scenarios recently presented in the literature. In the last section we consider a scenario apparently never considered in the literature, namely a capture within a 4-body gravitational system. We justify the idea using references to a recent lunar capture in the religious and mythologic traditions. Then we discuss a possible validation of such a scenario by mathematical modelling and integration of the dynamical equations via the very accurate ODE code due to Trigiante and Brugnano.

0.2 Some recent theories on lunar formation

In the following we list a number of theories that have appeared in the recent literature. Our survey is certainly not complete, but is based upon a lengthy search of the literature mainly via internet. We refer to articles, papers and conference presentations.

1 - Szebehely and R. T. Evans, The University of Texas, Austin, Tex., USA On the capture of the Moon, Springer, 21, 3, 1980

This paper studies the possibility of lunar capture depending on variations of the solar mass under certain well specified conditions and assumptions regarding the behavior of the three-body dynamical system formed by the Sun, Earth and Moon. It is found that a large amount of decrease in the solar mass (approximately 37%) would be required to allow capture if the model of the planar restricted problem of three bodies is assumed, if the masses of the Earth and Moon did not change and if the angular momentum of the Sun-Earth system did not change. Such large mass-changes of the Sun can not be associated with radiation mass losses but only with catastrophic events, such as stellar close approaches.

2 - S. Fred Singer - University of Virginia, Charlottesville, Virginia, USA, Origin of the moon by tidal capture and some geophysical consequences, Earth, Moon, and Planets, Springer, 5, 1-2, 1972

Of the many proposed modes of origin of the Moon, some violate physical laws; many are in conflict with observations; all are improbable. Perhaps the least improbable - based on recent tidal theory calculations and on the interpretation of lunar rock data - is capture of the Moon as it passed near the Earth in a direct (prograde) orbit, shortly after the formation of Moon and Earth, about 4.5 billion years ago. (Capture of the Moon from an initially retrograde orbit as proposed some years ago, leads to physically unacceptable consequences.) The effects of capture on the Earth would have been cataclysmic, leading to intensive heating of its interior, to volcanism, and to the immediate formation of an atmosphere and hydrosphere. Thus capture of a Moon may have given rise to the unique properties of the Earth (in the Solar System) and to the early evolution of life, about 3.5 billion years ago.

3 - Gary Peterson, Ph.D., San Diego State University, 2006
Capture of the Moon

We were taught in introductory geology courses that the Moon was made of material that was ejected from the proto-Earth some 4+ billion years ago, while Gary Peterson, the Independent Planetologist, is proposing a different, perhaps controversial mechanism. To wit, he will set forth the hypothesis that the Moon is a planetesimal body that was gravitationally captured early in Earth history.

4 - Kiyoshi Nakazawa, Teruyoshi Komuro and Chushiro Hayashi,
Origin of the Moon. Capture by gas drag of the Earth's primordial atmosphere
Earth, Moon, and Planets, Springer, 28, 3, 1983

We propose a new scenario of the lunar origin, which is a natural extension of planetary formation processes studied by us in Kyoto. According to these studies, the Earth grew up in a gaseous solar nebula and, consequently, the sphere of its gravitational influence (i.e., the Hill sphere of the Earth) was filled by a gas forming a dense primordial atmosphere. In the later stages, this atmosphere as well as the solar nebula was dissipated gradually, owing to strong activities of the early-Sun in a T Tauristage. In the present and the subsequent papers, we study a series of dynamical processes where a low energy (i.e., slightly unbound) planetesimal is trapped within the terrestrial Hill sphere, under the above-mentioned circumstances that the gas density of the primordial atmosphere is gradually decreasing. It is clear that two conditions must be satisfied for the lunar origin: first, an unbound planetesimal entering the Hill sphere dissipates its kinetic energy and comes into a bound orbit before it escapes from the Hill sphere and, second, the bound planetesimal never falls onto the surface of the Earth. In this paper we study the first condition by calculating the orbital motion of a planetesimal in the Hill sphere, which is affected both by solar gravity and by atmospheric gas drag. The results show that a low energy planetesimal with lunar mass or less can be trapped in the Hill sphere with a high probability, if it enters the Hill sphere at stages before the atmospheric density is decreased to about 1/50 of the initial value. In the subsequent paper, the second condition will be studied and it will be shown that a tidal force, among other forces, is very important for a trapped planetesimal to avoid collision with Earth and stay eternally in the Hill sphere as a satellite.

5 - Antonio F B. de A Prado, Instituto Nacional de Pesquisas Espaciais 12227-010 São José dos Campos, SP. Brazil,prado@dem.inpe.br

Analytical and Numerical Approaches to Study the Gravitational Capture in the Four-Body Problem Report July-September 2006, Vol. XXVIII, No. 3

The objective of this paper is to study the problem of gravitational capture in the bicircular restricted four-body problem. A gravitational capture occurs when a massless particle changes its two-body energy around one celestial body from positive to negative without the use of non-gravitational forces. We mainly study the effect of the fourth-body included in the dynamics. The Earth-Moon system with the inclusion of the Sun is used for the numerical simulations. The results show the savings obtained in this more realistic model when compared with the more traditional restricted three-body problem model. It is clear that large savings are obtained thanks to the effect of the Sun, if a proper geometry is used for the maneuver.

6 - James K. Miller, Jet Propulsion Laboratory, CALTECH, Pasadena, CA
lunar transfer trajectory design and the four-body problem

The existence of a ballistic trajectory from the Earth to orbit about the Moon was long considered to be impossible based on analysis of the three-body problem. In 1990 a ballistic trajectory from the Earth to lunar orbit was discovered while analyzing a plan to salvage the Muses A (Hiten) spacecraft. This trajectory utilized the Sun's gravity in conjunction with the Earth and Moon's gravity and was thus the first example of a practical four-body trajectory design. This paper presents a review of lunar transfer trajectories that go beyond three-body theory and the Jacobi integral. These include Hiten, lunar A and the Genesis return trajectory from the vicinity of the Moon to Earth. It is shown that these trajectories may be analyzed by piecing together segments where three-body motion dominates.

7 - Sergey Astakhov, R and D, UniqueICs, Saratov, Russia NIC, Forschungszentrum Juelich, Germany
Chaos-assisted capture in the Hill 4-body problem: Kuiper-belt binary

8 - Antonio Fernando Bertachini de Almeida Prado Ernesto Vieira Neto, Instituto Nacional de Pesquisas Espaciais São José dos Campos - SP - 12227-010 - Brazil Phone (123)41-8977 - Fax (123)21-8743

The objective of this paper is to study the problem of gravitational capture in the regularized restricted three-body problem. A gravitational capture occurs when a massless particle changes its two-body energy around one celestial body from positive to negative without the use of non-gravitational forces. We studied the importance of several of the parameters involved for a capture in the Earth-Moon system, including the time required for the capture and the effects of the periapse distance. Then, we generalize those results for other binary systems, like the Sun -Earth, Sun-Mars and Sun-Jupiter systems. Next, we cover the whole interval of the mass parameter m (the mass ratio of the two primaries) and we study the gravitational capture in the interval (0.0, 0.5) for m. The elliptical restricted problem is also considered as an option for the model.

9 - Jerry K. Cline

Satellite aided capture Celestial Mechanics and Dynamical Astronomy, Springer, 19, 4, 1979

A two body, patched conic analysis is presented for a planetary capture mode in which a gravity assisted by an existing natural satellite of the planet aids in the capture. An analytical condition sufficient for capture is developed and applied to some planet/satellite systems: Earth/Moon, Jupiter/Ganym+ede, Jupiter/Callisto, Saturn/Titan and Neptune/Triton. Co-planar, circular planetary orbits are assumed. Three sources of bodies to be captured are considered: spacecraft launched from Earth, bodies entering the solar system from interstellar space, and bodies already in orbit around the Sun. Results show that the Neptune/Triton system has the most capability for satellite aided capture of those studied. It can easily capture bodies entering the Solar System from interstellar space. Its ability to capture spacecraft launched from Earth is marginal and can only be decided with better definition of physical properties. None of the other systems can capture bodies from these two sources, but all can capture bodies already in orbit around the Sun under appropriate conditions.

10 - R. R. Winters and R. J. Malcuit

The lunar capture hypothesis revisited Earth, Moon, and Planets, Springer, 17, 4, 1977

Recent work on planetary formation processes has suggested that ancient planetary bodies could have been warmer and, therefore, more easily deformable soon after formation than at present. By use of the estimates for the elastic parameters believed to be appropriate for a warm ancient Moon and Earth, it is shown that the energy of deformation of the planetary bodies during a close gravitational encounter was sufficient to effect capture.

11 - Adrian Brunini and Conicet

Capture of planetesimals by the giant planets Earth, Moon, and Planets, Springer, 71, 3, 1995

We investigate the possibility of gravitational capture of planetesimals as temporary or permanent satellites of Uranus and Neptune during the process of planetary growth. The capture mechanism is based on the enhancement of the Hill's sphere of action not only due to the mass acquired by the planet, but also by variation of the planet-Sun distance as a consequence of the scattering of planetesimals by the planets of the outer solar system. Our calculations indicate that satellite capture was very important, especially during the first stages of the accretion process, contributing in a significant way to the planetary growth.

12 - Adrian Brunini

On the satellite capture problem Celestial Mechanics and Dynamical Astronomy, Springer, 64, 1-2, 1996

The stability and capture regions in phase space for retrograde and direct planetary satellites are investigated in the frame of the Circular Restricted Three-Body Problem. We show that a second integral of motion furnishes an accurate description for the stability limit of retrograde satellites. The distribution of heliocentric orbital elements is studied, and possible candidates to be temporary Jovian satellites are investigated. Previous results, limited to orbits satisfying the Mirror Theorem, are extended to give a complete set of capture conditions in phase-space.

13 - F. Szenkovits and Z. Makò, Astron. Dept. of the Eotvos Univ. 2005

Pulsating zero velocity surfaces and capture in the elliptic restricted three-body problem

Zero velocity surfaces are deduced in the gravitational restricted three-body problem by using the Jacobi integral. These surfaces are the boundaries of the Hill's regions: regions where the motion of the third, massless particle around the two primaries is possible. V. Szebehely generalized this result for the planar elliptic restricted three body problem. In a recent paper (Mak´o and Szenkovits (2004)) the authors presented a generalization of this result for the spatial elliptic restricted three-body problem, where the existence of an invariant relation was proved. From this invariant relation the equation of the zero velocity surfaces can be deduced. In this paper we discuss the pulsation and the change of the type of these zero velocity surfaces and we present applications to the phenomenon of the gravitational capture. In the model of the spatial elliptic restricted three-body problem criteria of the capture are deduced by using the pulsating Hill's regions.

14 - O. C. Winter and E. Vieira Neto, Grupo de Dinâmica Orbital and Planetologia, UNESP, CP 205, Guaratinguetá, CEP 12500-000, SP, Brazil, August 2001

Time analysis for temporary gravitational capture

In a previous work, Vieira Neto and Winter (2001) numerically explored the capture times of particles as temporary satellites of Uranus. The study was made in the framework of the spatial, circular, restricted three-body problem. Regions of the initial condition space whose trajectories are apparently stable were determined. The criterion adopted was that the trajectories do not escape from the planet during an integration of 105 years. These regions occur for a wide range of orbital initial inclinations. In the present work we study the reason for the existence of such stable regions. The stability of the planar retrograde trajectories is due to a family of simple periodic orbits and the associated quasi-periodic orbits that oscillate around them. These planar stable orbits had already been studied (Hénon 1970; Huang and Innanen 1983). Their results are reviewed using Poincaré surface of sections. The stable non-planar retrograde trajectories, are found to be tridimensional quasi-periodic orbits around the same family of periodic orbits found for the planar case. It was not found any periodic orbit out of the plane associated to such quasi-periodic orbits.

15 - A. Prado

Gravitational capture by the major primary in the restricted three-body

problem.

The ballistic gravitational capture is a characteristic of some dynamical systems in celestial mechanics, as in the restricted three-body problem that is considered in this paper. The basic idea is that a spacecraft (or any particle with negligible mass) can change from a hyperbolic orbit with a small positive energy around a celestial body into an elliptic orbit with a small negative energy without the use of any propulsive system. The force responsible for this modification in the orbit of the spacecraft is the gravitational force of another body involved in the dynamics. In this way, this force is used as a zero cost control, equivalent to a continuous thrust applied in the spacecraft. The present paper studies in some detail the ballistic gravitational capture performed by the first primary in a three body system. Analytical equations for the forces involved in this maneuver are derived to estimate their magnitude and to show the best directions of approach for the maneuver. The model used to study this problem is the planar restricted three-body problem. Several systems of primaries are considered for the simulations shown in this paper. The paper also shows an explanation of the phenomenon based on the calculation of the forces involved in the dynamics as a function of time and in their integration with respect to time. Analytical equations are derived to study this problem under the assumption of radial motion, which leads to the derivation of an equation that estimates the reduction of C3. Then, the forces acting on the ballistic gravitational capture problem are obtained in closed forms. There are two forces that act as disturbing forces in the direction of motion: the gravitational force due to the Moon and the centrifugal force. These forces can decelerate the spacecraft, working opposite to its motion. This is equivalent to applying a continuous propulsion force against the motion of the spacecraft. In the radial direction the gravitational force due to the Moon and the centrifugal force work in opposite directions, but the resultant force always works against the motion of the spacecraft.

16 - Craig B. Agnor and Douglas P. Hamilton

Neptune's capture of its moon Triton in a binary-planet gravitational encounter Nature, 411, pp. 192-194, May 11, 2006

Triton is Neptune's principal satellite and is by far the largest retrograde satellite in the Solar system (its mass is 40 per cent greater than that of Pluto). Its inclined and circular orbit lies between a group of small inner prograde satellites and a number of exterior irregular satellites with both prograde and retrograde orbits. This unusual configuration has led to the belief that Triton originally orbited the Sun before being captured in orbit around Neptune. Existing models for its capture however, all have significant bottlenecks that make their effectiveness doubtful. Here we report that a three-body gravitational encounter between a binary (of 103-kilometre-sized bodies) and Neptune is a far more likely explanation for Triton's capture. Our model predicts that Triton was once a member of a binary with a range of plausible characteristics, including ones similar to the Pluto/Charon pair.

17 - Ernesto Vieira Neto, Antonio Fernando Bertachini de Almeida Prado

A Study of the Gravitational Capture in the Restricted-Problem

The objective of this paper is to study the problem of gravitational capture in the regularized restricted three-body problem. A gravitational capture occurs when a massless particle changes its two body energy around one celestial body from positive to negative without the use of non-gravitational forces. We studied the importance of several of the parameters involved for a capture in the Earth-Moon system, including the time required for the capture. Then, we generalize those results for other binary systems, like the Sun-Earth, Sun-Mars and Sun-Jupiter systems. Next, we cover the whole interval of the mass parameter m (the mass ratio of the two primaries) and we study the gravitational capture in the interval (0.0 0.5) for m.

18 - W. S. Koon, M.W. Lo, J. E. Marsden and S. D. Ross

Resonance and capture of Jupiter comets

A number of Jupiter family comets such as Oterma and Gehrels 3 make a rapid transition from heliocentric orbits outside the orbit of Jupiter to heliocentric orbits inside the orbit of Jupiter and vice versa. During this transition, the comet can be captured temporarily by Jupiter for one to several orbits around Jupiter. The interior heliocentric orbit is typically close to the 3:2 resonance while the exterior heliocentric orbit is near the 2:3 resonance. An important feature of the dynamics of these comets is that during the transition, the orbit passes close to the libration points L1 and L2, two of the equilibrium points for the restricted three-body problem for the Sun-Jupiter system. Studying the libration point invariant manifold structures for L1 and L2 is a starting point for understanding the capture and resonance transition of these comets. For example, the recently discovered heteroclinic connection between pairs of unstable periodic orbits (one around the L1 and the other around L2) implies a complicated dynamics for comets in a certain energy range. Furthermore, the stable and unstable invariant manifold 'tubes' associated to libration point periodic orbits, of which the heteroclinic connections are a part, are phase space conduits transporting material to and from Jupiter and between the interior and exterior of Jupiter's orbit.

Above we have given a sample, certainly not complete, of the recent research on capture. These papers deal mainly with the three-body problem, where capture occurs only under special conditions. Quite remarkable is the paper by the Japanese Nakazawa et al., who obtain capture if there is braking by sufficiently dense and extended atmosphere, which might have existed billion of years ago. For such a case they have the very remarkable result that the orbit of the captured Moon, after an initial phase of complex behaviour, gets circularized quite soon, actually in only 10-20 years!

No paper have we found that proposes a four body capture for the Moon. Some 4-body problems appear in the context of a small body, e.g. an airship, moving from Earth to Moon, the 4 bodies being then the small body, Earth, Moon and Sun. In the next section we give our rationale for considering exactly this process, and moreover for hypothesizing that it was accomplished within human memory, at the sudden end of the last Ice Age, about 9450 BC. We notice that Van Flandern, a top expert in orbital analysis, in an email to Spedicato (October 27, 2007) has commented that 4-body capture is indeed possible, but requires fine tuning to get a lunar orbit bounded to Earth and to circularize it. According to him this would be an unusual process, a statement that we fully agree with. Also one should explain the reason of similarities between lunar rocks and Pacific basalts, a fact that was one of the reasons why authors as De Grazia claimed that Moon originated from the expulsion of terrestrial material from the Pacific area. Of course the scenario we present in the following need further developments to deal with such questions.

0.3 Recent origin of the Moon in a 4-body capture event

Velikovsky in an unpublished paper, available on the site containing his unpublished papers and due to Jan Sammer, briefly considered the possibility that the Moon was recently captured, in view of certain references to this question in Bible and other sources.

Here we quote some references, all obtained by us in casual reading of books dealing with ancient traditions.

1 - the Chimu were a people of the coastal civilization in Peru, more ancient than the Incas civilization, which took over only around the 14th century AD. For the Chimu the Moon appeared at a certain time in the sky and had a father, apparently the god Pachamacac

2 - the Malekula are a tribe of the New Guinea mountains (where about 700 languages are spoken and there is a great genetic variation among tribes). They claim that there was a time when vapors arose and it was almost impossible to see even at short distance. When the sky cleared they saw that the sea level had arisen and many lands had disappeared. Moreover in the sky the Moon had appeared.

3 - the Hindu claim that the Moon appeared after the sea had boiled.

Human memory may go back several thousand years, possibly even about 9-10 thousand years as the memory of the great catastrophe that destroyed Atlantis shows, if it is correct that that event corresponds to the end of the last Ice Age, as suggested by Muck, Barbiero, Spedicato (8, 9, 10) and others. In a recent monograph (11) we have proposed that the most likely cause of the end of the Ice Age, and of the rapid melting of ices, was the close passage of a large body. Such passage due to tidal effects deformed our planet, broke the thin crust of the oceans bottom, let huge amounts of magma pour out producing immense amount of rains. The clouds coming from the oceans let people think the sea was boiling, therefore explaining a detail in 2 and 3. Ices melted, sea water level increased, another detail explained in the Malekula perfectly correct memory.

The vision of the new Moon in the sky after these events can be explained if the passing body had a satellite that was lost to Earth, a fact that is perfectly possible in a 4-body context. Then there was a time with the Moon and a time without, as Job claims in a Bible passage.

The considered process is rather smooth, the passing by of the main body being certainly the main cause of the great catastrophe, the biggest one in Plato's list. However we are faced with the fact that the geological structures called varves are due to tidal effects of our satellite and extend in time well before the end of the last Ice Age. This problem can be solved if the capture of the Moon meant the loss also of another previous satellite and there are some arguments that such a satellite was....Mars. Indeed we have the following items:

A - in Censorinus *De die natali* we read that *the Arcadians claim, but I do not believe it, that before the Moon was the year had not 12 but 3 months*

B - an important symbol in Islamic world, whose origin is not Arabic but Turkish (the Turkish area has some of the oldest traditions in the world only partly explored, as the Epic of Manas of the Kirghisians consisting of some 6 million verses), is the well known LARGE HALF MOON HAVING INSIDE A SMALL FIVE POINTED STAR.

The statement in Censorinus, which was an incredible bonus for me because I believed impossible to have an estimate of the distance of a previous satellite, can be interpreted by the existence, previous to the Moon, of another satellite having about 3 cycles per year, hence providing 3 months per year. By Kepler's third law we can estimate that such a satellite was much more far away than the Moon, about 2 million km from us.

The Turkish symbol can be interpreted as the new Moon appearing larger in the sky than the previous satellite and with well defined phases. The star inside, having 5 points, suggests Mars, which is the 5th body from the Sun. Being smaller it suggests that it loomed in the sky smaller than the Moon.

Several features of Mars suggest a previous close relation with Earth, as the rotation period similar to our planet and the similar axis angle on the ecliptic. Moreover the recent evidence of water only recently lost by Mars and in a catastrophic way is quite compatible with the catastrophic removal that we prospect here. Notice that Mars would have been in the habitable zone, hence life was probably existing on it.

Now Mars is about 2 times larger than Moon but is darker, having lower albedo. Due to the distance about 5 times more, its light was reduced a factor 25 by distance, increased a factor 4 by surface size, reduced maybe another factor 2 by albedo. Hence total luminosity was probably about 10 times less than present luminosity of the Moon and angular diameter was less than half.

Therefore Moon appeared as a real *hue lean* in Chinese, i.e. *lamp in the sky.* Therefore it was seen as a gift by gods and a positive element in the world after the great catastrophe that terminated Ice Age and the Atlantis civilization. There are, to my knowledge, no records of the Moon having a bad notation in ancient records.

Our scenario can be validated by rather simple, in principle, mathematical simulation. The problem is to choose the initial parameters correctly, which can be done heuristically, perhaps by imbedding the problem in another bigger one. We expect in a next future to have the results from the following analysis:

1 - a 4-body model for the capture of the Moon, assuming as parameters different values of the mass of the passing by body, of its minimum distance from Earth orbit (order should be even less than 100.000 km) and of initial average distance of its satellite lost to Earth

2 - a 5-body model for the removal as well of Mars, the additional parameter here

being the position of Mars relative to the two main bodies at moment of passage.

The main difficulty is that one must start the integration of the motion equations (under the simplification of point like mass, this to be removed in a further improvement of the model) with the external body quite far away so that one can estimate the orbital corrections due to the interaction of the two bodies, and there is no simple way of providing the initial conditions on its position and velocity, as well as the initial position of Earth along its orbit.

Another extension could be considered, namely to follow the evolution of the removed Mars till present circularization of its orbit. But this would probably involve also other agents in the period that Velikovsky called catastrophist and De Grazia quantavolution. This is work for a team and for several years.

0.4 Appendix : the code GAM

The problem of analyzing a 4-body lunar capture will be dealt with by using the code GAM due to Brugnano and Trigiante for the solution of ODE (ordinary differential equations). A particular class of block-BVMs, namely the Generalized Adams Methods (GAMs), have been implemented in the code GAM. Block-BVMs have the property of being defined by a set of linear multistep methods (LMMs) and to belong to the class of Runge-Kutta methods. As a consequence they posses features of both LM and RK formulae. With RK-formulae they share good stability properties for high order methods, a stepsize variation strategy typical of one-step schemes (which is simpler than those used for LMMs), but they suffer from a high computational effort per step. On the other hand, they have the same representation of the local truncation error as LM-formulae, and this allows one to use a simple technique for varying the order while integrating the IVP.

The generalized Adams methods of order 3, 5, 7 and 9 may be identified by pairs (A_s^e, B_s^e), $s \in I$, $I = [3,5,7,9]$, of $N_s \times N_s + 1$ matrices used as linear operator $H^{N_s+1} \rightarrow H^N$.

Let the initial value problem be of the form

$$\begin{cases} \mathbf{y}' = \mathbf{f}(t, \mathbf{y}), \quad t \geq t_0 \;\; \mathbf{f} : \mathbb{R} \times H \rightarrow H, \\ \mathbf{y}(t_0) = \mathbf{y}_0, \end{cases} \tag{1}$$

where $H = \mathbb{R}^r$. The integration procedure is typical of a standard one-step implicit formula with the difference that now the method needs to be constant while advancing in time. At step n an integer s is chosen in the set I by a suitable criterion and the method (A_s^e, B_s^e) is used to approximate the true solution, say $\widehat{\mathbf{y}}(t)$, on the time block $W_n = [t^{(n-1)}, t^{(n)}]$. More precisely, if we want to advance the solution starting at the time t^{n-1}, we need to fix a stepsize h_n and therefore define W_n by means of the uniform mesh

$t^{(n-1)} = t_0^{(n)} < t_1^{(n)} < \ldots < t_{N_s}^{(n)} = t^{(n)}$, $t_i^{(n)} = t_{i-1}^{(n)} + h_n$. Finally we use the fixed GAM to obtain on that mesh a numerical solution represented by the vector $\mathbf{y}^{(n)} = [\mathbf{y}_1^{(n)}, \ldots, \mathbf{y}_{N_s}^{(n)}]$, where we do not include $\mathbf{y}_0^{(n)}$ since we assume that it coincides with the last component of $\mathbf{y}^{(n-1)}$ which has been previously computed.

Denoting by $\mathbf{a}_s$ and $\mathbf{b}_s$ the first columns of A_s^e and B_s^e, respectively, and performing the partitions

$$A_s^e = [\mathbf{a}_s, A_s], \qquad B_s^e = [\mathbf{b}_s, B_s],$$

the integration of (1) then requires the solution at each step of the following nonlinear problem:

$$A_s\mathbf{y}^{(n)} - h_n B_s F(\mathbf{y}^{(n)}) = \boldsymbol{\delta}_s^{(n)}, \quad s \in I, \tag{2}$$

where

$$F(\mathbf{y}^{(n)}) = [\mathbf{f}(t_1^{(n)}, \mathbf{y}_1^{(n)}), \ldots, \mathbf{f}(t_{N-s}^{(n)}, \mathbf{y}_{N_s}^{(n)})]^T,$$

and $\boldsymbol{\delta}_s^{(n)} = -\mathbf{a}_s\mathbf{y}_0^{(n)} + h_n\mathbf{b}_s\mathbf{f}(t_0^{(n)}, \mathbf{y}_0^{(n)})$ contains the known terms that were involved in the previous computed solution. An equivalent but more compact and sometimes more useful way to describe a general GAM is to directly use in (2) the extended matrices A_s^e B_s^e, that is

$$A_s^e\mathbf{y}^{e(n)} - h_n B_s^e F(\mathbf{y}^{e(n)}) = \mathbf{0}, \quad s \in I, \tag{3}$$

where now $\mathbf{y}^{(n)} = [\mathbf{y}_0^{(n)}, \ldots, \mathbf{y}_{N_s}^{(n)}]$ includes the known solution $t_0^{(n)}$ and the function F has an extra argument.

We now examine in more detail how the coefficient of the matrices A_e and B_e are defined for the class of GAMs. To simplify the notation, we omit the indices n and s in the above expression. The ith component of the system 3is really a k-step linear method with $k_1^{(i)}$ initial and $k_2^{(i)}$ final conditions, $k = k_1^{(i)} + k_2^{(i)}$, which may be written as

$$\mathbf{y}_i - \mathbf{y}_{i-1} = \sum_{j=-k_1^{(i)}}^{k_2^{(i)}} \beta_{ij}\mathbf{f}_{i+j}. \tag{4}$$

This formula computes $\mathbf{y}_i$ by means of approximations to the solution at $k_1^{(i)}$ left points and $k_2^{(i)}$ right points. The coefficients β_{ij} are uniquely determined by imposing the condition that the formula has maximum order $p = k+1$, while if the index ν is defined as

$$\nu = \begin{cases} \frac{k+2}{2}, & \text{per } k \text{ dispari,} \\ \frac{k}{2}, & \text{per } k \text{ pari,} \end{cases} \tag{5}$$

the integers $k_1^{(i)}$ (and therefore $k_2^{(i)} = k - k_1^{(i)}$ are defined as follows:

$$k_1^{(i)} = \begin{cases} i, & \text{per } i = 1, \dots, \nu - 1, \\ \nu, & \text{per } i = \nu, \dots, N - \nu, \\ i - N + 2\nu, & \text{per } i = N - \nu + 1, \dots, N. \end{cases} \tag{6}$$

We used the GAMs obtained in correspondence with an even number of steps and therefore with odd order. As we see from (6), whatever the dimension $N \geq k$ of the matrices A and B each GAM consists of a finite number of different formulae: the first $\nu - 1$ formula are called initial methods while the last ν are called final methods. Between the initial and final method we find the so-called basic method which is repeated $N-k+1$ times.

We have fixed the following dimensions for the four selected GAMs: $N_3 = 4$, $N_5 = 6$, $N_7 = 8$, $N_8 = 9$. The coefficients β_{ij} of the initial, basic and final methods are reported in the tables (1), while it is easily seen the matrices A_s are bidiagonal with 1 and -1 as diagonal and lower-diagonal entries, respectively. The local truncation error as defined as

$$\boldsymbol{\tau}(h) = A^e\widehat{\mathbf{y}}^e - hB^eF(\widehat{\mathbf{y}}^e),$$

and has components

$$\boldsymbol{\tau}(h) = c_i \widehat{\mathbf{y}}^{(p+1)}(\xi_i) h^{p+1}, \qquad i = 1, \ldots, N,$$

where $\xi_i \in [t_0, tN]$ and c_i are the error constants of te method.

p		nf							
2	b	1/2	1	1					
3	b	1/12	5	8	-1				
	f_1		-1	8	5				
4	i_1	1/24	9	19	-5	1			
	b		-1	13	13	-1			
	f_1		1	-5	19	9			
5	i_1	1/720	251	646	-264	106	-19		
	b		-19	346	456	-74	11		
	f_1		11	-74	456	346	-19		
	f_2		-19	106	-264	646	251		
6	i_1	1/1440	475	1427	-798	482	-173	27	
	i_2		-27	637	1022	-258	77	-11	
	b		11	-93	802	802	-93	11	
	f_1		-11	77	-258	1022	637	-27	
	f_2		27	-173	482	-798	1427	475	
7	i_1	1/60480	19087	65112	-46461	37504	-20211	6312	-863
	i_2		-863	25128	46989	-16256	7299	-2088	271
	b		271	-2760	30819	37504	-6771	1608	-191
	f_1		-191	1608	-6771	37504	30819	-2760	271
	f_2		271	-2088	7299	-16256	46989	25128	-8863
	f_3		-863	6312	-20211	37504	-46461	65112	19087

Tabella 1: The coefficients β_i for the block-GAMs up to order 9

The use of the code for the considered 4-body lunar capture will be done in the next future and the numerical results will be presented in later papers, because of the complexity of the model and the need to identify the initial dynamic values of the problem, possibly vi a nonlinear optimization procedure..

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Acknowledgements

Work partly supported by MIUR Funds Project for Diffusion of Science and Culture. Dr. Petruzzi work done under assegno support.

Redazione

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La Redazione ottempera agli obblighi previsti dall'art. 1 del D.L.L. 31.8.1945, n. 660 e successive modifiche

Stampato nel 2008
presso la Cooperativa
Studium Bergomense a r.l.
di Bergamo