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Enhanced optimal tracking error portfolio via quantile regression with SSD constraints

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Abstract
Constructing an index-tracking portfolio involves closely replicating the performance of a benchmark index while minimizing deviations from it. In this paper, we propose a novel enhanced index tracking model that combines a quantile-regression-based deviation measure with linear second-order stochastic dominance (SSD) constraints. The objective is to control the tail risk of the tracking error while guaranteeing an enhancement of the portfolio return distribution relative to the benchmark. The proposed formulation leads to a linear optimization problem that remains computationally tractable under realistic portfolio constraints. The model is empirically evaluated using real-world data to assess the contribution of SSD constraints and to compare its performance with that of classical quantile regression. The empirical results show that both models outperform the benchmark index. However, when the investable universe is restricted through preselection, the proposed model delivers significant improvements in risk-return performance and tail-risk control.

Keywords Index tracking · Quantile regression · Enhanced index tracking · Second-order stochastic dominance

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1 Introduction

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The Efficient Market Hypothesis (EMH) posits that asset prices fully and rapidly reflect all publicly available information, implying that systematic excess returns cannot be earned using such information alone; see (Malkiel & Fama, 1970). This view provides the theoretical foundation for passive investment strategies aimed at replicating the market portfolio. However, the presence of market frictions, such as transaction costs, liquidity constraints, and behavioral biases, suggests that price adjustments may be imperfect and time-varying. Market efficiency may therefore fail to hold uniformly over time, leaving scope for persistent inefficiencies and motivating investment strategies that seek modest improvements over pure passive replication (Cochrane, 2008; Lo & MacKinlay, 1988). Within this perspective, portfolio management strategies are commonly classified as passive or active, depending on whether they aim to replicate a benchmark or deliberately deviate from it to generate excess returns (Treynor & Black, 1973). Quantitative passive strategies are typically implemented through index tracking models, whose objective is to replicate benchmark returns as closely as possible. Early formulations, such as Treynor and Black (1973); Rudd (1980), cast index tracking as the minimization of the variance of the difference between portfolio and benchmark returns, commonly referred to as the *tracking error* (TE). This framework was later extended within a mean-variance setting to incorporate expected excess returns (Roll, 1992), marking an initial step toward *Enhanced Index Tracking* (EIT) strategies, which lie between purely passive and fully active portfolio management. These models provide a flexible and computationally tractable foundation for benchmark-relative portfolio construction.

A key practical aspect of index tracking is the sensitivity of performance to transaction costs, particularly when benchmarks include illiquid securities or constituents with small weights. Replicating such indices may require frequent rebalancing, resulting in substantial trading costs and highlighting the trade-off between tracking accuracy and turnover. While early formulations account for proportional transaction costs (Connor & Leland, 1995; Rudd, 1980), they typically do not explicitly control turnover or portfolio sparsity. Consequently, the explicit modeling of transaction costs and implementability constraints has become central in realistic tracking formulations.

Modern enhanced index tracking models are predominantly formulated as discrete stochastic optimization problems. This framework allows transaction costs and portfolio constraints to be incorporated explicitly under various objective functions. Early contributions to this literature include Worzel et al. (1994), who introduced an asymmetric risk measure later adopted in subsequent studies, Consiglio and Zenios (2001) and Jobst and Zenios (2003), for international and corporate bond portfolios respectively. These early studies laid the groundwork for subsequent contributions, such as Beasley et al. (2003) and Dose and Cincotti (2005). In these models, tracking error is treated as a discrete random variable, and the objective typically minimizes a deviation measure, possibly augmented with enhancement constraints aimed at achieving average excess returns relative to the benchmark. More generally, a tracking portfolio is considered enhanced when its return distribution exhibits improved downside characteristics, such as better left-tail behavior or controlled dispersion within a predefined tolerance range (Valle et al., 2017).

[1] This paper is scheduled to appear in a special issue of the Annals of Operations Research in honor of Professor John Mulvey of Princeton University for his many and lasting contributions to the field of operations research, particularly in optimization methods and modeling applications in business and government settings.

Recent research approaches this general framework from different angles. Deviation-based models, including quantile-based formulations, allow investors to focus explicitly on extreme outcomes of the tracking error distribution and to penalize downside deviations asymmetrically. In contrast, stochastic dominance-based enhancement models provide distribution-wide guarantees by ensuring that portfolio returns dominate those of the benchmark for broad classes of risk-averse investors. These two strands of the literature have largely evolved separately. Deviation-based tracking models offer asymmetric, tail-focused control but do not ensure dominance of the entire return distribution. Conversely, stochastic dominance-based models impose ex ante distribution-wide guarantees but may be overly restrictive, potentially leading to infeasibility or excessive conservatism under realistic portfolio constraints. Existing approaches therefore do not jointly address tail-risk control and distribution-wide enhancement within a unified and tractable index tracking framework.

The contribution of this paper is to bridge this gap. We propose a novel enhanced index tracking model that combines a quantile-regression-based tracking objective with linear second-order stochastic dominance (SSD) constraints. The resulting formulation integrates asymmetric tail-risk control with distribution-wide dominance guarantees in a single linear optimization framework.

To position our contribution within the literature, we next review deviation-based tracking models and stochastic dominance-based enhancement approaches, referring to Silva and Almeida Filho (2023) for a recent survey of the tracking error literature.

1.1 Literature review

Deviation measures quantify discrepancies between portfolio and benchmark returns and typically define the objective function of index tracking models. Different deviation measures correspond to distinct notions of tracking risk and lead to different mathematical formulations and portfolio characteristics. Early contributions relied on the variance of tracking error, resulting in quadratic optimization problems within the mean-variance framework (Roll, 1992; Rudd, 1980; Treynor & Black, 1973). Although analytically convenient, variance penalizes positive and negative deviations symmetrically and assigns zero incremental penalty to portfolios that systematically underperform the benchmark by a constant amount. These properties may be undesirable in enhanced tracking contexts, where downside deviations are of primary concern.

To overcome these limitations, alternative symmetric measures such as mean absolute deviation, maximum deviation, and median absolute deviation have been proposed (Beasley et al., 2003; Rudolf et al., 1999; Worzel et al., 1994). A key advantage is that these measures admit linear programming reformulations, facilitating the inclusion of transaction costs and cardinality constraints (Consiglio & Zenios, 2001; Mansini et al., 2003). Nevertheless, they remain symmetric by construction. Semi-deviation measures and lower partial moments introduce asymmetry by penalizing only one side of the tracking error distribution. In the context of index tracking, downside semi-deviation measures focus exclusively on underperformance relative to the benchmark, thereby introducing an asymmetric treatment of tracking risk (Gaivoronski et al., 2005; Sant'Anna et al., 2022). While more conservative than symmetric measures, these approaches remain expectation-based and may not adequately isolate extreme tail events. Tail-oriented measures address this limitation directly. Conditional Value-at-Risk (CVaR) at confidence level τ, introduced by Rockafellar and Uryasev (2002), measures the expected loss conditional on the worst $1 - \tau$ fraction of outcomes. In the context of index tracking, the loss variable typically corresponds to underperformance relative to the benchmark, and CVaR explicitly targets severe underperformance (see Jobst et al. (2006); Sehgal and Mehra (2019, 2023); Guastaroba et al. (2020) and references therein). Another advantage of CVaR is its convexity in the portfolio weights; under discrete probability measures, it admits an equivalent linear programming reformulation, facilitating the incorporation of turnover and integer constraints. Closely related, quantile regression (QR), introduced by Koenker and Bassett (1978), provides a flexible asymmetric deviation measure that targets specific quantiles of the tracking error distribution. In index tracking, quantile-based loss functions allow asymmetric control of under and overperformance while retaining linear programming tractability (Mezali & Beasley, 2013; Sehgal & Mehra, 2023).

The relationship between deviation measures, risk measures, and investor preferences has been formalized in the risk quadrangle framework of Rockafellar and Uryasev (2013), which links risk, deviation, error, and regret measures. Within this perspective, quantile-based deviation measures emerge as theoretically consistent tools for enhanced index tracking. Recent developments include penalized asymmetric deviation models (Torri et al., 2024), which control dispersion while limiting deviations from benchmark weights to mitigate estimation risk and turnover. Alternative performance criteria have also been explored. For example, Guastaroba et al. (2016) propose a mixed-integer linear model maximizing the Omega ratio Keating and Shadwick (2002), and Bruni et al. (2014) develop a bi-objective linear framework balancing excess return and risk.

The use of information from the entire distribution of tracking errors has motivated a strand of the literature that proposes portfolio enhancement strategies based on second-order stochastic dominance (SSD) constraints. Stochastic dominance is a concept from decision theory that provides criteria for ranking the distributions of random variables.[2] In particular, second-order stochastic dominance characterizes the preferences of risk-averse decision-makers by establishing a preference ordering over distributions: one distribution dominates another if it yields higher expected utility for all increasing and concave utility functions. This criterion captures a preference for distributions that reduce downside risk without sacrificing expected performance, which is consistent with the notion of enhanced tracking portfolios widely adopted in the literature. An early proposal to incorporate SSD constraints in an index tracking framework is due to Roman et al. (2013), who construct portfolios whose return distributions second-order stochastically dominate that of the benchmark. Their formulation relies on a finite-sample approximation that renders both the objective function and the SSD constraints linear in the portfolio weights, resulting in a multi-objective linear optimization problem solved via the cutting-plane algorithm developed by Fábián et al. (2011). Despite their appealing economic interpretation, stochastic dominance constraints may be overly restrictive and can lead to infeasible optimization problems. To address this issue, several authors propose relaxed or partial dominance conditions (Bruni et al., 2014; Sharma et al., 2017). More recent studies incorporate insights from behavioral finance into stochastic optimization models. In particular, Mitra et al. (2018) enhance index tracking by integrating market sentiment indicators into SSD-constrained formulations, thereby aligning portfolio construction more closely with investor behavior.

1.2 Contribution

The main contribution of this paper is the development of a unified enhanced index tracking framework that integrates asymmetric tail-risk control with distribution-wide stochastic dominance constraints. Specifically, we combine a quantile-regression-based tracking objective

[2] See, for instance, Levy (1992) for a comprehensive discussion and historical perspective.

with linear second-order stochastic dominance (SSD) constraints within a single optimization model. The quantile-based objective enables explicit control of downside tracking risk, while SSD constraints ensure that the portfolio return distribution dominates that of the benchmark for risk-averse investors. The resulting formulation can be expressed as a linear program and remains tractable under realistic portfolio constraints, including proportional transaction costs and turnover limits. To the best of our knowledge, this is the first linear index tracking framework that integrates a quantile-regression objective with linear SSD constraints in a unified and computationally tractable model. An extensive out-of-sample analysis shows that the proposed approach closely replicates benchmark performance while delivering statistically and economically significant improvements in tail-sensitive risk-return measures, particularly when the investable universe is restricted through preselection.

The remainder of the paper is organized as follows. Section 2 introduces the index tracking portfolio optimization problem. Section 2.1 presents the quantile regression objective used for tail-risk control, while Section 2.2 describes the SSD constraints and their implementation. Section 3 reports and analyzes the empirical results. Finally, Section 4 concludes and outlines directions for future research.

2 Model formulation

In this section, we introduce the optimization model for the index tracking problem. After presenting the model framework and the class of optimization problems to which it belongs, we devote specific subsections to explaining the choice of the deviation measure and the SSD constraints. Let $\mathbf{r} = [r_1, \ldots, r_I]^\top$ denote the random vector of I asset returns between two dates t and $t + 1$, and let y be the corresponding benchmark return, both defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The vector $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_I]^\top \in \mathbb{R}^I$ represents the portfolio weights chosen at time t. We assume that at each rebalancing date, the investor starts from a pre-existing portfolio with weights $\boldsymbol{\beta}^{(0)} = \left[\beta_1^{(0)}, \ldots, \beta_I^{(0)}\right]^\top$ and adjusts positions to obtain $\boldsymbol{\beta}$. In this setting $\mathbf{r}^\top \boldsymbol{\beta}$ represents the return realized over $[t, t + 1]$ by the tracking portfolio formed at time t; the tracking error of portfolio $\boldsymbol{\beta}$ is defined as $\varepsilon = y - \mathbf{r}^\top \boldsymbol{\beta}$. With this convention, positive values of ε correspond to portfolio underperformance relative to the benchmark, while negative values indicate excess returns.

The investor seeks a portfolio whose tracking error ε is tightly concentrated around zero in the purely passive case. In enhanced tracking, the objective is instead to apply an asymmetric penalty that discourages underperformance ($\varepsilon > 0$) while permitting moderate overperformance ($\varepsilon < 0$). More generally, such objectives can be formalized as the minimization of a functional $\mathbb{D}[\varepsilon]$, which represents a deviation measure[3] of the random variable ε. We will return in the next section to the precise definition of the functional form of $\mathbb{D}[\varepsilon]$ used in this work, contextualizing this choice within the existing class of deviation measures tailored to enhanced index tracking applications, as it is fundamental in shaping the characteristics of the tracking error model. A second design goal, which is specific to the enhanced index tracking approach, is to impose an additional profit target. A common approach is to impose a minimum expected excess return K^*, which is formalized through the constraint $\mathbb{E}\left[\mathbf{r}^\top \boldsymbol{\beta} - y\right] \geq K^*$, where K^* denotes a pre-specified target level of expected excess return. This condition requires the replicating portfolio to deliver at least K^* on average. This expectation constraint provides a parsimonious, linear way to target positive average over-

[3] For an axiomatic definition of deviation measures, see, for instance, Stoyanov et al. (2008).

performance while preserving tractability, without explicitly controlling extreme deviations. Distributional improvement in the sense of risk-averse preferences is instead enforced by the SSD constraints discussed in the next section.

In order to take into account transaction costs and to reflect real-world portfolio restrictions on the investor's decision, such as budget constraints, bounds on individual asset weights, and limits on portfolio turnover, the investor's choice is subject to a set of constraints reflecting transaction costs, budget feasibility, and turnover limits. In this work we assume transaction costs to be deterministic and proportional to the total value of traded positions, with a constant rate α applied uniformly across assets for each transaction. The total cost is added as linear penalties in the objective function, and does not enter the deviation measure or the enhancement constraint. This modeling choice treats transaction costs as implementation frictions, while preserving tractability. By introducing non-negative variables $\omega_i^+ \geq 0$ and $\omega_i^- \geq 0$, representing, respectively, increases and decreases in the weight of asset i, the total transaction cost is computed as $\alpha \sum_{i=1}^{I}(\omega_i^+ + \omega_i^-)$, subject to the linear relations $\omega_i^+ - \omega_i^- = \beta_i - \beta_i^{(0)}$, for $i = 1, \ldots, I$. In addition, portfolio turnover is controlled by imposing an upper bound on total rebalancing activity, $\sum_{i=1}^{I} \frac{\omega_i^+ + \omega_i^-}{2} \leq \theta$, where θ represents a maximum admissible turnover level. The variables ω_i^+ and ω_i^- are collected in the vectors $\boldsymbol{\omega}^+ = [\omega_1^+, \ldots, \omega_I^+]$ and $\boldsymbol{\omega}^- = [\omega_1^-, \ldots, \omega_I^-]$, respectively.

The index tracking problem with enhancement, transaction costs, and turnover constraints can thus be formulated as

$$\min_{\boldsymbol{\beta}, \boldsymbol{\omega}^+, \boldsymbol{\omega}^-} \quad \mathbb{D}[\varepsilon] + \alpha \sum_{i=1}^{I}(\omega_i^+ + \omega_i^-), \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{I} \beta_i = 1, \tag{2}$$

$$\mathbb{E}\Big[\mathbf{r}^\top \boldsymbol{\beta} - y\Big] \geq K^*, \tag{3}$$

$$\omega_i^+ - \omega_i^- = \beta_i - \beta_i^{(0)}, \qquad i = 1, \ldots, I, \tag{4}$$

$$\sum_{i=1}^{I} \frac{\omega_i^+ + \omega_i^-}{2} \leq \theta, \tag{5}$$

$$0 \leq \beta_i \leq 1, \qquad i = 1, \ldots, I, \tag{6}$$

$$\omega_i^+, \omega_i^- \geq 0, \qquad i = 1, \ldots, I. \tag{7}$$

where constraint (2) ensures full investment of available capital, while constraint (3) enforces the minimum enhancement requirement. Bounds in (6) restrict individual asset exposures to lie within the interval [0, 1]. Problem (1)–(7) is a stochastic optimization model with linear constraints that defines a common structural framework for enhanced index tracking models. Its computational tractability depends on the specific form of the functional $\mathbb{D}$ and on the joint probability measure of the random variables $r_1, \ldots, r_I, y$.

In empirical applications, the return distribution at time t is unknown and must be inferred from observed data. Although multidimensional return distributions may be specified using parametric or semi-parametric models, the resulting optimization problems typically involve numerical integration, which quickly becomes intractable in high dimensions. For this reason, a common assumption in the tracking error literature is to adopt a data-driven approach, in which the underlying probability space is defined through the empirical probability measure constructed from the available sample of returns. Let $\mathbf{R} \in \mathbb{R}^{N \times I}$ denote the matrix of asset returns observed over the previous N periods, and let $\mathbf{Y} \in \mathbb{R}^N$ denote the corresponding vector of benchmark returns. The empirical dataset is thus represented by the matrix $[\mathbf{R}, \mathbf{Y}]$, whose j-th row $(\mathbf{R}_j, Y_j)$ corresponds to the j-th joint observation. The empirical probability measure induced by this dataset is defined as $\widehat{\mathbb{P}} = \frac{1}{N}\sum_{j=1}^{N} \delta_{(\mathbf{R}_j, Y_j)}$, where $\delta_{(\mathbf{R}_j, Y_j)}$ denotes the Dirac measure concentrated at $(\mathbf{R}_j, Y_j)$. This measure is used throughout the remainder of the paper to evaluate expectations and deviation measures. Under this empirical measure, all expectations and deviation measures reduce to finite sums over the observed scenarios, yielding a deterministic optimization problem. Formally, this corresponds to a sample average approximation of the original stochastic optimization problem, whereby the underlying probability measure is replaced by its empirical counterpart.

2.1 Quantile-based deviation measure for tracking error

In this section we describe the *Quantile Regression* (QR), which will be used as the deviation measure $\mathbb{D}$ in the tracking problem (1)–(7). Consider an integrable random variable X and define the function f, called *quantile loss*, by

$$f(\xi; X, \tau) = \tau\, \mathbb{E}\big[(X-\xi)_+\big] + (1-\tau)\, \mathbb{E}\big[(X-\xi)_-\big], \tag{8}$$

where $(x)_+ = \max\{x, 0\}$ and $(x)_- = \max\{-x, 0\}$, $\xi \in \mathbb{R}$ is a deterministic parameter, and $\tau \in [0, 1]$. The function $f(\xi; X, \tau)$ is a weighted sum of the expected positive and negative deviations of X from the level ξ. The parameter τ controls the asymmetry between the penalties assigned to positive and negative deviations of X from ξ: larger values of τ place greater weight on positive deviations $(X-\xi)_+$, while smaller values emphasize negative deviations $(X-\xi)_-$. A key property of $f(\xi; X, \tau)$ is that its set of minimizers coincides with the set of τ-quantiles of X, that is,

$$\arg\min_{\xi \in \mathbb{R}} f(\xi; X, \tau) = q_\tau(X), \tag{9}$$

where $q_\tau(X) := \inf\{x : F_X(x) \geq \tau\}$ and F_X denotes the cumulative distribution function of X. Moreover, the objective itself provides a dispersion measure that penalizes positive and negative deviations asymmetrically. The mapping $\xi \mapsto f(\xi; X, \tau)$ is convex on $\mathbb{R}$ and therefore admits at least one minimizer. This follows directly from the convexity of $\xi \mapsto (X-\xi)_+$ and $\xi \mapsto (X-\xi)_-$ and the linearity of the expectation operator. If X is discrete with N possible realizations $\{x_j\}_{j=1}^{N}$ with equal probability masses, then (8) reduces to the deterministic problem

$$f(\xi; X, \tau) = \frac{1}{N}\sum_{j=1}^{N} \big[\tau\,(x_j-\xi)_+ + (1-\tau)\,(x_j-\xi)_-\big], \tag{10}$$

which can be linearized by introducing auxiliary variables $u_j = (x_j-\xi)_+$ and $\nu_j = (x_j-\xi)_-$, yielding the linear program

$$\begin{aligned} \min_{\xi, u, \nu} \quad & \frac{1}{N}\sum_{j=1}^{N}\big[\tau u_j + (1-\tau)\nu_j\big] \\ \text{s.t.} \quad & x_j - \xi = u_j - \nu_j, \qquad && j = 1, \ldots, N, \\ & u_j \geq 0,\ \nu_j \geq 0, && j = 1, \ldots, N. \end{aligned} \tag{11}$$

This formulation yields a deterministic linear optimization problem that can be efficiently solved using standard linear programming techniques.

Applying this formulation to the empirical tracking error vector, the loss function can be written as

$$\tau\,\mathbb{E}\big[(\mathbf{Y}-\mathbf{R}\boldsymbol{\beta}-\xi)_+\big]+(1-\tau)\,\mathbb{E}\big[(\mathbf{Y}-\mathbf{R}\boldsymbol{\beta}-\xi)_-\big], \tag{12}$$

which depends on the vector of portfolio weights $\boldsymbol{\beta}$. With this convention, positive values of the tracking error indicate portfolio underperformance relative to the benchmark, while negative values correspond to excess returns. Consequently, larger values of τ penalize benchmark underperformance more strongly. The resulting minimization problem

$$\begin{aligned}\min_{\boldsymbol{\beta},\,\xi,\,\boldsymbol{u},\,\boldsymbol{\nu}} \quad & \frac{1}{N}\sum_{j=1}^{N}\big(\tau u_j+(1-\tau)\nu_j\big)\\ \text{s.t.}\quad & Y_j-\mathbf{R}_j\boldsymbol{\beta}-u_j+\nu_j-\xi=0, \quad && j=1,\ldots,N,\\ & u_j\ge 0,\ \ \nu_j\ge 0, && j=1,\ldots,N,\end{aligned} \tag{13}$$

is equivalent to estimating $(\beta_0,\boldsymbol{\beta})$ in a linear τ-quantile regression model of the form $\mathbf{Y}=\beta_0\mathbf{1}+\mathbf{R}\boldsymbol{\beta}$, where $\beta_0=\xi$ and $\mathbf{1}$ denotes the N-dimensional vector of ones, as introduced in Koenker and Bassett (1978). In other words, the goal is to determine the intercept β_0 and slope vector $\boldsymbol{\beta}$ such that $\beta_0\mathbf{1}+\mathbf{R}\boldsymbol{\beta}$ provides an estimate of the conditional τ-quantile function of y given $\mathbf{r}$. Problem (1)–(7) can now be reformulated as the following linear model:

$$\begin{aligned}\min_{\boldsymbol{\beta},\xi,\,\boldsymbol{u},\,\boldsymbol{\nu},\boldsymbol{\omega}^+,\boldsymbol{\omega}^-}\quad & \frac{1}{N}\sum_{j=1}^{N}\big[\tau u_j+(1-\tau)\,\nu_j\big]+\alpha\sum_{i=1}^{I}\left(\omega_i^+ +\omega_i^-\right),\\ \text{s.t.}\quad & \sum_{i=1}^{I}\beta_i=1,\\ & Y_j-\mathbf{R}_j\boldsymbol{\beta}-u_j+\nu_j-\xi=0 && j=1,\ldots,N,\\ & \frac{1}{N}\sum_{j=1}^{N}\big[\mathbf{R}_j\boldsymbol{\beta}-Y_j\big]\ge K^*,\\ & \omega_i^+-\omega_i^-=\beta_i-\beta_i^{(0)}, && i=1,\ldots,I,\\ & \sum_{i=1}^{I}\frac{\omega_i^+ +\omega_i^-}{2}\le\theta,\\ & 0\le\beta_i\le ub, && i=1,\ldots,I,\\ & \omega_i^+,\omega_i^-\ge 0, && i=1,\ldots,I.\\ & u_j\ge 0,\ \ \nu_j\ge 0, && j=1,\ldots,N,\end{aligned} \tag{14}$$

2.2 Stochastic dominance constraints for enhancement

In addition to controlling tracking error risk through a quantile-based deviation measure, enhanced index strategies aim to improve the distributional properties of portfolio returns relative to the benchmark. Stochastic dominance is a fundamental concept in decision theory and economics (Levy, 1992), which provides a natural framework to formalize such enhancement objectives, as it allows portfolios to be compared on the basis of their entire return distributions rather than through scalar performance measures. In portfolio theory, stochastic dominance has been widely analyzed (Dentcheva & Ruszczyński, 2006; Ortobelli Lozza et al., 2013; Post & Kopa, 2017), and it has also been applied to index-tracking settings (Bruni et al., 2014; Roman et al., 2013). In particular, first-order stochastic dominance (FSD) captures preferences of all non-satiable decision makers, while second-order stochastic dominance (SSD) characterizes preferences of all non-satiable and risk-averse decision makers. Because enhanced index strategies are typically designed for risk-averse investors in a benchmark-relative setting, SSD represents a particularly appropriate criterion for defining enhancement relative to a benchmark.

Formally, given two random variables X and Z with cumulative distribution functions F_X and F_Z, X is said to first-order stochastically dominate Z, denoted $X \succeq_{(1)} Z$, if

$$F_X(z) \leq F_Z(z), \qquad \forall z \in \mathbb{R}. \tag{15}$$

while X is said to second-order stochastically dominate Z, denoted $X \succeq_{(2)} Z$, if

$$\int_{-\infty}^{z} F_X(u)\, du \leq \int_{-\infty}^{z} F_Z(u)\, du, \qquad \forall z \in \mathbb{R}. \tag{16}$$

Equivalently, $X \succeq_{(2)} Z$ if and only if $\mathbb{E}[u(X)] \geq \mathbb{E}[u(Z)]$ for all non-decreasing concave utility functions $u(\cdot)$ for which the expectations are finite. In particular, since the linear utility $u(x) = x$ is non-decreasing and concave, SSD implies $\mathbb{E}[X] \geq \mathbb{E}[Z]$, so that an improvement in expected return is a necessary condition for second-order stochastic dominance. This characterization establishes a direct link between SSD and risk aversion. In the context of enhanced index tracking, stochastic dominance is imposed in a benchmark-relative sense: the return distribution of the tracking portfolio is required to dominate that of the benchmark index. Throughout this section, dominance relations therefore compare the portfolio return $\mathbf{r}^\top \boldsymbol{\beta}$ to the benchmark return y, so that second-order stochastic dominance formalizes enhancement as a preference for the tracking portfolio over the benchmark for all risk-averse investors.

Under the empirical probability measure introduced in the previous subsection, stochastic dominance relations between the empirical return vectors $\mathbf{R}\boldsymbol{\beta}$ and the vector $\mathbf{Y}$ of the benchmark's returns can be expressed through linear constraints. Assuming equiprobable scenarios under the empirical measure induced by the data matrix $[\mathbf{R}, \mathbf{Y}]$, the first- and second-order stochastic dominance relations admit equivalent linear (mixed-integer for FSD, linear for SSD) formulations (Kopa, 2010; Kuosmanen, 2004). The empirical FSD condition $\mathbf{R}\boldsymbol{\beta} \succeq_{(1)} \mathbf{Y}$ holds if and only if there exists a permutation matrix $\mathbf{P} = \{p_{k,c}\} \in \{0, 1\}^{N \times N}$, where k and c represent row and columns respectively, such that

$$\begin{aligned} &\mathbf{R}\boldsymbol{\beta} \geq \mathbf{P}\mathbf{Y}, \\ &\sum_{k=1}^{N} p_{k,c} = 1, \quad \sum_{c=1}^{N} p_{k,c} = 1, \quad p_{k,c} \in \{0, 1\}, \end{aligned} \tag{17}$$

where $\mathbf{R}\boldsymbol{\beta}$ and $\mathbf{Y}$ denote the vectors of empirical portfolio and benchmark returns and where inequalities between vectors are understood component wise. Similarly, the empirical SSD condition $\mathbf{R}\boldsymbol{\beta} \succeq_{(2)} \mathbf{Y}$ holds if and only if there exists a doubly stochastic matrix $\mathbf{M} = \{m_{k,c}\} \in [0, 1]^{N \times N}$ satisfying the following component wise inequality

$$\mathbf{R}\boldsymbol{\beta} \geq \mathbf{M}\mathbf{Y},$$
$$\sum_{k=1}^{N} m_{k,c} = 1, \quad \sum_{c=1}^{N} m_{k,c} = 1, \quad 0 \leq m_{k,c} \leq 1. \tag{18}$$

The above characterization holds under equiprobable scenarios and is independent of the ordering of empirical returns due to the permutation-invariant nature of the doubly stochastic representation (Kopa, 2010). The SSD formulation replaces the binary constraints of FSD with continuous ones, resulting in a linear and computationally more tractable model. Stochastic dominance constraints can now be integrated into the QR-based index tracking problem to enforce enhancement relative to the benchmark. In particular, SSD constraints ensure that the return distribution of the tracking portfolio dominates that of the benchmark for all risk-averse investors. The enhanced index tracking problem with the quantile loss objective function and first-order stochastic dominance (FSD) constraints leads to a mixed-integer linear program, due to the presence of binary variables in the permutation matrix **P**. By contrast, replacing FSD with second-order stochastic dominance (SSD) constraints yields a purely linear optimization problem, although the number of constraints grows quadratically with the sample size N. In addition to their higher computational burden, FSD constraints are typically highly restrictive and may significantly reduce the feasible region, potentially leading to infeasibility in empirical applications. SSD constraints, on the other hand, provide a less restrictive dominance criterion that is consistent with risk-averse preferences and lead to a linear optimization problem. For these reasons, and in line with the preferences of risk-averse investors, the empirical analysis in the next section focuses on the SSD-based formulation, which offers a practical and economically meaningful framework for enhancement in index tracking. The resulting enhanced index tracking model can be formulated as the following linear optimization problem:

$$\begin{aligned}
\min_{\boldsymbol{\beta},\xi,\boldsymbol{u},\boldsymbol{v},\boldsymbol{\omega}^{+},\boldsymbol{\omega}^{-},\mathbf{M}} \quad & \frac{1}{N}\sum_{j=1}^{N}\left[\tau u_j + (1-\tau)\,v_j\right] + \alpha \sum_{i=1}^{I}\left(\omega_i^{+} + \omega_i^{-}\right), \\
\text{s.t.} \quad & \sum_{i=1}^{I}\beta_i = 1, \\
& Y_j - \mathbf{R}_j\boldsymbol{\beta} - u_j + v_j - \xi = 0, && j = 1,\ldots,N, \\
& \frac{1}{N}\sum_{j=1}^{N}\left[\mathbf{R}_j\boldsymbol{\beta} - Y_j\right] \geq K^{*}, \\
& \omega_i^{+} - \omega_i^{-} = \beta_i - \beta_i^{(0)}, && i = 1,\ldots,I, \\
& \sum_{i=1}^{I}\frac{\omega_i^{+} + \omega_i^{-}}{2} \leq \theta, \\
& \mathbf{R}\boldsymbol{\beta} \geq \mathbf{M}\mathbf{Y}, \\
& \sum_{k=1}^{N} m_{k,c} = 1, \quad \sum_{c=1}^{N} m_{k,c} = 1, \\
& 0 \leq m_{k,c} \leq 1, && k, c = 1,\ldots,N, \\
& 0 \leq \beta_i \leq 1, && i = 1,\ldots,I, \\
& \omega_i^{+}, \omega_i^{-} \geq 0, && i = 1,\ldots,I. \\
& u_j \geq 0,\ v_j \geq 0, && j = 1,\ldots,N,
\end{aligned} \tag{19}$$

3 Empirical application

In this section, we analyze optimal portfolios constructed using the proposed enhanced tracking error model. We solve the associated optimization problem under different parameter configurations over an investible universe composed by the constituents of an index. Our objective is to assess how the resulting optimal strategies differ from those obtained by removing the enhancement defined by constraints (3), (18). We also compare the results with those obtained from a minimum-CVaR (Min-CVaR) model. Model parameters are chosen to preserve the tractability of the optimization problem: although all constraints are linear, the computational complexity of the problem grows quadratically with the length of the calibration window (N) and linearly with the number of assets (I), where the former determines the dimension of the permutation matrix, while the latter increases the number of linear constraints. The enhancement parameter plays a significant role in reducing the feasible set of the optimization problem and therefore requires careful calibration to avoid making the problem overly restrictive or infeasible. Results are evaluated out-of-sample using a rolling-window approach, which determines the time shift between the calibration and testing datasets. The numerical computations are performed using MATLAB 2024b and MOSEK optimization toolbox, on an Intel Core i5-1235U processor (12M Cache, up to 4.40 GHz).

3.1 Data and experiment setup

To evaluate the model, we focus on the main U.S. equity index. Accordingly, we construct a dataset of adjusted daily closing prices for the Standard&Poor 500 (S&P500) constituents spanning from January 1, 2004, to December 31, 2023. The dataset comprises 4,995 observations for a total of 978 assets[4]. For the empirical analysis, the $S\&P500$ index serves as the benchmark. We adopt a rolling-window backtesting procedure with monthly rebalancing (21 trading days). Specifically, at each rebalancing date t (every 21 trading days), we estimate the model using the most recent N daily observations and solve the corresponding optimization problem to obtain portfolio weights w_t. The portfolio is held constant until the next rebalancing date. Out-of-sample performance is evaluated using the daily realized returns generated during each holding period, yielding a daily out-of-sample return series for each strategy and parameter configuration. The maximum portfolio turnover parameter is set to $\theta = 0.25$, and the proportional constant for transaction costs is set to 2 basis points for each transaction, which implies 4 basis points for a round-trip trade ($\alpha = 0.0002$).

We pay particular attention to the choice of the calibration window length. If the window is too short, the stochastic dominance constraints may overly restrict the feasible set, leading to frequent infeasibility. Conversely, increasing the window length substantially raises the number of constraints in the optimization problem, resulting in a significant increase in computational burden. Based on this trade-off, we set the calibration window to 500 observations. To evaluate the robustness of our results to this specification choice, we additionally report findings based on an alternative calibration window in Appendix A. Overall, these additional tests yield evidence that is fully consistent with our main findings. The rebalancing frequency is instead fixed at 21 trading days in order to maintain a consistent tracking objective; varying this frequency would shift the model toward a different tactical perspective and is therefore beyond the scope of the present analysis.

[4] The analysis is based on the historical time series of all securities that were index's constituents at any time during the sample period, including those that subsequently exited the index.

The enhancement parameter is set to $K^* = 0.0005$, corresponding to a minimum expected daily excess return of 5 basis points. This value should be interpreted as a calibration device controlling the tightness of the enhancement mechanism, rather than as a realistic overperformance target at an annual horizon. In the rare cases in which the optimization problem is infeasible, we carry forward the previous period's portfolio weights; these cases occur in less than 1% of rebalancing dates. Since the model is calibrated using daily returns, the constraint is imposed at the daily frequency and serves to control the strength of the enhancement mechanism rather than to reflect an expected annual overperformance. We evaluate the model at multiple quantile levels ($\tau = 0.05, 0.5, 0.95$), covering both asymmetric penalization ($\tau = 0.05, 0.95$) and the symmetric median case ($\tau = 0.5$). Since $\varepsilon = y - \mathbf{r}^\top \boldsymbol{\beta}$ is positive under benchmark underperformance, larger values of τ penalize underperformance more heavily, whereas smaller values emphasize overperformance. Strategies are evaluated using a set of standard statistics computed from the distribution of daily out-of-sample returns. These measures are classified into risk measures and reward-risk (RR) ratios. The exact definitions of all statistics, together with brief explanations and relevant references, are reported in Appendix B. Risk is assessed through complementary measures capturing different dimensions of downside exposure. In particular, we consider the Value at Risk at the 95% level ($\text{VaR}_{0.95}$), the Conditional Value at Risk at the same level ($\text{CVaR}_{0.95}$) and the Maximum Drawdown (MDD), which is instead used to quantify the most severe cumulative loss observed over the evaluation period. Reward-risk ratios summarize expected performance relative to risk by measuring the excess return achieved per unit of risk. The main differences among these ratios arise from the specific risk measure employed in the denominator. Accordingly, the Sharpe Ratio (SR) is based on total return volatility, the Mean-to-CVaR ratio at the 95% level ($\text{MtC}_{0.95}$) relies on $\text{CVaR}_{0.95}$, and the Sortino–Satchell Ratio (SSR) uses the lower semi-deviation. By construction, the SSR emphasizes downside risk and is therefore particularly informative when negative returns are of primary concern. Jensen's alpha (JA) measures the abnormal return of a strategy after adjusting for systematic risk and is defined as the intercept of a regression of the strategy's excess returns on benchmark excess returns within the Capital Asset Pricing Model (CAPM) framework. Positive values of JA indicate risk-adjusted overperformance, whereas negative values indicate underperformance. Closely related, the Information Ratio (IR) evaluates the excess return generated per unit of tracking error and can be interpreted as the Sharpe ratio computed on out-of-sample active returns. Unlike the Sharpe Ratio, which reflects total portfolio volatility, the IR focuses exclusively on benchmark-relative risk. Finally, we report the average portfolio Turnover (AvgTO), defined as the sample mean of the out-of-sample optimal portfolio turnover at each rebalancing date.

To evaluate the effect of combining the quantile loss objective with second-order stochastic dominance (SSD) constraints and the enhancement mechanism, we compare the full model with a relaxed version that excludes both the SSD constraints and the enhancement constraint (3). In addition, given the close relationship between the quantile loss objective and the minimization of CVaRτ, we also consider a benchmark model based on CVaRτ minimization under portfolio constraints, but without any enhancement mechanism. In summary, we compare the following optimization strategies:

- The proposed quantile regression model with enhancement and second-order stochastic dominance constraints (hereafter QR-SSD_τ);
- The quantile regression model without any enhancement constraints (hereafter QR_τ);
- The Min-CVaR index-tracking model without enhancement (hereafter Min-CVaR_τ)

As an additional analysis, we investigate the sensitivity of the optimal strategies to the size of the investible universe. In addition to the full dataset, we apply preselection procedures to restrict the portfolio universe to 200 and 400 assets. Following the evidence in Qu et al. (2017), which suggests that the most attractive assets tend to belong to high-return, low-risk subsets, stocks are preselected based on their past SR. This approach allows us to reduce portfolio cardinality while preserving computational tractability. As a robustness check with respect to the choice of the preselection criterion, we also consider alternative rankings based on the MtC ratio and the IR. The resulting portfolios exhibit qualitatively similar performance patterns. Results for the 500-day moving window are reported in Appendix C, while those for the alternative 250- and 125-day windows are presented in Appendix A.

An alternative approach would be to introduce explicit cardinality constraints or penalization terms directly into the objective function. However, these modifications would substantially increase the complexity of the optimization problem. For instance, while a lasso-type penalization could preserve linearity, it is ineffective in the absence of short-selling. Other regularization schemes, such as ridge penalties or explicit cardinality constraints, would instead lead to a nonlinear optimization formulation. For these reasons, we adopt a preselection-based approach to control portfolio cardinality while preserving tractability.

3.2 Empirical results

The upper panel of Figure 1 shows that, when the full investment universe is considered, portfolio concentration, measured by the normalized Herfindahl-Hirschman Index (HHI^*), exhibits a general upward trend. This effect is also evident in the top panel of Figure 2, which highlights over time a decline in the number of assets of the out-of-sample optimal portfolios[5]. In the full-universe case, the QR-SSD strategy tends to display higher concentration levels than the competing strategies over almost the entire out-of-sample period. From an analysis of the feasibility of this parameter setting, we notice that second-order stochastic dominance constraints, turnover and enhancement constraints do not pose substantial feasibility issues. When focusing attention on the optimal strategies obtained using a pre-selection method based on the SR, diversification is lower than in the full-universe case, as shown in the bottom panel of Figure 1, and it appears broadly similar across strategies within each period.

An increase in concentration is observed during the 2009 financial crisis, especially for the preselection cases with 200 and 400 assets, which exhibit a sharp increase in the HHI^*, likely due to limited diversification opportunities during that period. The portfolio constructed from the full asset universe benefits from broader diversification, which helps mitigate concentration effects during the crisis. By contrast, when the investible universe is restricted, diversification potential is reduced, leading to the observed sharp decrease in cardinality in 2009-2010.

By contrast, restricting the investible universe reduces diversification opportunities and accelerates the decline in portfolio cardinality (see Figure 2).

Main statistics for the series of out-of-sample returns of optimal strategies and the index are reported in Table 12 in Appendix D. We report, the mean (μ), standard deviation (σ), skewness (γ), kurtosis (κ) and the α-percentiles q_α with $\alpha \in [1\%, 5\%, 50\%, 95\%, 99\%]$. The ex-post optimal portfolio returns exhibit negative skewness and large kurtosis. However the returns of all optimal portfolios exhibit higher mean values and greater skewness compared to the benchmark portfolios.

Table 1 summarizes the performance measures and ratios for the three different strategies applied at the entire investible universe. The proposed QR-SSD models outperform the benchmark across most indicators. The only exception concerns VaR, which is slightly higher

5 Figure 2 reports the number of assets with portfolio weights greater than or equal to 10^{-6}.

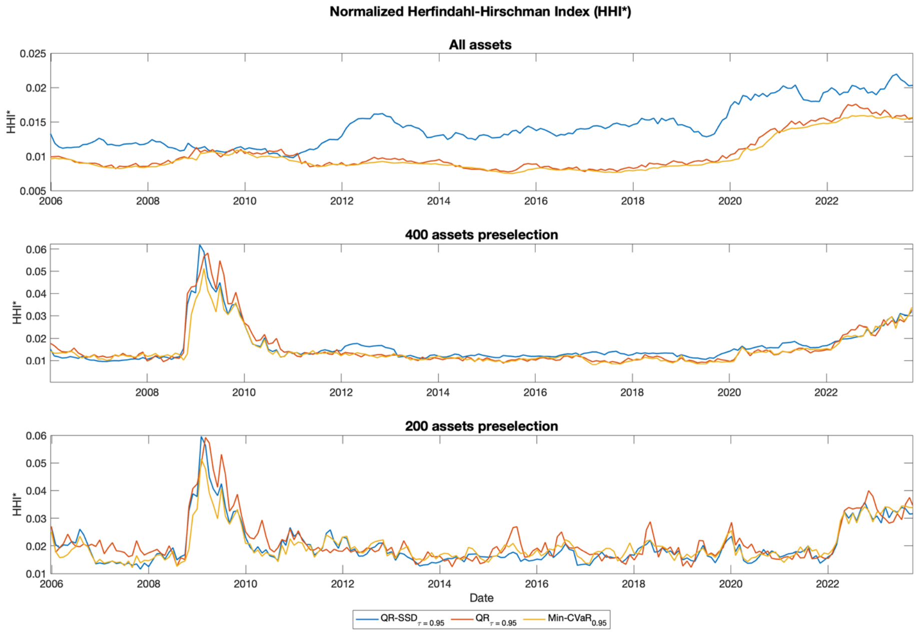

Fig. 1 Time series evolution of the normalized Herfindahl-Hirschman Index (HHI*) for the optimal portfolios evaluated out-of-sample. The top panel refers to the entire investment universe, while the bottom panels are based on a preselection of 200 and 400 assets, respectively

Fig. 2 Time series evolution of the cardinality of the out-of-sample optimal portfolios. The top panel considers the entire investment universe, while the bottom panels are based on a preselection of 200 and 400 assets, respectively

in the no-preselection case. Table 2 summarizes instead the performance measures applied at the preselected investible universe. As the investment universe reduces, from the full universe to the best 200 assets, the IR decreases, indicating a lower ability to generate excess return per unit of active risk relative to the benchmark. In contrast, the SSR and SR increase, suggesting a stronger ability to generate risk-adjusted returns, whether focusing solely on downside risk (SSR) or total volatility (SR). This pattern is consistent with the nature of the preselection procedure. Selecting assets with the highest Sharpe ratios tends to retain securities with stronger standalone performance. This will improve the absolute risk-return profile of the portfolios, while reducing their ability to generate benchmark-relative overperformance.

As the size of the investment universe decreases, the risk measures, VaR, CVaR and MDD tend to decline[6].

This effect is largely driven by the preselection procedure, which retains assets with the highest Sharpe ratios and therefore concentrates the investment universe on securities with stronger historical risk-return characteristics. Although restricting the universe reduces diversification opportunities, the risk–return filtering introduced by the preselection step appears to dominate this effect, resulting in lower tail losses and smaller drawdowns.

In particular, the QR-SSD portfolios exhibit a notable reduction in MDD when moving from the full universe to the 200-asset universe, suggesting that the preselection step contributes to improved downside resilience. For example, for $\tau = 0.95$, the QR-SSD portfolio exhibits a reduction in MDD from approximately 54% in the full universe to about 45% when the universe is restricted to 200 assets. Finally, we observe a turnover reduction as the investible universe increases, which may indicate that stronger diversification yields more stable strategies, requiring fewer rebalancing operations and consequently leading to lower trading costs.

Table 3 reports the differences in performance indicators across the three models considered: the QR-SSD, the QR, and the Min-CVaR model. These differences are subsequently tested for statistical significance with respect to CVaR, MtC, the SR, the IR, and the SSR, for each pair of portfolios (coupled by equal value of τ): QR-SSD versus the Min-CVaR portfolio, QR-SSD versus QR and QR versus Min-CVaR. To test the null hypothesis $\text{CVaR}_j = \text{CVaR}_i$ versus the alternative $\text{CVaR}_j < \text{CVaR}_i$ and $\text{MtC}_j = \text{MtC}_i$ versus $\text{MtC}_j > \text{MtC}_i$ with $j \in \{\text{QR-SSD, QR}\}$, $i \in \{\text{QR, Min-CVaR}\}$ and $i \neq j$.

Specifically, inference on CVaR and the MtC ratio relies on the test proposed by Lotfi et al. (2025), which employs block bootstrapping to estimate the sampling distribution of CVaR and MtC differences and computes p-values using their derived asymptotic null distribution. Differences in Sharpe ratios are tested using the robust procedure of Ledoit and Wolf (2008), which is based on a studentized block bootstrap designed to accommodate serial dependence and non-normal returns. Finally, for the Sortino and Information ratios, we adopt the nonparametric bootstrap test of Aboy and Magadia (2021), which constructs confidence intervals for performance differences without imposing distributional assumptions. In all cases, inference is implemented via block bootstrap resampling with replacement using 5,000 replications, preserving time alignment to account for serial dependence and addressing heteroskedasticity through block resampling of returns. The block length is set equal to two, chosen on the basis of the low serial correlation observed in daily active returns from the inspection of the

[6] We also examine two sub-periods, namely the Global Financial Crisis (2007-2010) and the COVID-19 pandemic (2020-2021), to assess the ability of the QR-SSD model to control tail risk during market downturns. During the COVID-19 period, the QR-SSD model exhibits slightly superior performance relative to the other models, whereas the evidence is less pronounced during the Global Financial Crisis. Nevertheless, requiring enhancement during crisis periods does not appear to compromise tail-risk control. Results for these periods are available upon request.

Table 1 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over the full investible universe (no preselection), using a two-year calibration window (500 observations). All values are reported $\times 10^{-2}$

No Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.9037	3.1588	0.8761	2.2033	-	0.0721	56.7754	-	-
QR-SSD τ=0.95	1.9106	3.1051	1.4487	3.5610	11.1832	0.1181	53.8934	0.0173	5.0122
QR-SSD τ=0.50	1.9206	3.1023	1.4366	3.5356	10.9091	0.1173	53.5798	0.0169	6.2052
QR-SSD τ=0.05	1.9158	3.0996	1.4442	3.5551	10.8098	0.1178	53.8424	0.0171	8.4867
QR τ=0.95	1.8806	3.1278	1.2109	2.9812	9.8937	0.1000	57.1599	0.0102	9.3938
QR τ=0.50	1.8733	3.1234	1.1881	2.9253	9.4092	0.0977	56.8249	0.0094	9.1543
QR τ=0.05	1.8793	3.1252	1.1818	2.9060	9.0820	0.0971	56.5630	0.0093	9.9971
Min-CVaR τ=0.95	1.8819	3.1309	1.2065	2.9671	9.5177	0.0997	57.0181	0.0101	3.9335
Min-CVaR τ=0.50	1.8764	3.1280	1.2067	2.9667	9.6845	0.0994	56.9819	0.0101	4.5709
Min-CVaR τ=0.05	1.8775	3.1177	1.2200	2.9959	10.1998	0.1005	56.6357	0.0104	6.3080

Table 2 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest Sharpe Ratio, using a two-year calibration window (500 observations). All values are reported $\times 10^{-2}$

Maximum Sharpe Ratio Preselection									
($\times 10^{-2}$)	**$VaR_{0.95}$**	**$CVaR_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.9037	3.1588	0.8761	2.2033	-	0.0721	56.7754	-	-
200 assets									
QR-SSD τ=0.95	1.8117	2.9362	1.6188	3.9367	7.2701	0.1287	45.4918	0.0199	14.6956
QR-SSD τ=0.50	1.8245	2.9388	1.5938	3.8791	6.9986	0.1262	45.8398	0.0192	15.7786
QR-SSD τ=0.05	1.8208	2.9457	1.5683	3.8288	6.6149	0.1246	47.2509	0.0185	16.4136
QR τ=0.95	1.8362	3.0023	1.5243	3.7077	6.5303	0.1216	47.8297	0.0181	16.9895
QR τ=0.50	1.8148	2.9424	1.4880	3.6279	6.0842	0.1179	46.0601	0.0161	17.1821
QR τ=0.05	1.8467	2.9698	1.4388	3.5296	5.3226	0.1145	47.6634	0.0151	17.1402
Min-CVaR τ=0.95	1.8183	2.9503	1.4893	3.6373	6.1025	0.1189	46.1988	0.0163	16.7867
Min-CVaR τ=0.50	1.8151	2.9827	1.4623	3.5655	5.9016	0.1173	45.9092	0.0159	16.8938
Min-CVaR τ=0.05	1.8582	3.0306	1.4192	3.4741	5.5126	0.1134	48.6529	0.0153	17.2452

Table 2 continued

Maximum Sharpe Ratio Preselection									
($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.9037	3.1588	0.8761	2.2033	-	0.0721	56.7754	-	-
400 assets									
QR-SSD τ=0.95	1.8517	2.9827	1.4765	3.6220	7.0444	0.1189	47.8917	0.0164	9.2386
QR-SSD τ=0.50	1.8585	2.9853	1.4347	3.5214	6.3810	0.1152	47.8020	0.0152	10.6572
QR-SSD τ=0.05	1.8481	2.9842	1.4704	3.6111	6.8213	0.1182	48.7653	0.0162	12.5833
QR τ=0.95	1.8268	3.0243	1.3524	3.3219	6.1951	0.1103	50.8847	0.0132	15.4896
QR τ=0.50	1.8109	2.9606	1.3442	3.2984	5.9516	0.1084	47.8674	0.0121	15.5267
QR τ=0.05	1.8474	2.9790	1.3063	3.2101	5.0868	0.1051	48.5509	0.0112	15.5979
Min-CVaR τ=0.95	1.8319	2.9838	1.2534	3.0889	4.6319	0.1018	50.2757	0.0097	13.9651
Min-CVaR τ=0.50	1.8054	2.9736	1.2976	3.1861	5.3090	0.1052	49.0708	0.0109	14.2038
Min-CVaR τ=0.05	1.8199	3.0640	1.3436	3.2907	5.7402	0.1107	52.1069	0.0135	15.5629

Table 3 The table reports pairwise differences in performance indicators between the models listed in the first column. For the Sharpe, Information, and Sortino–Satchell ratios, the null hypothesis is that the difference equals zero against the two-sided alternative of a nonzero difference. For CVaR and MtC, the null hypothesis is that the difference equals zero against the one-sided alternative that the first model outperforms the second. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$. All values are reported $\times 10^{-2}$

($\times 10^{-2}$)	$\Delta\mathbf{CVaR}_{0.95}$	$\Delta\mathbf{MtC}_{0.95}$	Δ**SR**	Δ**IR**	Δ**SSR**
200 assets preselection					
QR-SSD vs Min-CVaR $\tau = 0.95$	-0.014	0.129***	0.299***	1.168***	0.010***
QR-SSD vs Min-CVaR $\tau = 0.50$	-0.044	0.132**	0.314**	1.097**	0.009**
QR-SSD vs Min-CVaR $\tau = 0.05$	-0.085**	0.149**	0.355*	1.102	0.011*
QR-SSD vs QR $\tau = 0.95$	-0.066	0.094	0.229	0.740	0.007
QR-SSD vs QR $\tau = 0.50$	-0.004	0.106**	0.251***	0.914**	0.008***
QR-SSD vs QR $\tau = 0.05$	-0.024	0.130**	0.299**	1.292*	0.010*
QR vs Min-CVaR $\tau = 0.95$	0.052	0.035	0.070	0.428	0.003
QR vs Min-CVaR $\tau = 0.50$	-0.040	0.026	0.062	0.183	0.001
QR vs Min-CVaR $\tau = 0.05$	-0.061	0.020	0.056	-0.190	0.001
400 assets preselection					
QR-SSD vs Min-CVaR $\tau = 0.95$	-0.001	0.223***	0.533***	2.413***	0.017***
QR-SSD vs Min-CVaR $\tau = 0.50$	0.012	0.137**	0.335*	1.072	0.010
QR-SSD vs Min-CVaR $\tau = 0.05$	-0.080	0.127*	0.320*	1.081	0.008
QR-SSD vs QR $\tau = 0.95$	-0.042	0.124*	0.300*	0.849	0.009
QR-SSD vs QR $\tau = 0.50$	0.025	0.090	0.223	0.429	0.007
QR-SSD vs QR $\tau = 0.05$	0.005	0.164**	0.401**	1.735*	0.013**
QR vs Min-CVaR $\tau = 0.95$	0.041	0.099**	0.233*	1.563**	0.009*
QR vs Min-CVaR $\tau = 0.50$	-0.013	0.047*	0.112*	0.643	0.003
QR vs Min-CVaR $\tau = 0.05$	-0.085**	0.037	-0.081	-0.653	-0.006

Table 3 continued

($\times 10^{-2}$)	Δ**CVaR**$_{0.95}$	Δ**MtC**$_{0.95}$	Δ**SR**	Δ**IR**	Δ**SSR**
No preselection					
QR-SSD vs Min-CVaR $\tau = 0.95$	-0.026	0.242***	0.594***	1.665	0.018**
QR-SSD vs Min-CVaR $\tau = 0.50$	-0.026	0.230***	0.569***	1.225	0.018**
QR-SSD vs Min-CVaR $\tau = 0.05$	-0.018	0.224***	0.559***	0.610	0.017***
QR-SSD vs QR $\tau = 0.95$	-0.023	0.238***	0.580***	1.289	0.018***
QR-SSD vs QR $\tau = 0.50$	-0.021	0.248***	0.610***	1.500	0.020***
QR-SSD vs QR $\tau = 0.05$	-0.026	0.262***	0.649***	1.728	0.021***
QR vs Min-CVaR $\tau = 0.95$	-0.003	0.004	0.014	0.376	0.000
QR vs Min-CVaR $\tau = 0.50$	-0.005	-0.019	-0.041	-0.275	-0.002
QR vs Min-CVaR $\tau = 0.05$	0.007	-0.038	-0.090*	-1.118*	-0.003

autocorrelogram of out-of-sample returns. Statistical significance is assessed by computing confidence intervals for the differences in the selected ratios and rejecting the null hypothesis whenever the interval excludes zero.

Differences in CVaR are generally not statistically different from zero, indicating that QR-SSD models do not significantly improve CVaR compared to other strategies. This result is not unexpected, given the similarity of the objective functions. The primary objective of the analysis is to assess whether the SSD constraints contribute to enhanced overall performance, rather than merely improving the CVaR metric itself. Overall, the differences in MtC, SR, IR, and SSR between the QR-SSD and Min-CVaR strategies are positive and statistically significant for $\tau = 0.95$, where the focus is on penalizing deviations below the benchmark in a manner consistent with the enhancement constraint.

This indicates that, for $\tau = 0.95$, QR-SSD models tend to generate portfolios with superior financial characteristics across all investible universes.

For $\tau = 0.5$ and $\tau = 0.05$, the statistical significance of the performance differences increases with the size of the investible universe.

Comparing QR-SSD and QR, we find that the differences are generally positive and statistically significant, when the full investment universe is considered. This suggests that the inclusion of SSD constraints and the enhancement condition improves the portfolio's risk profile, with the improvement being particularly pronounced when no preselection of assets is applied. When the full investment universe is considered, the opportunity set is broader and the SSD constraints operate over a richer cross-section of assets. In contrast, when the universe is preselected or restricted, the feasible set is already narrower and portfolios tend to be more concentrated. Finally, QR and Min-CVaR exhibit comparable performance, as both effectively control tail risk without delivering incremental enhancement.

4 Conclusions

This paper presents a linear quantile regression model with second-order stochastic dominance (SSD) constraints. The linear formulation requires building permutation matrices and constraints that grow quadratically with the calibration window, which forces a trade-off between informational richness and computational feasibility. The QR-SSD framework offers a transparent, linear, and implementable tool to incorporate stochastic dominance preferences into portfolio construction. The approach is designed for managers who seek to penalize downside deviations while pursuing excess returns relative to the benchmark. The inclusion of a turnover constraint further enhances operational realism by limiting excessive rebalancing and the associated trading costs. We perform an extensive empirical assessment to test the impact on traditional performance measures on out-of-sample returns. Our key empirical findings can be summarized as follows:

- The QR-SSD model tends to generate portfolios with higher Sharpe and Sortino ratios and systematically higher MtC compared to Min-CVaR and the classical QR;
- These positive differences are generally statistically significant in the out-of-sample tests and across different investible universe sizes;
- Improvements are more pronounced in the MtC ratio than in CVaR reductions, which suggests that QR-SSD enhances the return-tail risk trade-off rather than merely reducing extreme losses;
- The preselection procedure improves the absolute risk-return profile of the portfolios, leading to lower tail risk and smaller maximum drawdowns, at the cost of a reduction in the information ratio, reflecting a reduction in active management opportunities.

In conclusion, the proposed approach provides a practical and effective framework to embed stochastic dominance preferences into linear portfolio optimization.

Disclaimer

The opinions, analyses, and methodologies described in this article do not necessarily reflect those of the asset management company with which one of the author is affiliated. The methodology presented should not be construed as being used, endorsed, or promoted by the company for investment management purposes or any other operational purposes.

Appendix A Alternative time windows

To evaluate the robustness of our results, we report findings based on an alternative calibration windows (125 and 250 days) and preselection strategies (SR, MtC and IF) in Tables 4, 5, 6, 7, 8, 9.

Table 4 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest Sharpe Ratio, using a one-year calibration window (250 observations). All values are reported $\times 10^{-2}$

Maximum Sharpe Ratio Preselection $(\times 10^{-2})$	**VaR**$_{0.95}$	**CVaR**$_{0.95}$	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8483	3.0936	0.9127	2.1957	-	0.0698	56,7754	-	-
200 assets									
QR-SSD τ=0.95	1.7806	2.9362	1.2413	3.0535	3.2313	0.0957	49.5892	0.0094	19.6972
QR-SSD τ=0.50	1.7989	2.9502	1.2469	3.0638	3.3509	0.0960	50.8121	0.0097	19.1629
QR-SSD τ=0.05	1.7982	2.9647	1.3566	3.3222	4.1951	0.1031	49.7935	0.0132	18.0317
QR τ=0.95	1.8087	3.0091	1.3420	3.2908	3.9668	0.1036	53.5080	0.0133	18.3903
QR τ=0.50	1.8197	2.9688	1.2319	3.0194	3.2535	0.0947	51.5055	0.0095	18.7557
QR τ=0.05	1.7964	2.9689	1.3854	3.3928	4.5131	0.1060	52.6222	0.0141	18.5241
Min-CVaR τ=0.95	1.7909	2.9700	1.2780	3.1375	3.6349	0.0990	48.8567	0.0109	18.8480
Min-CVaR τ=0.50	1.8123	2.9754	1.3427	3.2841	4.3694	0.1040	47.0097	0.0129	18.8619
Min-CVaR τ=0.05	1.7865	2.9708	1.4039	3.4215	4.4749	0.1084	51.2816	0.0147	18.4669

Table 4 continued

Maximum Sharpe Ratio Preselection									
$(\times 10^{-2})$	$\mathbf{VaR}_{0.95}$	$\mathbf{CVaR}_{0.95}$	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8483	3.0936	0.9127	2.1957	-	0.0698	56,7754	-	-
400 assets									
QR-SSD τ=0.95	1.8001	2.9593	1.2331	3.0410	3.7469	0.0956	50.2626	0.0095	14.8871
QR-SSD τ=0.50	1.8057	2.9535	1.2663	3.1174	4.1580	0.0987	49.4529	0.0104	15.4878
QR-SSD τ=0.05	1.8013	2.9509	1.2769	3.1461	3.9906	0.0993	51.3255	0.0106	15.4176
QR τ=0.95	1.7905	2.9784	1.1809	2.9108	2.8575	0.0927	53.5785	0.0081	17.0260
QR τ=0.50	1.7922	2.9452	1.1409	2.8102	2.7022	0.0890	51.4542	0.0066	17.0242
QR τ=0.05	1.7788	2.9500	1.2164	2.9918	3.3772	0.0952	53.5485	0.0088	16.8404
Min-CVaR τ=0.95	1.7633	2.9306	1.1552	2.8348	2.7183	0.0905	47.5033	0.0068	17.3781
Min-CVaR τ=0.50	1.7827	2.9340	1.1769	2.8861	2.9940	0.0922	48.7332	0.0075	17.3569
Min-CVaR τ=0.05	1.7764	2.9726	1.1833	2.8986	2.8103	0.0932	51.0269	0.0081	17.2771

Table 5 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest Information Ratio, using a one-year calibration window (250 observations). All values are reported $\times 10^{-2}$

Maximum Information Ratio Preselection									
($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8483	3.0936	0.8739	2.1957	-	0.0698	56.7754	-	-
200 assets									
QR-SSD τ=0.95	1.8615	2.9866	1.3318	3.2431	4.9759	0.1023	50.1927	0.0127	19.8115
QR-SSD τ=0.50	1.8440	2.9895	1.3608	3.3167	5.3824	0.1042	50.9782	0.0136	19.1527
QR-SSD τ=0.05	1.8564	3.0137	1.4308	3.4941	5.7509	0.1088	49.8550	0.0161	17.9751
QR τ=0.95	1.8829	3.0158	1.3867	3.3802	5.1397	0.1054	53.2700	0.0148	18.2930
QR τ=0.50	1.8745	3.0104	1.3370	3.2621	5.0378	0.1016	50.6125	0.0132	18.5637
QR τ=0.05	1.8608	3.0131	1.3929	3.3963	5.3755	0.1059	51.6447	0.0149	18.4279
Min-CVaR τ=0.95	1.8453	2.9943	1.3540	3.2984	5.2683	0.1038	51.6369	0.0135	19.0499
Min-CVaR τ=0.50	1.8606	3.0143	1.3448	3.2828	5.3758	0.1034	52.2375	0.0135	19.0870
Min-CVaR τ=0.05	1.8662	3.0354	1.3655	3.3360	5.1937	0.1041	53.1085	0.0144	18.7153

Table 5 continued

Maximum Information Ratio Preselection									
($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8483	3.0936	0.8739	2.1957	-	0.0698	56.7754	-	-
400 assets									
QR-SSD τ=0.95	1.8239	3.0100	1.3544	3.3116	7.5964	0.1064	51.7525	0.0137	15.9264
QR-SSD τ=0.50	1.8224	3.0117	1.3236	3.2399	6.9634	0.1042	52.8940	0.0128	16.5230
QR-SSD τ=0.05	1.8104	3.0014	1.3848	3.3881	7.2983	0.1085	51.3557	0.0145	16.1191
QR τ=0.95	1.8121	3.0368	1.2792	3.1320	6.3848	0.1007	53.1358	0.0118	16.8144
QR τ=0.50	1.8031	3.0038	1.3001	3.1839	6.7609	0.1021	52.0851	0.0120	16.8907
QR τ=0.05	1.8216	3.0013	1.3118	3.2121	6.5360	0.1029	52.3901	0.0123	16.8824
Min-CVaR τ=0.95	1.8070	3.0079	1.2580	3.0830	6.1039	0.0991	52.3705	0.0108	17.5217
Min-CVaR τ=0.50	1.8185	3.0084	1.2248	3.0046	5.7327	0.0965	52.6730	0.0098	17.5375
Min-CVaR τ=0.05	1.8122	3.0091	1.3491	3.2952	7.5981	0.1057	53.0290	0.0136	17.2222

Table 6 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest MtC Ratio, using a one-year calibration window (250 observations). All values are reported $\times 10^{-2}$

Maximum MtC Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8483	3.0936	0.8739	2.1957	-	0.0698	56.7754	-	-
200 assets									
QR-SSD τ=0.95	1.7970	2.9335	1.1824	2.9029	2.6042	0.0912	50.5641	0.0077	19.8454
QR-SSD τ=0.50	1.7895	2.9424	1.2449	3.0598	3.3076	0.0962	50.9945	0.0096	19.3321
QR-SSD τ=0.05	1.8018	2.9675	1.3363	3.2774	3.9926	0.1019	50.2582	0.0126	18.0592
QR τ=0.95	1.8299	3.0067	1.2505	3.0685	3.1021	0.0965	53.7654	0.0106	18.6198
QR τ=0.50	1.8143	2.9624	1.2565	3.0758	3.4707	0.0966	51.3951	0.0102	18.8373
QR τ=0.05	1.8134	2.9487	1.3096	3.2043	3.6629	0.0997	53.3940	0.0116	18.6777
Min-CVaR τ=0.95	1.8238	2.9882	1.2333	3.0188	2.8860	0.0960	53.0473	0.0098	19.2532
Min-CVaR τ=0.50	1.8391	3.0188	1.2233	3.0095	3.1834	0.0960	50.2044	0.0099	19.3684
Min-CVaR τ=0.05	1.7753	2.9637	1.3424	3.2782	3.8467	0.1036	52.5819	0.0127	18.6768

Table 6 continued

Maximum MtC Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8483	3.0936	0.8739	2.1957	-	0.0698	56.7754	-	-
400 assets									
QR-SSD τ=0.95	1.8193	2.9492	1.2464	3.0729	3.8162	0.0968	49.6563	0.0097	14.7746
QR-SSD τ=0.50	1.8006	2.9502	1.2769	3.1428	4.2325	0.0992	49.7084	0.0106	15.5111
QR-SSD τ=0.05	1.7816	2.9522	1.2984	3.1983	4.2337	0.1009	51.6494	0.0113	15.4033
QR τ=0.95	1.7730	2.9859	1.2233	3.0125	3.3379	0.0962	54.5399	0.0095	16.8531
QR τ=0.50	1.7898	2.9521	1.1719	2.8848	3.1178	0.0914	51.5006	0.0076	16.9524
QR τ=0.05	1.7616	2.9497	1.2100	2.9794	3.3058	0.0944	53.4370	0.0087	16.8560
Min-CVaR τ=0.95	1.7582	2.9136	1.2188	2.9820	3.2501	0.0955	47.2484	0.0085	17.3567
Min-CVaR τ=0.50	1.7748	2.9371	1.1986	2.9443	3.2152	0.0937	50.0877	0.0082	17.3738
Min-CVaR τ=0.05	1.7848	2.9909	1.2320	3.0312	3.4871	0.0976	52.8867	0.0098	17.1229

Table 7 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest Sharpe Ratio, using a six-month calibration window (125 observations). All values are reported $\times 10^{-2}$

Maximum Sharpe Ratio Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8314	3.0576	0.9127	2.2874	-	0.0718	56.7754	-	-
200 assets									
QR-SSD τ=0.95	1.7967	2.8408	1.3682	3.3456	3.0401	0.1032	49.7682	0.0110	23.8569
QR-SSD τ=0.50	1.8033	2.8875	1.3565	3.3308	3.2558	0.1031	50.7190	0.0113	22.6795
QR-SSD τ=0.05	1.8094	2.9031	1.3745	3.3705	3.2580	0.1028	51.9700	0.0120	21.1271
QR τ=0.95	1.8228	2.9479	1.4467	3.5613	3.9127	0.1082	55.4186	0.0147	20.7215
QR τ=0.50	1.8029	2.9088	1.4614	3.5861	4.1064	0.1096	51.1145	0.0146	20.8615
QR τ=0.05	1.8150	2.9238	1.4561	3.5709	3.8627	0.1090	52.8373	0.0147	20.9060
Min-CVaR τ=0.95	1.8468	3.2313	1.2177	2.9653	2.3031	0.0966	62.5766	0.0114	23.1608
Min-CVaR τ=0.50	1.8331	3.1012	1.3234	3.2352	2.9276	0.1030	59.2666	0.0131	22.3721
Min-CVaR τ=0.05	1.8594	3.1303	1.2257	3.0159	2.1541	0.0953	57.2142	0.0105	22.0776

Table 7 continued

Maximum Sharpe Ratio Preselection									
$(\times 10^{-2})$	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8314	3.0576	0.9127	2.2874	-	0.0718	56.7754	-	-
400 assets									
QR-SSD τ=0.95	1.7676	2.8510	1.2643	3.1076	2.6331	0.0953	50.9008	0.0081	17.5494
QR-SSD τ=0.50	1.7892	2.8840	1.1860	2.9262	2.1020	0.0901	53.0249	0.0063	17.4774
QR-SSD τ=0.05	1.7716	2.8660	1.2120	2.9898	2.1791	0.0919	51.1138	0.0068	16.9013
QR τ=0.95	1.7737	2.9333	1.1884	2.9417	2.2195	0.0905	59.3792	0.0070	18.5706
QR τ=0.50	1.7387	2.8796	1.3270	3.2607	3.3939	0.1015	52.6842	0.0103	18.5926
QR τ=0.05	1.7467	2.9025	1.2917	3.1734	2.9607	0.0980	54.6158	0.0096	18.6998
Min-CVaR τ=0.95	1.8138	3.2621	1.1697	2.7641	1.8967	0.0950	63.7863	0.0103	21.0885
Min-CVaR τ=0.50	1.7881	3.3184	1.0386	2.3940	1.0753	0.0857	74.4651	0.0066	20.5860
Min-CVaR τ=0.05	1.8230	3.9562	1.7322	2.9909	2.2484	0.1560	60.3675	0.0406	24.9034

Table 8 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest Information Ratio, using a six-month calibration window (125 observations). All values are reported $\times 10^{-2}$

Maximum IR Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8314	3.0576	0.9127	2.2874	-	0.0718	56.7754	-	-
200 assets									
QR-SSD τ=0.95	1.8182	2.9217	1.5514	3.7715	5.8866	0.1178	50.2415	0.0174	23.1568
QR-SSD τ=0.50	1.8109	2.9396	1.5434	3.7596	5.9549	0.1167	51.3545	0.0175	22.2681
QR-SSD τ=0.05	1.8225	2.9800	1.5110	3.7078	5.4001	0.1136	52.5969	0.0171	20.6835
QR τ=0.95	1.8800	3.0011	1.6255	3.9571	6.4172	0.1217	51.0973	0.0209	20.6833
QR τ=0.50	1.8329	2.9606	1.5106	3.6807	5.6370	0.1128	54.2280	0.0168	20.7038
QR τ=0.05	1.8342	2.9786	1.4582	3.5833	4.9667	0.1095	52.3385	0.0155	20.8826
Min-CVaR τ=0.95	1.8970	3.1111	1.5047	3.6897	4.6688	0.1150	53.1246	0.0189	22.7552
Min-CVaR τ=0.50	1.8652	3.0354	1.5651	3.8413	5.6410	0.1189	54.2135	0.0196	21.7700
Min-CVaR τ=0.05	1.8493	3.0695	1.5644	3.8475	5.6196	0.1191	54.0965	0.0201	21.3605

Table 8 continued

Maximum IR Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8314	3.0576	0.9127	2.2874	-	0.0718	56.7754	-	-
400 assets									
QR-SSD τ=0.95	1.7746	2.9726	1.2408	3.0176	4.3473	0.0947	55.4935	0.0090	18.6855
QR-SSD τ=0.50	1.7736	2.9798	1.3374	3.2547	5.9010	0.1029	53.9471	0.0119	18.3432
QR-SSD τ=0.05	1.7923	2.9771	1.3452	3.2913	5.9084	0.1034	53.4579	0.0121	18.0054
QR τ=0.95	1.7828	3.0063	1.3946	3.4087	6.5468	0.1075	54.3825	0.0140	18.1888
QR τ=0.50	1.7665	2.9916	1.4018	3.4231	6.8961	0.1073	56.0946	0.0140	18.1837
QR τ=0.05	1.7907	2.9879	1.4289	3.4944	7.2276	0.1097	53.6322	0.0148	18.1619
Min-CVaR τ=0.95	1.7852	2.9733	1.2732	3.1089	4.9067	0.0976	53.9217	0.0100	19.5279
Min-CVaR τ=0.50	1.7625	2.9688	1.3386	3.2573	5.9802	0.1025	52.9449	0.0118	19.2982
Min-CVaR τ=0.05	1.7743	2.9870	1.3946	3.4028	6.7392	0.1069	55.2892	0.0138	18.9337

Table 9 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest MtC Ratio, using a six-month calibration window (125 observations). All values are reported $\times 10^{-2}$

Maximum MtC preselection ($\times 10^{-2}$)	**$VaR_{0.95}$**	**$CVaR_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8314	3.0576	0.9127	2.2874	0.0000	0.0718	56.7754	0.0000	0.0000
200 assets									
QR-SSD τ=0.95	1.7674	2.8361	1.3423	3.2897	2.8008	0.1016	51.6556	0.0102	23.9714
QR-SSD τ=0.50	1.8047	2.8756	1.3621	3.3382	3.2265	0.1033	51.9025	0.0113	22.6468
QR-SSD τ=0.05	1.7915	2.8881	1.4225	3.4945	3.6007	0.1068	51.7198	0.0132	21.1016
QR τ=0.95	1.8342	2.9546	1.3049	3.2271	2.8361	0.0977	58.0556	0.0106	21.2245
QR τ=0.50	1.8096	2.9089	1.4212	3.4971	3.7998	0.1067	51.7978	0.0134	21.0578
QR τ=0.05	1.8143	2.9180	1.3926	3.4299	3.3655	0.1044	52.3657	0.0127	21.1443
Min-CVaR τ=0.95	1.8577	3.0261	1.4432	3.5238	3.6586	0.1106	53.9290	0.0158	22.6298
Min-CVaR τ=0.50	1.8095	3.1047	1.3246	3.2317	3.3094	0.1039	60.6749	0.0132	22.6444
Min-CVaR τ=0.05	1.8296	2.9944	1.3189	3.2355	2.7772	0.1000	57.5001	0.0116	21.7477

Table 9 continued

Maximum MtC preselection $(\times 10^{-2})$	**$VaR_{0.95}$**	**$CVaR_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.8314	3.0576	0.9127	2.2874	0.0000	0.0718	56.7754	0.0000	0.0000
400 assets									
QR-SSD $\tau=0.95$	1.7743	2.8543	1.2824	3.1506	2.8100	0.0970	51.5133	0.0087	17.6070
QR-SSD $\tau=0.50$	1.7658	2.8702	1.2727	3.1385	2.8814	0.0971	52.8116	0.0086	17.2931
QR-SSD $\tau=0.05$	1.7635	2.8673	1.2819	3.1663	2.8226	0.0974	51.6851	0.0088	16.7838
QR $\tau=0.95$	1.7635	2.9246	1.2638	3.1232	2.8843	0.0960	59.0800	0.0091	18.5469
QR $\tau=0.50$	1.7638	2.8852	1.3308	3.2760	3.4478	0.1012	53.4615	0.0105	18.6248
QR $\tau=0.05$	1.7704	2.9041	1.2521	3.0790	2.6033	0.0956	54.0770	0.0085	18.4983
Min-CVaR $\tau=0.95$	1.7890	3.1293	1.1299	2.7128	1.6436	0.0899	58.6884	0.0075	20.5232
Min-CVaR $\tau=0.50$	1.7699	3.1119	1.0809	2.5925	1.3848	0.0864	63.2130	0.0057	20.1872
Min-CVaR $\tau=0.05$	1.7914	3.0463	1.1614	2.8158	2.1372	0.0915	57.1042	0.0075	19.5544

Appendix B Performance measures

We describe the performance measures used to assess the out-of-sample behavior of the optimal strategies. Let r denote a real-valued random variable representing portfolio returns, defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with mean $\mu := \mathbb{E}[r]$ and variance $\sigma^2 := \mathbb{E}[(r - \mu)^2]$. Benchmark returns are represented by the random variable y. Let $k \in \mathbb{R}$ denote a fixed target return and let r_f denote the (deterministic) risk-free rate.

- $\mathrm{VaR}_\tau(L)$: The Value-at-Risk at confidence level τ is defined as the τ-quantile of the loss variable $L = -r$:
$$\mathrm{VaR}_\tau(L) := \inf\{\ell \in \mathbb{R} : \mathbb{P}(L \leq \ell) \geq \tau\}.$$
- $\mathrm{CVaR}_\tau(L)$: The Conditional Value-at-Risk at confidence level τ is defined as the expected loss conditional on losses exceeding $\mathrm{VaR}_\tau(L)$ and admits the equivalent representation
$$\mathrm{CVaR}_\tau(L) := \min_{\xi \in \mathbb{R}} \left\{ \xi + \frac{1}{1-\tau} \mathbb{E}\big[(L-\xi)_+\big] \right\},$$
Higher values of $\mathrm{VaR}_\tau(L)$ and $\mathrm{CVaR}_\tau(L)$ therefore indicate larger downside risk.
- $D^-(r;k)$: Downside deviation of returns relative to a target level k:
$$D^-(r;k) = \sqrt{\mathbb{E}\big[(r-k)^2 \, \mathbb{I}_{\{r<k\}}\big]},$$
which corresponds to the square root of the lower partial moment of order two. In our analysis, we set $k = 0$.
- **Sharpe Ratio (SR)**:
$$SR = \frac{\mu - r_f}{\sigma}.$$
- **Sortino Ratio (SSR)**:
$$SSR = \frac{\mu - r_f}{D^-(r;k)}.$$
- **Information Ratio (IR)**:
$$IR = \frac{\mu - \mu_y}{\sigma_{r-y}},$$
where $\mu_y := \mathbb{E}[y]$ and σ_{r-y} denotes the standard deviation of the tracking error $r - y$.
- **Mean-to-CVaR Ratio ($\mathbf{MtC}_\tau$)**:
$$MtC_\tau = \frac{\mu - r_f}{CVaR_\tau(L)}.$$
- **Maximum Drawdown (MDD)**: Let $\left\{r_t^{(out)}\right\}_{t=1}^{T}$ denote the realized daily out-of-sample return series over the evaluation period, and the associated cumulative wealth process is defined as:
$$W_t := \prod_{s=1}^{t}(1 + r_s^{(out)})$$

The absolute maximum drawdown used in this work is then defined as

$$MDD = \max_{t \in \{1,\ldots,T\}} \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}.$$

- **Average Turnover (AvgTO)**: Portfolio turnover at date t is defined as

$$TO_t = \frac{1}{2} \sum_{j=1}^{I} \left| \beta_{j,t} - \beta_{j,t-1} \right|,$$

where I is the number of assets and $\beta_{j,t}$ denotes the portfolio weight of asset j at time t. The Average Turnover (AvgTO) is computed as the mean turnover over the out-of-sample horizon:

$$\text{AvgTO} = \frac{1}{T} \sum_{t=1}^{T} TO_t.$$

- **Jensen's Alpha (JA)**: Measures the abnormal return of a portfolio relative to a benchmark after adjusting for systematic risk:

$$JA = \mu - \left[r_f + \beta_p \left(\mu_y - r_f \right) \right],$$

where β_p is the portfolio beta with respect to the benchmark.

- **Normalized Herfindahl–Hirschman Index (HHI*)**: A measure of portfolio concentration:

$$HHI^* = \frac{HHI - \frac{1}{I}}{1 - \frac{1}{I}}, \qquad HHI = \sum_{j=1}^{I} \beta_j^2.$$

Lower values of HHI^* indicate greater diversification, while higher values correspond to more concentrated portfolios.

Appendix C Alternative preselections

In Table 10, 11 we report the results for the 500-days calibration window with two alternative preselections: IR and MtC respectively.

Table 10 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest Information Ratio, using a two-year calibration window (500 observations). All values are reported $\times 10^{-2}$

Maximum IR Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.9037	3.1588	0.8761	2.2033	-	0.0721	56.7754	-	-
200 assets									
QR-SSD τ=0.95	1.8617	3.0053	1.5126	3.6884	7.1974	0.1207	50.2718	0.0178	15.4522
QR-SSD τ=0.50	1.8615	3.0049	1.5116	3.6861	7.1057	0.1206	50.6293	0.0177	16.2306
QR-SSD τ=0.05	1.8650	3.0138	1.4926	3.6468	6.7425	0.1186	51.4593	0.0173	16.5033
QR τ=0.95	1.8694	3.0787	1.4139	3.4621	6.2643	0.1137	54.1134	0.0159	17.0006
QR τ=0.50	1.8665	3.0200	1.4534	3.5513	6.7326	0.1164	50.3855	0.0162	17.2847
QR τ=0.05	1.8754	3.0418	1.3879	3.4138	5.5695	0.1105	53.5796	0.0145	17.1257
Min-CVaR τ=0.95	1.8366	3.0064	1.4858	3.6261	6.9645	0.1188	50.8092	0.0170	16.9060
Min-CVaR τ=0.50	1.8682	3.0324	1.4403	3.5252	6.7028	0.1158	51.0828	0.0160	17.0016
Min-CVaR τ=0.05	1.8664	3.0626	1.3624	3.3333	5.7113	0.1094	53.4937	0.0140	17.2440

Table 10 continued

Maximum IR Preselection ($\times 10^{-2}$)	$\mathbf{VaR}_{0.95}$	$\mathbf{CVaR}_{0.95}$	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.9037	3.1588	0.8761	2.2033	-	0.0721	56.7754	-	-
400 assets									
QR-SSD τ=0.95	1.8732	3.0692	1.4442	3.5467	8.7879	0.1164	52.3136	0.0167	9.9473
QR-SSD τ=0.50	1.8819	3.0739	1.4386	3.5320	9.0167	0.1158	52.0755	0.0165	11.4077
QR-SSD τ=0.05	1.8566	3.0443	1.5090	3.6960	8.1581	0.1215	51.7932	0.0183	13.6307
QR τ=0.95	1.8608	3.0819	1.3153	3.2123	7.3766	0.1063	53.3057	0.0129	15.5891
QR τ=0.50	1.8458	3.0568	1.3202	3.2352	7.8291	0.1066	51.6458	0.0127	15.6616
QR τ=0.05	1.8576	3.0697	1.3589	3.3429	8.3306	0.1101	53.6059	0.0140	15.7343
Min-CVaR τ=0.95	1.8676	3.0689	1.3047	3.2072	7.6827	0.1056	52.4171	0.0124	14.7468
Min-CVaR τ=0.50	1.8595	3.0671	1.3108	3.2176	8.0461	0.1061	52.5580	0.0125	14.8463
Min-CVaR τ=0.05	1.8479	3.0861	1.3286	3.2518	7.9745	0.1080	53.4304	0.0133	15.7815

Table 11 The table reports daily performance indicators, as defined in Appendix B, for the benchmark and the out-of-sample optimal portfolios evaluated over two preselected asset universes (200 and 400 assets), selected based on the highest MtC Ratio, using a two-year calibration window (500 observations). All values are reported $\times 10^{-2}$

Maximum MtC Ratio Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.9037	3.1588	0.8761	2.2033	-	0.0721	56.7754	-	-
200 assets									
QR-SSD τ=0.95	1.8110	2.9269	1.6372	3.9822	7.4114	0.1293	43.8776	0.0202	14.7791
QR-SSD τ=0.50	1.8218	2.9269	1.6030	3.9007	7.0326	0.1268	44.5794	0.0192	15.8457
QR-SSD τ=0.05	1.8183	2.9313	1.5803	3.8491	6.6677	0.1250	46.3420	0.0186	16.3616
QR τ=0.95	1.8126	2.9926	1.5339	3.7326	6.6319	0.1225	48.8294	0.0182	17.0031
QR τ=0.50	1.8089	2.9357	1.5296	3.7256	6.5358	0.1217	46.2583	0.0172	17.2087
QR τ=0.05	1.8172	2.9555	1.5216	3.7275	6.0925	0.1211	46.9273	0.0173	17.0604
Min-CVaR τ=0.95	1.8151	2.9433	1.5284	3.7428	6.4647	0.1221	44.3531	0.0173	16.7936
Min-CVaR τ=0.50	1.8141	2.9708	1.4896	3.6430	6.3671	0.1194	47.3200	0.0166	16.8723
Min-CVaR τ=0.05	1.7977	2.9809	1.5266	3.7200	6.5428	0.1228	47.8000	0.0178	17.1955

Table 11 continued

Maximum MtC Ratio Preselection ($\times 10^{-2}$)	**VaR$_{0.95}$**	**CVaR$_{0.95}$**	**MtC**	**SR**	**IR**	**SSR**	**MDD**	**JA**	**AvgTO**
$S\&P500$	1.9037	3.1588	0.8761	2.2033	-	0.0721	56.7754	-	-
400 assets									
QR-SSD τ=0.95	1.8557	2.9835	1.4849	3.6437	7.1588	0.1194	47.6836	0.0166	9.2207
QR-SSD τ=0.50	1.8599	2.9853	1.4659	3.5964	6.8613	0.1176	47.6185	0.0161	10.4778
QR-SSD τ=0.05	1.8468	2.9862	1.4651	3.6000	6.7482	0.1177	48.7028	0.0161	12.5412
QR τ=0.95	1.8312	3.0231	1.3707	3.3656	6.4428	0.1117	50.9278	0.0138	15.5045
QR τ=0.50	1.8119	2.9601	1.3284	3.2606	5.7199	0.1068	47.8092	0.0116	15.5737
QR τ=0.05	1.8372	2.9828	1.2971	3.1887	4.9984	0.1043	49.1600	0.0110	15.5719
Min-CVaR τ=0.95	1.8200	2.9844	1.2721	3.1355	4.9275	0.1031	50.2651	0.0103	13.9970
Min-CVaR τ=0.50	1.7967	2.9852	1.2818	3.1489	5.0814	0.1038	49.9812	0.0106	14.1975
Min-CVaR τ=0.05	1.8280	3.0686	1.3352	3.2648	5.5212	0.1097	52.8280	0.0133	15.6008

Appendix D Statistics

Table 12 The table reports the main daily summary statistics of the benchmark and the out-of-sample returns of the optimal portfolios. Specifically, μ denotes the mean return, σ the standard deviation, γ the skewness, κ the kurtosis, and q_α the α-quantile. All values are reported $\times 10^{-2}$

$(\times 10^{-2})$	μ	σ	γ	κ	q_1	q_5	q_{50}	q_{95}	q_{99}
$S\&P500$	0.028	1.256	-0.518	15.401	-3.679	-1.904	0.069	1.687	3.341
200 Assets									
$QR\text{-}SSD_{\tau=0.95}$	0.048	1.207	-0.210	15.321	-3.530	-1.812	0.074	1.676	3.153
$QR\text{-}SSD_{\tau=0.50}$	0.047	1.208	-0.230	14.961	-3.455	-1.824	0.071	1.684	3.128
$QR\text{-}SSD_{\tau=0.05}$	0.046	1.207	-0.262	14.644	-3.501	-1.821	0.073	1.675	3.086
$QR_{\tau=0.95}$	0.046	1.234	-0.244	16.730	-3.563	-1.836	0.077	1.679	3.180
$QR_{\tau=0.50}$	0.044	1.207	-0.290	15.487	-3.478	-1.815	0.077	1.670	3.118
$QR_{\tau=0.05}$	0.043	1.211	-0.340	14.441	-3.540	-1.847	0.078	1.686	3.159
$Min\text{-}CVaR_{\tau=0.95}$	0.044	1.208	-0.265	15.530	-3.466	-1.818	0.072	1.670	3.154
$Min\text{-}CVaR_{\tau=0.50}$	0.044	1.223	-0.249	16.079	-3.509	-1.815	0.080	1.673	3.176
$Min\text{-}CVaR_{\tau=0.05}$	0.043	1.238	-0.441	16.682	-3.630	-1.858	0.074	1.686	3.197
400 Assets									
$QR\text{-}SSD_{\tau=0.95}$	0.044	1.216	-0.311	15.064	-3.525	-1.852	0.074	1.674	3.119
$QR\text{-}SSD_{\tau=0.50}$	0.043	1.216	-0.301	15.118	-3.487	-1.858	0.069	1.681	3.150
$QR\text{-}SSD_{\tau=0.05}$	0.044	1.215	-0.315	15.046	-3.486	-1.848	0.075	1.675	3.122
$QR_{\tau=0.95}$	0.041	1.231	-0.216	16.113	-3.529	-1.827	0.066	1.693	3.259
$QR_{\tau=0.50}$	0.040	1.207	-0.275	15.192	-3.476	-1.811	0.064	1.675	3.189
$QR_{\tau=0.05}$	0.039	1.212	-0.337	15.574	-3.441	-1.847	0.065	1.678	3.140
$Min\text{-}CVaR_{\tau=0.95}$	0.037	1.211	-0.320	15.770	-3.581	-1.832	0.059	1.680	3.126
$Min\text{-}CVaR_{\tau=0.50}$	0.039	1.211	-0.299	15.898	-3.448	-1.805	0.067	1.679	3.158
$Min\text{-}CVaR_{\tau=0.05}$	0.041	1.251	-0.227	17.008	-3.645	-1.820	0.065	1.690	3.271
All Assets									
$QR\text{-}SSD_{\tau=0.95}$	0.045	1.263	-0.250	15.057	-3.653	-1.911	0.073	1.719	3.329
$QR\text{-}SSD_{\tau=0.50}$	0.045	1.261	-0.241	15.039	-3.652	-1.921	0.079	1.700	3.339
$QR\text{-}SSD_{\tau=0.05}$	0.045	1.259	-0.246	14.870	-3.672	-1.916	0.077	1.711	3.363
$QR_{\tau=0.95}$	0.038	1.271	-0.204	15.944	-3.660	-1.881	0.065	1.709	3.452
$QR_{\tau=0.50}$	0.037	1.269	-0.222	15.888	-3.649	-1.873	0.066	1.704	3.461
$QR_{\tau=0.05}$	0.037	1.271	-0.207	16.010	-3.696	-1.879	0.062	1.720	3.455

Table 12 continued

$(\times 10^{-2})$ $S\&P500$	μ 0.028	σ 1.256	γ -0.518	κ 15.401	q_1 -3.679	q_5 -1.904	q_{50} 0.069	q_{95} 1.687	q_{99} 3.341
Min-$CVaR_{\tau=0.95}$	0.038	1.273	-0.202	16.212	-3.659	-1.882	0.066	1.703	3.489
Min-$CVaR_{\tau=0.50}$	0.038	1.272	-0.200	16.113	-3.671	-1.876	0.064	1.711	3.467
Min-$CVaR_{\tau=0.05}$	0.038	1.270	-0.201	16.121	-3.658	-1.877	0.061	1.726	3.450

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