Department of Management and Information Technology

Working Paper

Series “Mathematics and Statistics”

n. 7/MS – 2005

A Statistical Approach to Heterogeneous Monitoring Networks

by

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A statistical approach to heterogeneous monitoring networks

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Abstract
In this paper our objective is to propose a flexible model able to integrate different environmental data subject to heterogeneity. In particular we consider PM$_{10}$ data coming from monitoring networks for air quality assessment; in this case the heterogeneity can arise because of the different instruments used in the monitoring station and the sampling strategies that change in time and space. To do this we propose a Geostatistical Dynamical model based on the state – space approach introduced by Fassò and Nicolis in [1] which is an extension of the DDC model presented in [2]. We assume that the observed data are random fields composed by a linear function of the “true” levels and error components, where the “true” concentrations of PM$_{10}$ are unobservable processes and represent the state equation of the model. Considering the PM$_{10}$ data of the Piemonte region during the year 2003, we show some preliminary results.

Keywords: spatio-temporal modelling; Kalman filter; calibration; Geostatistical Dynamical Calibration model (GDC), heterogeneous monitoring networks.

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1. Introduction

In recent years, as a result of studies that have verified the negative effects on human health, atmospheric pollution has been of great concern for many countries of the world. For example in Europe this focus is a consequence of the Treaty Establishing the European Community, Title XIX, and specifically of Council Directives 96/62/EC, on ambient air quality, and 99/30/EC, on limit values of air pollutants, including particulate matters with a diameter under 10 micron (PM$_{10}$). These Directives are being gradually adopted by member states, for example in Italy the Decreto Ministeriale number 60 (2$^{\text{th}}$ april 2002) states both the air quality standards on the daily and yearly scale and the measurement quality standards for instruments measuring PM$_{10}$. The reference method for the sampling and the measurement of PM$_{10}$ is based on the collection on a filter of the PM$_{10}$ fraction of ambient particulate matter and the determination of its mass; this is the gravimetric principle.

Despite the fact that the carrying out of these laws is going on, the data for the recent years are characterized by various inhomogeneities. With regard to North of Italy, Piemonte, Lombardia and Emilia Romagna are the three regions that cover the “Pianura Padana” area and for each of them policies and techniques have been quite different.

First of all the monitor types accuracy and precision may change over time and space. For this reason it is necessary to apply suitable transformations to make the data equivalent to those gathered by the reference system. For example, some networks have been based on automatic monitors based on tapered element oscillating microbalance (TEOM). Moreover we may have different time series lengths due to missing values and/or different sampling frequencies and/or different station lifetimes. In fact it could happen that a station changes its sampling strategy (e.g. from every two hours to hourly) or that some monitoring instruments collect data slower than others (e.g. only with daily frequency) while other monitors collect data more often (e.g. with hourly frequency). For example, the former is the case of the Lombardia region where many stations change their sampling frequency (for instance, “Milano Juvara” station collected PM$_{10}$ data every two hours until 25/11/2004 and then every hours), the last is the case of the Piemonte region where we can find gravimetric monitors with daily data and TEOM monitors with hourly data.

For all these reasons we have heterogeneous PM$_{10}$ measurements and the aim of this paper is to reconstruct an homogenous space-time series of comparable data allowing both
retrospective and trend analysis, as well as performing meaningful quality standard attainments.

In our model we consider the spatial correlation between data gathered by different monitors: this is a geostatistical approach. Geostatistics is the branch of statistics that studies data spatially dispersed and was originally used in the mining industry, regarding spatial sampling of rock formations and the problem of the total quantity on an ore or mineral in a field, given concentrations of the ore or mineral at a finite set of sampling point. Today geostatistics is used for environmental application (geology, atmospheric science, hydrology, climatology, etc.) and for any discipline that work with data collected from different spatial locations and need to develop models that indicate when there is dependence between measurements (for an extensive discussion of geostatistic see [3]).

In particular, our model is a geostatistical extension of the Dynamical Displaced Calibration model of Fassò and Nicolis presented in [2]. In order to reduce the dimensionality of the model, we decompose the “true” unobservable process in \( p \) principal fields following the approach of Mardia et al. [4] and Wikle and Cressie [5,8].

2. Model setup and fitting

2.1 The general Geostatistical Dynamical model

In this section we present a flexible model which is able to handle data coming from different networks and with the heterogeneity problems explained in the introduction. Since now we stress the importance of such a model for the creation of a single framework of homogenous data regarding, for a big administrative area, for example “Valle Padana”

Assume that data \( y(t) = \{ y(t,s_1),...,y(t,s_{p_j}) \} \) are obtained from an observable and spatially continuous process \( y(t,s_j) \), where \( t \in \{1,2,...,n\} \), a discrete index of time (for example hours), and the generic \( s_j \in \{1,...,n\} \) is a pointer into the spatial domain \( D = (z_1,...,z_n), z_j \in \mathbb{R}^2 \). Note that the dimension \( p_j \) is time varying due to the sampling frequency heterogeneity and the missing values problem.

Easily with the notation \( y(t,s) \) we will indicate the measurement at time \( t \) in location \( z_s \in D \).

Obviously \( y(t,s) \) is subject to uncertainty and can be biased by some type of error (for example, measurement error); for this reason it can be read as the practical determination of
an underlying process. Regarding the PM$_{10}$ data, we investigate the real level of air pollution
(underlying process) but we observe $y(t,s)$, that is the PM$_{10}$ concentration level (in mg/m$^3$) in
station $s$ at time $t$ (observed data subject to uncertainty).

The “true” phenomenon to be monitored is given by a time varying linear function of an
unobserved process denoted by $\mu(t)$. This underlying global process is $K$-dimensional and
Markovian, so we have

$$\mu(t) = H\mu(t-1) + \eta(t)$$

The process $\mu(t)$ determines the network measurements in the sense that

$$E(y(t,s)/\mu(t)) = A(t,s) + \sum_{i=1}^{24} B(t,s)\mu(t-i) + G(s)x(t,s)$$

where the vector $A = A^*$ is the additive bias depending on some parameters $\tau$ and the matrix
$B(t,s)$ contains the following elements:

- a $K$-dimensional set of EOF or principal components PCA$^1$ (see [4] and [5]);
- a component for missing data handling;
- a component for change in sampling frequency.

Hence we have

$$B(t,s) = M(t)B^*_\tau F(t)\Phi(t)$$

where $M(t)$ is a matrix for missing data handling, $B^*_\tau$ concerns parameters $\tau, F(t)$ is composed
of frequency-time averaging weights and $\Phi(s)$ is the PCA loading matrix corresponding to
location $s$.

We then have the following $p_\tau$-dimensional measurement equation (in matrix form):

$$y(t) = A^*_\tau + \sum_{i=1}^{24} B_i \mu(t-i) + G(t)x(t) + \epsilon(t)$$

where $G(t,s) = M(t)Q(s)$ is a matrix multiplied by $M(t)$ as above, and $x(t)$ are generic
covariates. These kinds of variables are helpful to explain the “true” phenomenon and they
regard, for example, meteorological, chemical or physical aspects directly connected to the air
pollution process. These variables can be measured in the same location $z_s$ of $y(t,s)$ (site
specific covariates) or be related to a higher level spatial dimension (area specific covariates),
such as administrative regions.

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$^1$ Section 2.2 is dedicated to Principal Component Analysis (PCA).
The measurement error components \( \varepsilon(t,s) \) is given by the generic equation

\[
\varepsilon(t,s) = \mu(t,s) + \varepsilon_0(t,s)
\]

where

- the first left-side term, \( \mu(t,s) \), is the spatially descriptive component or the so-called small-scale residual component which describes the spatial correlation not explained by the PCA component \( \mu(t) \). This has been often supposed independent over time and spatially correlated with spatial covariance matrix given by \( \Sigma_{\mu_0} \), which is usually related to a stationary and isotropic spatial process (see [3]). In particular the \textit{J-Bessel} is a good semivariogram model for interpreting the data spatial correlation because of its flexible parameterization which considers also periodic terms. The \textit{J-Bessel} model is given by

\[
\Omega(\hat{r}) = \theta_\nu \left[ 1 - \frac{2^\nu \Gamma(\nu + 1)}{\nu!} \Theta(\nu \sqrt{\nu \theta_\nu / \theta_\nu}) \right]
\]

for each \( h \) (distance between two locations) with \( \theta_\nu, \theta_{d}, \theta_r \geq 0 \) and \( \Omega_{\theta_\nu} \) subject to

\[
\min_{B > 0, \gamma(B) = \theta_\nu, \gamma(B) < 0} B = \theta_r , \quad \text{where} \quad \Gamma \text{ is the Gamma function, } \Theta \text{ is the J-Bessel function of the first type parameter } \nu \text{ (see [7]).}
\]

- The second term \( \varepsilon_0(t,s) \) is a Gaussian spatially and time independent pure measurement error, uncorrelated on \( \mu_0(t,s) \), with standard deviation equal to \( \sigma_\varepsilon \).

The unobserved components \( \mu(t) \) is defined by a \((K+1)\)-dimensional first order stationary Markovian process

\[
\mu(t) = H_\mu \mu(t-1) + \eta_\mu(t)
\]

where the innovations \( \eta \) are defined by an independent process with mean zero.

A special case for the matrix \( A(t,s) \) is discussed in section 2.4.

2.2 The Principal Component Analysis (PCA)

Principal Component Analysis is a multivariate statistical tool. The central idea of PCA is to reduce the dimensionality of a data set in which there are a lot of correlated variables, while retaining as much as possible the variation present in the data set. This reduction is achieved
by creating a new data set of principal components which are uncorrelated and which are ordered so that the first few components retain most of the variability of the original variables. Computation of the principal components reduces to the solution of an eigenvalue-eigenvector problem for a positive-semidefinite symmetric matrix.

Using \( y(t,s) \) at each location \( s \) (\( s=1,2,\ldots,n \)) and time point \( t \) (\( t=1,2,\ldots,T \)) we can define the \( k \)-th principal component (or field) as \( \mu_k(t) = \phi_k^t y(t,s) \), \( k=1,2,\ldots,K \), \( K \leq n \), where \( \phi_k = (\phi_k(s_1),\ldots,\phi_k(s_n))^t \) is a vector of the loading matrix \( \Phi \). So \( \phi_1 \) is the vector that allows \( \text{var}[\mu_1(t)] \) to be maximized subject to the constraint \( \phi_1^t \phi_1 = 1 \); \( \phi_2 \) is the vector that maximizes \( \text{var}[\mu_2(t)] \) subject to the constraint \( \phi_2^t \phi_2 = 1 \) and \( \text{cov}[\mu_1(t),\mu_2(t)] = 0 \); in general, \( \phi_k \) maximizes \( \text{var}[\mu_k(t)] \) subject to the orthogonal (\( \phi_k^t \phi_j = 1 \)) and the uncorrelation constraint (\( \text{cov}[\mu_j(t),\mu_k(t)] = 0 \), \( j \neq k \)).

This is equivalent to solving the eigensystem

\[
C_y \Phi = \Phi \Lambda
\]

where \( C_y = E[y(t),y(t)] \), \( \Phi = (\phi_1,\ldots,\phi_n)^t \) with \( \phi_k = (\phi_k(s_1),\ldots,\phi_k(s_n))^t \), \( k = 1,2,\ldots,n \), \( \Lambda = \text{diag}(\lambda_1,\ldots,\lambda_n) \) and \( \lambda_i = \text{var}[\mu_i(t)] \), \( i = 1,2,\ldots,n \). The solution is obtained by a symmetric decomposition of the covariance matrix \( C_y = \Phi \Lambda \Phi^t \).

Once the PCA decomposition is computed the problem of choosing an appropriate \( K \leq n \) arises. It’s a common practice to choose the first \( K \) components such that the cumulative percentage of variance explained is high enough.

### 2.3 The Piemonte region: a case study

The PM\(_{10}\) monitoring network of the Piemonte region is composed by different measurement monitors: Low Volume Gravimeter (LV or LVG), High Volume Gravimeter (HV), TEOM, BETA, and NEFELOMETRO monitors\(^2\). Figure 1 shows the spatial locations of the monitoring stations together with the type of monitor.

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\(^2\)The data considered in this work have been gathered by the Piemonte AriaWeb informative system that is a branch of the “Sistema Regionale di Rilevamento della qualità dell’Aria (SRQA)”. 

In our analysis we consider only LVG and TEOM monitors and the PM$_{10}$ daily concentrations for the period from 1st January 2003 to 31st December 2003. Since some monitoring stations were affected by a large number of missing values, we selected those stations with more than 90% of the validated data.
The following Table 1 contains the selected stations (17 LVG and 2 TEOM) with some descriptive statistics concerning the PM$_{10}$ concentrations: percentage of validated data calculated on the annual expected number of data (%VD), mean, standard deviation (SD), min, max and median.

Three of the selected stations are located in Turin city, another three monitors are located in the surrounding hilly area (Borgaro, Pinerolo and Buttiglier Alta Alta) while the remaining stations are spread all around the Piemonte region in an area with most monitors on the plain of the Po River and some in the neighbouring Alpine valley which are subject to local climatic factors.

<table>
<thead>
<tr>
<th>Station label</th>
<th>% VD</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Low Volume Gravimetric (LVG)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AL - Nuova Orti (AL)</td>
<td>93.7</td>
<td>55.1</td>
<td>27.8</td>
<td>13</td>
<td>167</td>
<td>48</td>
</tr>
<tr>
<td>Alba (CN)</td>
<td>93.4</td>
<td>42.3</td>
<td>26.9</td>
<td>7</td>
<td>159</td>
<td>35</td>
</tr>
<tr>
<td>Borgaro (TO)</td>
<td>92.6</td>
<td>42.0</td>
<td>23.9</td>
<td>4</td>
<td>115</td>
<td>35.5</td>
</tr>
<tr>
<td>Borgosesia (VC)</td>
<td>99.7</td>
<td>36.6</td>
<td>21.3</td>
<td>4</td>
<td>113</td>
<td>30.5</td>
</tr>
<tr>
<td>Bra (CN)</td>
<td>94.5</td>
<td>57.3</td>
<td>28.4</td>
<td>10</td>
<td>144</td>
<td>51</td>
</tr>
<tr>
<td>Buttiglier Alta (TO)</td>
<td>92.6</td>
<td>44.4</td>
<td>26.8</td>
<td>6</td>
<td>135</td>
<td>37</td>
</tr>
<tr>
<td>Buttiglieri d'Asti (AT)</td>
<td>95.1</td>
<td>43.5</td>
<td>25.5</td>
<td>6</td>
<td>133</td>
<td>39</td>
</tr>
<tr>
<td>Carmagnola</td>
<td>94.0</td>
<td>59.8</td>
<td>32.5</td>
<td>8</td>
<td>149</td>
<td>51</td>
</tr>
<tr>
<td>Casale Monferrato - Via De Negri (AL)</td>
<td>92.3</td>
<td>45.2</td>
<td>27.7</td>
<td>2</td>
<td>165</td>
<td>36</td>
</tr>
<tr>
<td>CN - Piazza II Reggimento Alpini (CN)</td>
<td>98.9</td>
<td>37.6</td>
<td>23.0</td>
<td>2</td>
<td>135</td>
<td>32</td>
</tr>
<tr>
<td>Novi Ligure (AL)</td>
<td>89.6</td>
<td>50.7</td>
<td>30.5</td>
<td>5</td>
<td>185</td>
<td>43</td>
</tr>
<tr>
<td>Pinerolo (TO)</td>
<td>90.7</td>
<td>39.4</td>
<td>23.5</td>
<td>6</td>
<td>135</td>
<td>34</td>
</tr>
<tr>
<td>TO - Piazza Rivoli (TO)</td>
<td>90.1</td>
<td>49.0</td>
<td>27.0</td>
<td>5</td>
<td>140</td>
<td>42</td>
</tr>
<tr>
<td>TO - Via Consolata (TO)$^3$</td>
<td>98.1</td>
<td>63.6</td>
<td>33.9</td>
<td>12</td>
<td>165</td>
<td>54</td>
</tr>
<tr>
<td>TO - Via Gaidano (TO)</td>
<td>95.6</td>
<td>42.0</td>
<td>30.8</td>
<td>1</td>
<td>162</td>
<td>32</td>
</tr>
<tr>
<td>Tortona (AL)</td>
<td>97.0</td>
<td>48.8</td>
<td>25.9</td>
<td>5</td>
<td>144</td>
<td>45</td>
</tr>
<tr>
<td>VC - Corso Gastaldi (VC)</td>
<td>94.0</td>
<td>57.6</td>
<td>31.5</td>
<td>13</td>
<td>312</td>
<td>47</td>
</tr>
<tr>
<td><strong>TEOM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ponzone (AL)</td>
<td>93.9</td>
<td>26.4</td>
<td>12.4</td>
<td>4</td>
<td>78</td>
<td>24</td>
</tr>
<tr>
<td>TO - Via Consolata (TO)</td>
<td>96.1</td>
<td>56.3</td>
<td>19.8</td>
<td>20</td>
<td>125</td>
<td>53</td>
</tr>
</tbody>
</table>

Tab. 1 – **Descriptive statistics concerning the PM$_{10}$ concentration in the selected stations (year 2003)**

$^3$ The LVG data of TO – Via Consolata station will be used only for the validation of the GDC model.
The monthly mean of PM$_{10}$ concentrations can be read also in Fig. 2 which shows the monthly averages for each station; the bold line in the graph refers to the mean level concentration (considering all the stations together). It can be seen that the plotted series depends on a seasonal component related to the months of the year: for example, in winter the PM$_{10}$ concentration level are higher than in other period. The reasons of this trend could be explained by other physical, chemical or meteorological variables (for example, temperature, pressure, etc.) depending to the atmospheric pollution process.

![Monthly PM$_{10}$ Concentration](image)

**Fig. 2 – Monthly averages time series**

### 2.4. Fitting the model to the Piemonte data

In this section we present the GDC model considering that in our case study we have two types of monitor (LVG and TEOM), giving respectively two kinds of measurement, $y_G(t,s)$ and $y_T(t,s')$. As we consider daily data the averaging component $F(t)$ may be suppressed.

In particular we use the PM$_{10}$ concentrations from LVG monitors to perform a dynamical calibration of the spatially displaced TEOM data, which are known to underestimate the “true” PM$_{10}$ concentration level.
Assuming that both data have a component error, the model can be expressed by the following equations both depending on the unobserved process $\mu(t,s)$:

$$y_c(t,s) = \Phi(s)\mu(t,s) + \epsilon_c(t,s)$$

$$y_r(t,s') = \alpha_0 + \alpha(t) + \beta \Phi(s')\mu(t,s') + \epsilon_r(t,s')$$

The first equation is simply an observation equation while the second may be interpreted as a time-varying linear calibration equation where:

- $\Phi(s)$ and $\Phi(s')$ are the loadings from the PCA decomposition for LVG and TEOM stations;
- $\alpha_0$ is the fixed additive bias;
- $\alpha(t)$ is the time varying component of the additive bias;
- $\beta$ is the multiplicative bias.

The measurement error components $\epsilon_c$ and $\epsilon_r$ are given by the generic equation

$$\epsilon(t,s) = \mu_0(t,s) + \epsilon_0(t,s)$$

where $\mu_0(t,s)$ is the small-scale component with covariance matrix $\Sigma_\mu_0$ and $\epsilon_0(t,s)$ is a Gaussian pure measurement error, uncorrelated on $\mu_0(t,s)$, with standard deviations equal to $\sigma_{\epsilon_c}$ and $\sigma_{\epsilon_r}$, respectively.

As in section 2.1 the unobserved components $\mu(t,s)$ and $\alpha(t)$ are defined by a $(K+1)$-dimensional first order stationary Markovian process

$$\mu(t) = H_\mu \mu(t-1) + \eta_\mu(t)$$

$$\alpha(t) = h_\alpha \alpha(t-1) + \eta_\alpha(t)$$

where the innovations $\eta$ are defined by an independent process with mean zero and block diagonal covariance matrix

$$\Sigma_\eta = \begin{pmatrix} \Sigma_\mu & 0 \\ 0 & \sigma_\alpha^2 \end{pmatrix}.$$ 

Here the vector of parameters to be estimated is given by

$$\omega = (\alpha, \beta, \sigma_{\epsilon_c}^2, \sigma_{\epsilon_r}^2, \Sigma_\mu, H_\mu, h_\alpha, \Sigma_\mu, \sigma_\alpha^2).$$
The estimation of the model parameters for the Piemonte PM$_{10}$, based on the kalman filter algorithm, follows the following steps:

- first we evaluate the principal components loadings, that is $\Phi(s)$ and $\Phi(s')$, given LVG and corrected TEOM$^4$ observations both transformed with the logarithmic function;
- next we estimate the $\omega$ parameters vector by the maximum likelihood function obtained by the kalman filter recursion conditionally on the matrix $\Phi(s)$ and $\Phi(s')$ given previously;
- finally, the kalman filter algorithm provides the estimates of the state equations $\alpha(t)$ and $\mu(t)$ and the calibrated values $\hat{\mu}(t,s)$ for each time and station.

2.5 Model identification

The first step, PCA analysis, applied to the Piemonte data, is summarized by Fig.3, where the cumulative percentage of explained variance shows that the first component accounts for about 70%. The subsequent components have substantially smaller and decreasing variances. We used the first $K = 11$ PCA components which give more than 95% of explained variance. Note that the first component may be interpreted as the common regional pollution level ([10]).

The second step, identification, is actually an iterated step. In order to find the simplest model with good fitting, we estimated a number of alternative models with various levels of complexity. Comparing the log-likelihoods, the calibrated versus observed LVG values for Consolata station and the residual properties, we found that the following important simplifications are in order.

$^4$ We use the 1.3 correction factor proposed by the APEG Report (see [6]).
- The small scale component in the error equation

\[ \varepsilon(t,s) = \mu_\varepsilon(t,s) + \varepsilon_\eta(t,s) \]

of § 2.1 may be ignored at least for our data. This conclusion is based on the fact that the spatial correlation matrix of the empirical errors \( \varepsilon(t,s) \) has no elements significantly different from zero at 95% level. This is consistent with a pure measurement error interpretation of \( \varepsilon(t,s) \).

- we have homoskedastic measurement errors, i.e the observation equations for LVG and TEOM have the same error variance, \( \sigma^2_{\varepsilon} = \sigma^2_{\varepsilon} \). This is consistent with the fact that both instruments are known to be accurate;

- the PCA components are spatially uncorrelated and have independent and homogeneous dynamics over time. This means that

\[ H_\mu = \text{diag}(h_\mu, \ldots, h_\mu), \quad 0 < h_\mu < 1 \]

- The same orthogonality properties of PCA components entail that the innovations are orthogonal, i.e.

\[ \Sigma_\mu = \text{diag}(\sigma^2_{\eta_1}, \ldots, \sigma^2_{\eta_\mu}). \]

- Moreover we found that \( \sigma^2_{\eta_1} = \ldots = \sigma^2_{\eta_i} \) and \( \sigma^2_{\eta_i} > \sigma^2_{\eta_j} \). This is consistent with the PCA above, where the first component was found to have a relevantly larger variance.
The dynamics of the time varying additive bias $\alpha(t)$ was found to be absent. This because the estimates for $h_\mu$ and $\sigma_\epsilon^2$ where close to one and zero respectively. Moreover fitting was better without $\alpha(t)$ in the model.

Considering that we had no covariates for our model, this result is a little bit unexpected, since the relation between TEOM and LVG is often reported to depend on temperature and, according to this, some seasonal variation of $\alpha(t)$ may be expected.

The following Table 2 contains the corresponding parameter estimates and standard errors. Note that the standard errors are quite small and the residuals of the model passed standard tests for approximate normality and white noise.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_0$</th>
<th>$h_\mu$</th>
<th>$\beta$</th>
<th>$\log(\sigma_\eta^2)$</th>
<th>$\log(\sigma_\epsilon^2)$</th>
<th>$\log(\sigma_\xi^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>3.63</td>
<td>0.84</td>
<td>1.08</td>
<td>0.27</td>
<td>-3.59</td>
<td>-2.61</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.08</td>
<td>0.08</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Tab. 2 - Estimated parameters of the GDC model for the Piemonte region

3. Conclusions

In this work we introduced a general model able to handle data coming from different heterogeneous monitoring networks. Heterogeneity arises when different instruments with different precision and accuracy are used or when the sampling frequency changes through the reference area and/or the period of interest.

We considered a simple example concerning daily data from Piemonte network with only two types of monitor. Using these we studied the spatio-temporal variability and found a simplified model.

According to this the spatial variation is well explained by a finite dimensional representation while the residual spatial small scale component may be omitted. This is consistent with a pure error model interpretation.

At least for these data it seems that the calibration equation, whenever based on a dynamical model, is time invariant.
4. References


