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Abstract

The present paper provides a vertical differentiated model of a broadcasting market with a two-sided approach. We calculate the equilibrium in terms of advertising levels, subscription fees and qualities provision, both in monopoly and in duopoly where the market is uncovered. Furthermore, welfare considerations are made for all market structure by considering viewers' surplus.

Key words: two-sided market, broadcasting, quality, welfare.

JEL codes: D42, D43, L15, L82

1 Introduction

Television broadcasting has a long history of salient regulation problems, and these problems have recently been emphasized by the convergence of internet, computer software and telecommunications firms. Conventionally, regulatory issues in media markets are divided into classes that consist of economic and non-economic features. The elements of the former class are related to the structure of the supply side: the definition of the relevant market, the assessment of the degree of concentration and competition, the impact of the ownership structure and the conditions of the access to the broadcasting service; however, the elements of the latter class mainly focus on the broadcasting contents and the control of advertising (Rowat 2007). The interplay among these economic and non-economic issues, which is a peculiar feature of broadcasting, deserves closer attention from policy makers and antitrust authorities.

In this respect, our paper provides a unified framework to address all of these issues and to analyze broadcasting competition. Particular attention is devoted to the quality, prices, share of the audience and consumers' surplus. More precisely, our work analyzes the role of competition in a two-sided market characterized by vertical differentiation.

Quality is the first ingredient of our model. Although quality is a relevant characteristic of the broadcasting market, it lacks a clear and common economic definition (Born and Prosser 2001). At first glance, quality could be associated with the technological innovations that have deeply affected broadcasting, such as higher-definition images or interactive services. From this perspective, the quality of broadcasting can be interpreted in the standard vertical approach. However, if we focus on content, quality is more difficult to define, as content's quality can be related to its accuracy, truth, impartiality and immediacy of information that helps form public opinion, expresses minority voices or performs a watchdog role for the public interest.[1] For instance, Collins (2007), debating about the role of public service broadcasting, associates quality with the purpose of providing not only entertainment but also education, learning and cultural excellence.[2]

However, it is worth noticing that viewers' perceptions of these features might differ. Indeed, the audience has a taste for a variety of broadcasting outputs, including cultural programs, popular genres, and sport events. Therefore, an increase in the content quality does not necessarily translate into an upward shift of the demand and audience. Hence, some dimensions of content's quality may encompass a horizontal feature.

Nevertheless, in a specific genre, all of the viewers prefer high-quality content to low-quality content, which implies vertical competition in the market, as an example a live soccer game with respect to a pre-recorded one. Given these considerations, in the present paper, we assume that broadcasters provide vertically differentiated output with respect to quality.

A second important aspect we would like to address is the role of competition in a two-sided market. We consider this type of market structure because broadcasting networks compete on two sides, namely, for audience and advertisers; their goal is to maximize profits. Advertising is typically considered a nuisance by the audience, and it represents a negative externality. However, the audience exerts a positive externality on advertisers. Therefore, competition has a broader meaning with re-

[1] Mepham (1990) argues that there is a general rule for assessing television quality, namely, "whether or not [its production] is governed by an ethic of truth-telling".

[2] Ellmann (2014) distinguishes between "soft" and "hard" attributes to media consumption. He defines hard or informative attributes of media those generating positive social externalities, while soft attributes are those with only a private value, such as graphic quality, sensationalism and entertainment.

spect to the standard industrial organization and may generate different results and policy implications. In our setup, whereas viewers are single-homing, advertisers are multi-homing, which implies that platforms have monopoly power in providing access to their single-homing customers. In this respect, platforms act as "bottlenecks" between advertisers and consumers by offering sole access to their respective set of consumers. This assumption is crucial to explaining the prevailing competition on the consumer side.[3] We also model advertisers as non-strategic: their payoffs do not depend on what other advertisers do, but on benefits that are related to the market demand. This behavior suits the case of informative advertising.

Finally, it is well known that media markets are characterized by a broad range of business models, which are under both private and public ownership:[4] free-to-air TV, where broadcast platforms are only financed through advertising revenues; pay TV, where broadcast stations are financed through subscription revenues; lastly, a mixed regime in which broadcast platforms are financed through both subscription fees and advertising. We consider a very general framework with platforms that are financed both by advertising and by subscription fees.[5]

As previously mentioned, we provide a model of platforms' competition in a framework of vertical differentiation. In a context where platforms endogenously provide their quality levels, we calculate the equilibrium values of advertising, the optimal subscription fees for viewers and the provision of quality. In particular, we take into account a single-channel and multi-channel monopoly as well as a duopoly. In our analysis, we want to stress the importance of having a market that is never covered ex-ante. Indeed, we believe that the potential demand has a relevant role and may alter the equilibrium configuration in terms of the prices, quality, audience size and advertising. Furthermore, the uncovered market configuration fits the case of a broadcasting market very well, and this market is characterized by continuous technological turmoil with the creation of new market segments. We also calculate the consumers' surplus for each market configuration to determine whether the interplay among the content's quality, subscription fees and advertising might benefit the audience

To anticipate the results, we show that viewers are always better off when they are

[3] For a further discussion on the role of the single-homing or multi-homing assumption, see Roger (2010).

[4] In Italy, for instance, there exists a public broadcaster financed by both subscription fees (canone RAI) and advertising revenues. Furthermore, there exist both free-to-air private operators such as Mediaset that are totally financed through advertising and private pay-TV providers financed through subscription fees and advertising revenues (e.g., Sky).

[5] Subscription fees are set in general terms and could be either positive or negative. Thus, the fees encompass the possibility of subsidization.

free to choose among channels of different qualities. In our two-sided framework there are two forces at stake. Higher quality induces consumers to pay higher subscription fees to join the platform. In turn, the platform can extract a surplus on the advertiser side and "invest" it in a reduction of subscription fees, implying that advertisers cross-subsidize single-homing consumers. Therefore, a sort of substitution between quality and advertising arises. In addition, we show that competition is beneficial to the audience, resulting in a viewers' surplus that is larger in the duopoly configuration than in the monopoly even when both provide high-quality and low-quality channels. Finally, we illustrate that the chance of catching extra viewers (who compose the uncovered market share) disciplines the platforms' behavior in a duopoly, which renders consumers' surplus higher.

1.1 Related Literature

Our paper belongs to the literature of vertically differentiated two-sided markets dealing with welfare issues. In this stream, Armstrong (2006) and Weeds (2013) provide a model with an endogenous quality provision in the two-sided context of digital broadcasters. By comparing the competition in two different regimes, namely, free-to-air and pay TV, they show that the program quality is higher for pay TV, which is also optimal from a social point of view. In a similar setting, Anderson (2007) analyzes the effect of an advertising cap on the quality provision of a monopoly broadcaster and on welfare. He shows that advertising time restrictions may improve the welfare but decrease the program quality.[6] Kind et al. (2007) perform a welfare analysis with an endogenous quality provision and find that a merger between TV channels may improve the welfare. Moreover, Lin (2011) extends the analysis to direct competition between different business models where one platform operates as a free-to-air TV service and the second one as a pay-TV service. In this framework, he shows that platforms vertically differentiate their programs according to the degree of viewers' dislike of advertising. In the same approach, Gonzales-Mestre and Martinez-Sanchez (2013) study how publicly owned platforms affect the program quality provision, the social welfare and the optimal level of advertising. In contrast to our model, all of the above contributions focus on the duopoly case and neglect monopoly behavior with the exception of Anderson (2007). Furthermore, the duopoly setting is always assumed to be covered, preventing any welfare consideration about the role of increasing demand. Conversely, we relax this assumption by introducing an uncovered market. We also provide a comparison between the uncovered and the covered

[6] Although without a specific reference to a quality provision, Dukes (2004) and Anderson and Coate (2005) show that monopoly media ownership may increase the welfare.

market structures from a welfare perspective.[7]

The paper is organized as follows. Section 2 illustrates the case of a multi-channel monopoly broadcaster and explains the set-up and equilibrium. Then, Section 3 focuses on the welfare comparison between a multi-channel monopoly broadcaster and a single-channel one. Section 4 introduces competition among broadcasters set-up and equilibrium, and Section 5 addresses the effects on welfare. Finally, we provide some conclusions in Section 6.

2 The Multi-Channel Broadcaster

For the sake of exposition we describe first the case of a multi-product monopoly platform and second the duopoly case. [8]

A multiproduct monopoly platform can provide vertical differentiated *channels* to a uniform distribution of individuals (viewers [9]) of mass 1. We refer to this platform as the multi-channel broadcaster.

Individuals are assumed to be single-homing. The utility of an individual accessing platform's channel i is:

$$u_i = V - \delta a_i + \beta\theta_i - s_i \tag{1}$$

and zero otherwise. V is the utility of accessing the platform independently of its quality. The channel's quality is denoted by the parameter θ_i which belongs to a technological range $\Theta = [\underline{\theta}, \overline{\theta}]$ with $\overline{\theta} > \underline{\theta} > 0$. Individuals have a private valuation for information expressed by the parameter $\beta \sim U[0,1]$ which can be interpreted as their willingness to pay for quality. Moreover, they incur in a nuisance cost δa_i due to the presence of advertising on the channels. [10] Finally s_i stands for the subscription charge.

If the platform provides two channels of different quality, θ_H and θ_L (with $\theta_H > \theta_L$) it obtains the following audience shares:

$$B_H = 1 - \beta_{HL} = 1 - \frac{\delta\left(a_H - a_L\right)}{\left(\theta_H - \theta_L\right)} - \frac{\left(s_H - s_L\right)}{\left(\theta_H - \theta_L\right)} \tag{2}$$

[7] In a different set up, concerning the credit card market, the issue of the endogenous quality of the service has been arised (see e.g., Rochet and Tirole (2007), Verdier (2010)).

[8] The model in this section builds on that of Battaggion and Drufuca (2014).

[9] We consider a broadcasting market, which well fits our setting. However, in principle this model might be refered also to a broader range media (newspapers, as example).

[10] This cost depends on the intensity of advertising on the channel a_i and on a parameter of viewers' aversion to ads δ. This parameter is assumed to be invariant across individuals.

$$B_L = \beta_{HL} - \beta_{0L} = \frac{\delta\left(a_H - a_L\right)}{\left(\theta_H - \theta_L\right)} + \frac{\left(s_H - s_L\right)}{\left(\theta_H - \theta_L\right)} - \left(\frac{\delta a_L - V}{\theta_L} - \frac{s_L}{\theta_L}\right) \tag{3}$$

where β_{HL} and β_{0L} characterize respectively the individual indifferent between the two channels and the one indifferent between accessing the low-quality channel or not accessing at all.

Advertisers are producers of mass 1 who access the platform to advertise their products to individuals. They sell products of quality α that are produced at constant marginal costs, which we set equal to zero. The product quality α is distributed on an interval $[0,1]$ according to a distribution function $F(\alpha)$.[11] Individuals have a willingness to pay α for a good of quality α. Each producer has monopoly power, and therefore, it can extract the full surplus from individuals by selling its product at a price equal to α. As is standard in this class of models, we assume that the advertising is informative and that consumers watching an advertisement always buy the good. Hence, we refer to producers as advertisers. Unlike viewers, advertisers are allowed to multi-home. Advertisers have to pay an advertising charge r_i, that is endogenously determined for each channel. Due to the assumption of single homing on the viewers' side, each channel behaves as a "monopoly" in carrying its audience to advertisers. Therefore, r_i is set by the platform in order to leave the marginal advertiser with zero profit:

$$\alpha_i = \frac{r_i}{B_i} \tag{4}$$

Thus, the amount of advertising for each channel is the share of advertisers with $\alpha > \alpha_i$:

$$a_H = 1 - F\left(\frac{r_H}{B_H}\right) \tag{5}$$

$$a_L = 1 - F\left(\frac{r_L}{B_L}\right) \tag{6}$$

The platform sets advertising spaces and subscription prices (unconstrained) and it can provide its channels' quality θ_H and θ_L by incurring a fixed cost K.[12] In other words, once the cost is incurred, the higher-quality outlet can be provided to

[11] In the discussion of our results, we will also consider the special case of a uniform distribution of advertisers.

[12] This assumption fits very well the structure of the ICT and media markets, where there is a prominent role of fixed costs compared to marginal ones (see e.g., Shapiro and Varian (1998), Areeda and Hovenkamp (2014))

individuals without any additional charges. Notice that since our costs are also fixed in quantity, they meet the requirement of separability.

The multi-product media platform collects revenues from both individuals and advertisers:

$$\Pi_{MP} = (B_H s_H + B_L s_L) + a_H r_H + a_L r_L - 2K \tag{7}$$

Then, according to the literature, we define the advertising revenues per individual as:

$$\rho(a_i) = \frac{a_i r_i}{B_i} = \frac{a_i F^{-1}(1-a_i) N B_i}{B_i} = a_i F^{-1}(1-a_i) \tag{8}$$

We assume $\rho(a_i)$ to be concave on the interval $a \in [0,1]$. Given that $\rho(a_i) = 0$ for $a_i = 0$ and $a_i = 1$, the function is single-peaked. Hence, profits rewrite as follow:

$$\Pi_{MP} = B_H(s_H + \rho(a_H)) + B_L(s_L + \rho(a_L)) - 2K \tag{9}$$

We consider a three-stage game. First the monopoly platform chooses the levels of quality. Second, it sets subscription fees and advertising spaces Finally, in the third stage, viewers and advertisers simultaneously decide whether to join a channel.

2.1 Subscription Fees and Advertising Intensities

Having defined the demand function of viewers and advertisers, for given prices we solve the game backwards, from stage three. This determines how advertising charges react to pay-per-view prices s_i and to advertising levels a_i:

$$r_H\left(s_H, s_L, a_H, a_L, \theta_H, \theta_L\right) = F^{-1}(1-a_H)\left(\frac{\theta_H - \theta_L - (s_H - s_L) - \delta(a_H - a_L)}{\theta_H - \theta_L}\right) \tag{10}$$

$$r_L\left(s_H, s_L, a_H, a_L, \theta_H, \theta_L\right) = F^{-1}(1-a_L)\left(\tfrac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L} - \tfrac{s_L + \delta a_L - V}{\theta_L}\right) \tag{11}$$

The multi-channel broadcaster relies on advertising revenues and subscription fees to fund its services.

$$\begin{cases} \max\limits_{a_H, a_L, s_H, s_L} \Pi_{MP} = \pi_L + \pi_H = B_H(s_H + \rho(a_H)) + B_L(s_L + \rho(a_L)) - 2K \\ s.t. a_H \geq 0 \\ a_L \geq 0 \end{cases}$$

The platform maximizes profits (9), with respect to advertising intensity (a_H, a_L) and subscription fees (s_H, s_L) for each channel, subject to a positivity constraint on advertising. The following Proposition summarizes results regarding advertising.

Proposition 1 *The multi-channel monopoly broadcaster chooses the same advertising intensity, independently of quality and subscription revenues:*

$$\rho'(a_i) = \delta$$

for $i = H, L$.

Proof. First order conditions with respect to the advertising spaces and subscription fees are respectively, for $i, j = H, L$ with $i \neq j$:

$$\frac{\partial \pi_{MP}}{\partial s_i} = \frac{\partial B_i}{\partial s_i}(s_i + \rho_i) + B_i(1 + \frac{\partial \rho_i}{\partial s_i}) + \frac{\partial B_j}{\partial s_i}(s_j + \rho_j) + B_j(\frac{\partial \rho_j}{\partial s_i}) = 0$$
$$\frac{\partial \pi_{MP}}{\partial a_i} = \frac{\partial B_i}{\partial a_i}(s_i + \rho_i) + B_i(\frac{\partial s_i}{\partial a_i} + \frac{\partial \rho_i}{\partial a_i}) + \frac{\partial B_j}{\partial a_i}(s_j + \rho_j) + B_j(\frac{\partial s_j}{\partial a_i} + \frac{\partial \rho_j}{\partial a_i}) \leqq 0$$

where $\rho_i = \rho(a_i)$ to simplify notation. Given the construction of advertising revenues per individual (see equation (8)), we have that $\frac{\partial \rho_i}{\partial a_j} = 0$. Moreover $\frac{\partial s_i}{\partial a_i} = 0$ and $\frac{\partial s_i}{\partial a_j} = 0$. Hence, first order conditions simplify as follows:

$$\frac{\partial \pi_{MP}}{\partial s_i} = \frac{\partial B_i}{\partial s_i}(s_i + \rho_i) + B_i(1 + \frac{\partial \rho_i}{\partial s_i}) + \frac{\partial B_j}{\partial s_i}(s_j + \rho_j) + B_j(\frac{\partial \rho_j}{\partial s_i}) = 0$$
$$\frac{\partial \pi_{MP}}{\partial a_i} = \frac{\partial B_i}{\partial a_i}(s_i + \rho_i) + B_i(\frac{\partial \rho_i}{\partial a_i}) + \frac{\partial B_j}{\partial a_i}(s_j + \rho_j) \leqq 0$$

It is easy to show that $\frac{\partial B_H}{\partial a_H} = \delta \frac{\partial B_H}{\partial s_H}$, $\frac{\partial B_L}{\partial a_L} = \delta \frac{\partial B_L}{\partial s_L}, \frac{\partial B_H}{\partial a_L} = \delta \frac{\partial B_H}{\partial s_L}$ and $\frac{\partial B_L}{\partial a_H} = \delta \frac{\partial B_L}{\partial s_H}$. FOCs rewrite as follows:

$$\frac{\partial \pi_{MP}}{\partial s_H} = \frac{\partial B_H}{\partial s_H}(s_H + \rho_H) + B_H + \frac{\partial B_L}{\partial s_H}(s_L + \rho_L) = 0 \tag{12}$$
$$\frac{\partial \pi_{MP}}{\partial s_L} = \frac{\partial B_L}{\partial s_L}(s_L + \rho_L) + B_L + \frac{\partial B_H}{\partial s_L}(s_H + \rho_H) = 0 \tag{13}$$
$$\frac{\partial \pi_{MP}}{\partial a_H} = \delta \frac{\partial B_H}{\partial s_H}(s_H + \rho_H) + B_H \rho'_H + \delta \frac{\partial B_L}{\partial s_H}(s_L + \rho_L) \leqq 0 \tag{14}$$
$$\frac{\partial \pi_{MP}}{\partial a_L} = \delta \frac{\partial B_L}{\partial s_L}(s_L + \rho_L) + B_L \rho'_L + \delta \frac{\partial B_H}{\partial s_L}(s_H + \rho_H) \leqq 0 \tag{15}$$

By substitution we get from (14) and (15)

$$B_H \rho'_H + \delta(-B_H) \leqq 0$$
$$B_L \rho'_L + \delta(-B_L) \leqq 0$$

If $a_i > 0$ for $i = H, L$, then

$$\rho'(a_H^*) = \delta \tag{16}$$
$$\rho'(a_L^*) = \delta \tag{17}$$

■

According to Proposition 1 an optimal decision is to set a fixed advertising space for each channel just depending on the disutility of the viewers, δ. Moreover, the multi-channel broadcaster does not set the maximum intensity of advertising ($a_i = 1$) or the amount that maximize revenues per viewer, i.e. $\rho'(a_i) = 0$. This result is in line with the literature dealing with the issue of bottlenecks, suggesting that competition just focuses on the audience side. From optimality conditions (12) and (13), given a_H^* and a_L^*, we obtain equilibrium subscription fees, s_H^* and s_L^*, and shares on viewers' side, B_H^* and B_L^*, as function of quality, revenues per viewer and advertising level:

$$s_H^* = \frac{\theta_H + V - a^*\delta - \rho(a^*)}{2} \tag{18}$$
$$s_L^* = \frac{\theta_L + V - a^*\delta - \rho(a^*)}{2} \tag{19}$$

$$B_H^* = \frac{1}{2} \tag{20}$$
$$B_L^* = \frac{1}{2} - \left(\frac{\theta_L - V + \delta a^* - \rho(a^*)}{2\theta_L}\right) \tag{21}$$

The above values show a *profit neutrality* result, where revenues from the advertising side are counterbalanced by a decrease of the subscription fees that is the same for each channel. Moreover, given that the subscription fees positively depend on the quality, a type of substitutability between advertising and quality emerges. The high-quality channel always covers half of the viewers' market, whereas the audience of the low-quality channel relies on quality, fees and advertising. If the monopoly would cover the whole market, it equally divides the audience between the two channels. Otherwise, the low-quality channel always has fewer viewers. Recall that the advertising revenues $\rho(a^*)$ depend on the distribution function of advertisers. Thus, we can obtain sharper intuition of our results by assuming a specific type of distribution. In particular, we consider the case of a uniform distribution of advertisers $\alpha \sim U(0, 1)$ and we obtain the following equilibrium values:

$$a_H^* = a_L^* = a^* = \frac{1-\delta}{2} \tag{22}$$

$$s_H^* = \frac{\theta_H + V - \frac{(1-\delta)(1+3\delta)}{4}}{2}$$
$$s_L^* = \frac{\theta_L + V - \frac{(1-\delta)(1+3\delta)}{4}}{2}$$

$$B_H^* = \frac{1}{2}$$
$$B_L^* = \frac{1}{2} - \frac{\theta_L - V - (\frac{1-\delta}{2})^2}{2\theta_L}$$

In the uniform case equilibrium fees and advertising intensity just depend on quality, disutility from advertising δ and V.

2.2 Quality

At stage 1, the multichannel platform chooses quality levels.[13] Its profits are:

$$\pi_{MP} = \frac{\theta_H}{4} + \frac{(V + \rho(a^*) - \delta a^*)(2\theta_L + V + \rho(a^*) - \delta a^*)}{4\theta_L} - 2K$$

Looking at first order conditions we get:

$$\frac{\partial \pi_{MP}}{\partial \theta_H} = \frac{1}{4} > 0 \tag{23}$$
$$\frac{\partial \pi_{MP}}{\partial \theta_L} = -\frac{1}{4\theta_L^2} \left(V + \rho(a^*) - \delta a^*\right)^2 < 0 \tag{24}$$

Hence, we get a result of maximal differentiation, as stated in the following Proposition

[13] The result of this stage follows the assumption of fixed cost of quality, K. However, we obtain similar outcomes different functional form for the cost of quality (see Appendix 7.1).

Proposition 2 *Given a technological constraint $\Theta = [\underline{\theta}, \overline{\theta}]$, when viewers differ in their willingness to pay for quality, the multi-channel broadcaster chooses to maximally differentiate quality: it chooses the minimal quality for the L channel while it sets the highest quality for the H one.*

$$\theta_H^* = \overline{\theta}$$
$$\theta_L^* = \underline{\theta}$$

Moreover, it charges different subscription fees for the two channels, according to the quality level:

$$s_H^*(\overline{\theta}) > s_L^*(\underline{\theta})$$

According to Proposition 2, profits become:

$$\pi_{MP}^* = \frac{\overline{\theta}}{4} + \frac{(V - \delta a^* + \rho(a^*))(2\underline{\theta} + V - \delta a^* + \rho(a^*))}{4\underline{\theta}} - 2K \tag{25}$$

In the uniform case, equilibrium values for advertising, subscription fees, audience and profits are respectively:

$$a_H^* = a_L^* = a^* = \frac{1-\delta}{2}$$

$$s_H^* = \frac{\overline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2}$$
$$s_L^* = \frac{\underline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2}$$

$$B_H^* = \frac{1}{2}$$
$$B_L^* = \frac{1}{2} - \frac{\underline{\theta} - V - (\frac{1-\delta}{2})^2}{2\underline{\theta}}$$

$$\pi_{MP}^* = \frac{\overline{\theta}}{4} + \frac{(V + (\frac{1-\delta}{2})^2)(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)}{4\underline{\theta}} - 2K$$

2.3 Viewers' Surplus

We turn now to the welfare implications. Let us start by considering the general formulation of the viewers' surplus:

$$\begin{aligned} SC_{MP} &= \int_0^{\beta_{0L}} (u_0)\, d\beta + \int_{\beta_{0L}}^{\beta_{LH}} (u_L)\, d\beta + \int_{\beta_{LH}}^{1} (u_H)\, d\beta \\ &= \tfrac{1}{2}\beta_{LH}^2\theta_L + \beta_{LH}\left(V - \delta a_L - s_L\right) - \tfrac{1}{2}\beta_{0L}^2\theta_L - \beta_{0L}\left(V - \delta a_L - s_L\right) \\ &\quad + \tfrac{1}{2}\left(1 - \beta_{LH}^2\right)\theta_H + \left(1 - \beta_{LH}\right)\left(V - s_H - \delta a_H\right) \end{aligned} \tag{26}$$

By substituting equilibrium values, we get

$$SC^*_{MP} = \frac{\overline{\theta}}{8} + \frac{(2\underline{\theta} + V + \rho(a^*) - \delta a^*)(V + \rho(a^*) - \delta a^*)}{8\underline{\theta}} \tag{27}$$

In the uniform case, provided that $a^* = \left(\frac{1-\delta}{2}\right)$, equation (27) rewrites as follows:

$$SC_{MP} = \frac{\overline{\theta}}{8} + \frac{(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)(V + (\frac{1-\delta}{2})^2)}{8\underline{\theta}}$$

which helps in assessing the effects of the nuisance parameter δ and of the technological range $\Theta = (\underline{\theta}, \overline{\theta})$. The disutility parameter affects the consumers' surplus in two ways. First, an increase in δ has a direct negative impact on the individual utility for a given advertising intensity. Second, there is an indirect impact through advertising. Indeed, at equilibrium, an increase in δ reduces the advertising intensity. In turn, the effect of lower advertising is twofold: the advertising cost per viewer $a^*\delta$ drops back and the advertising revenues per viewers $\rho(a)$ are reduced. The latter effect induces higher subscription fees due to profit-neutrality. Both the direct effect and the indirect one on subscription fees prevail, inducing a negative impact on the surplus:

$$\frac{\partial SC^*_{MP}}{\partial \delta} = \frac{1}{32\underline{\theta}}\left(\delta - 1\right)\left(\delta^2 - 2\delta + 4\underline{\theta} + 4V + 1\right) \leq 0$$

For $\delta < 1$ the above effect is strictly negative, while for $\delta > 1$ the effect is null due to the fact that the platform does not broadcast advertising in any channel.

$$\frac{\partial SC^*_{MP}}{\partial \overline{\theta}} = \frac{1}{8} > 0$$

$$\frac{\partial SC^*_{MP}}{\partial \underline{\theta}} = -\frac{1}{128\underline{\theta}^2}\left(\delta^2 - 2\delta + 4V + 1\right)^2 < 0$$

The above derivatives explain the positive effect of enlarging the technological range. Consumers benefit by widening differentiation between the two channels.

3 Multi-Channel vs Single-Channel Broadcaster

In order to asses the welfare analysis it should be relevant to compare our previous insights with case of a single-channel monopoly broadcaster. The equilibrium solution for the single-channel monopoly is along the line of the previous subsections. As the structure of the analysis does not vary, the mathematical analysis of this case can be found in the Appendix 7.2.[14] The results provide an equal ground for comparing the multi-channel case to the single-channel case. The results for the single-channel case are summarized in the following Proposition:

Proposition 3 *A single-channel monopoly platform which maximizes profits in an uncovered market, shows the following equilibrium levels of advertising, subscription fee and audience share:*

$$a_M^* = \frac{1-\delta}{2}$$

$$s_M^* = \frac{V + \theta_M^* - \rho(a_M^*) - \delta a_M^*}{2}$$

$$B_M^* = \frac{V + \theta_M^* + \rho(a^*) - \delta a^*}{2\theta_M^*}$$

Moreover, regarding quality, two possible equilibrium configurations emerge, depending on the technological range

- $\theta_M^* = \overline{\theta}$ *if* $\Theta_{RL} = \left[\underline{\theta}, \overline{\theta}\right]$ *with* $\underline{\theta} = \rho(a) - \delta a$
- $\theta_M^* = \underline{\theta}$ *if* $\Theta_{RH} = \left[\underline{\theta}, \overline{\theta}\right]$ *with* $\underline{\theta} > 0$ *and* $\overline{\theta} = \rho(a) - \delta a$

Proof. See Appendix 7.2 ■

We proceed by comparing viewers' surplus, subscription fees and audience shares in the multi-channel case and the single one.

Proposition 4 *In the multi-channel monopoly case, viewers' surplus is larger than in the single-channel case, independently of the technological range of quality.*

[14] The results for the single-channel case rely on our previous paper Battaggion and Drufuca (2014).

Proof. We consider first the case of $\Theta_{RL} = \left[\underline{\theta}, \overline{\theta}\right]$ with $\underline{\theta} = \rho(a) - \delta a$. We compare the multi-channel platform with the single-channel platform that provides the maximum quality. Viewers' surplus are respectively:

$$SC^*_{MP}(\underline{\theta}, \overline{\theta}) = \frac{\overline{\theta}}{8} + \frac{(2\underline{\theta} + V - \delta a^* + \rho(a^*))(V - \delta a^* + \rho(a^*))}{8\underline{\theta}}$$
$$SC^*_{M}(\overline{\theta}) = \frac{1}{8\overline{\theta}} \left(V + \rho(a^*) - \delta a^* + \overline{\theta}\right)^2$$

If $\overline{\theta} > \underline{\theta}$:

$$SC^*_{MP}(\underline{\theta}, \overline{\theta}) - SC^*_{M}(\overline{\theta}) = \frac{(\overline{\theta} - \underline{\theta})(V - \delta a^* + \rho(a^*))^2}{8\overline{\theta}\underline{\theta}} > 0$$

Analogously, in the case $\Theta_{RH} = \left[\underline{\theta}, \overline{\theta}\right]$ with $\underline{\theta} > 0$ and $\overline{\theta} = \rho(a) - \delta a$, we obtain the same result:

$$SC^*_{MP}(\underline{\theta}, \overline{\theta}) - SC^*_{M}(\underline{\theta}) = \frac{\overline{\theta} - \underline{\theta}}{8} > 0$$

■

According to Proposition 4, a multi-channel monopoly is welfare improving with respect to viewers' concerns relative to a single-channel monopoly. This finding is valid independently of the technological range of quality, i.e., independently of $\overline{\theta}$ and $\underline{\theta}$. Initially, it seems that viewers benefit from the presence of multiple channels of different quality. To discern the driving forces behind this result, we compare the equilibrium audiences and subscription fees. We make this comparison for two cases: the single-channel monopoly choosing $\overline{\theta}$ or the single-channel monopoly choosing $\underline{\theta}$.

Table 1: COMPARISON AMONG REGIMES

	Case Θ_{RL}	Case Θ_{RH}
Viewers' Surplus	$SC^*_{MP} > SC^*_M$	$SC^*_{MP} > SC^*_M$
Channels' Quality Levels	$\theta^*_H = \theta^*_M$ $\theta^*_L < \theta^*_M$	$\theta^*_H > \theta^*_M$ $\theta^*_L = \theta^*_M$
Viewers' Fees	$s^*_M = s^*_H$ $s^*_M > s^*_L$	$s^*_M < s^*_H$ $s^*_M = s^*_L$
Viewers' Market Shares	$B^*_{MP}(\underline{\theta}, \bar{\theta}) > B^*_M(\overline{\theta})$	$B^*_{MP}(\underline{\theta}, \bar{\theta}) = B^*_M(\underline{\theta})$
Advertisers' Market Shares	$a^*_H = a^*_L = a^*_M$	$a^*_H = a^*_L = a^*_M$

Note: In this table, we compare equilibrium values of the multichannel monopoly broadcaster and the single-channel one. The case with the single-channel choosing maximum quality (Case Θ_{RL}) is
shown in the first column, the case with minimum quality (Case Θ_{RH}) in the second column.

In the first case, we disentangle two effects: one pertains to the subscription fees and the other effect is related to the audience's share.[15] The multi-channel broadcaster serves a larger market share of viewers than the single-channel monopoly. Moreover, it charges a lower price for low-quality channels. Hence, in this case the welfare-improving effect is driven by prices and market shares. Similarly, we compare the subscription fees and the audience's share for the second case: we can state that viewers benefit from the possibility of a multi-channel choice with a high-quality option. However, there is no positive effect on the fees and shares. To highlight our findings, we illustrate our results in the case of a uniform distribution of advertisers, as summarized in the following Remark.

Remark 5 *We consider the case of a uniform distribution of advertisers. We show that viewers' surplus is higher if they are served by a multi-channel monopoly compared to a single-channel one. This result holds independently of the technological range of quality; that is, either if the single-channel chooses the minimum quality*

[15]Provided that $V + \rho(a^*) - \delta a^* > 0$.

(Θ_{RL}) or it chooses the maximum quality (Θ_{RH}). For what concern prices and audience's shares, we obtain the following equilibrium values (see Table 2), which confirm our previous insights on the different effects driving our results on surplus.

Table 2: EQUILIBRIUM VALUES (Uniform Case)

	Case Θ_{RL}	Case Θ_{RH}
Channels' Quality Levels	$\theta^*_H=\overline{\theta}$	$\theta^*_H=\overline{\theta}=\frac{(1-\delta)^2}{4}$
	$\theta^*_L=\underline{\theta}=\frac{(1-\delta)^2}{4}$	$\theta^*_L=\underline{\theta}$
	$\theta^*_M=\overline{\theta}$	$\theta^*_M=\underline{\theta}$
Viewers' Fees	$s^*_H=\frac{\overline{\theta}+V-\frac{(1-\delta)(1+3\delta)}{4}}{2}$	$s^*_H=\frac{\overline{\theta}+V-\frac{(1-\delta)(1+3\delta)}{4}}{2}$
	$s^*_L=\frac{\underline{\theta}+V-\frac{(1-\delta)(1+3\delta)}{4}}{2}$	$s^*_L=\frac{\underline{\theta}+V-\frac{(1-\delta)(1+3\delta)}{4}}{2}$
	$s^*_M=\frac{V+\overline{\theta}-\left(\frac{1-\delta}{2}\right)\left(\frac{1+3\delta}{2}\right)}{2}$	$s^*_M=\frac{V+\underline{\theta}-\frac{(1-\delta)(1+3\delta)}{4}}{2}$
Viewers' Market Shares	$B^*_{MP}(\underline{\theta},\overline{\theta})=\frac{V+\underline{\theta}+\frac{(1-\delta)^2}{4}}{2\underline{\theta}}$	$B^*_{MP}(\underline{\theta},\overline{\theta})=\frac{V+\underline{\theta}+\frac{(1-\delta)^2}{4}}{2\underline{\theta}}$
	$B^*_M(\overline{\theta})=\frac{V+\overline{\theta}+\frac{(1-\delta)^2}{4}}{2\overline{\theta}}$	$B^*_M(\underline{\theta})=\frac{V+\underline{\theta}+\frac{(1-\delta)^2}{4}}{2\underline{\theta}}$
Advertisers' Market Shares	$a^*_H=a^*_L=a^*_M=\frac{1-\delta}{2}$	$a^*_H=a^*_L=a^*_M=\frac{1-\delta}{2}$

Proof. For what concerns consumers' surplus, if $\Theta_{RL}=\left[\underline{\theta},\overline{\theta}\right]$ with $\underline{\theta}=\rho(a-\delta a)$, then:

$$SC^*_{MP}(\underline{\theta},\overline{\theta})=\frac{\overline{\theta}}{8}+\frac{(2\underline{\theta}+V+(\frac{1-\delta}{2})^2)(V+(\frac{1-\delta}{2})^2)}{8\underline{\theta}}$$

$$SC^*_M(\overline{\theta})=\frac{1}{8\overline{\theta}}\left(V+(\frac{1-\delta}{2})^2+\overline{\theta}\right)^2$$

Then if if $\overline{\theta}>\underline{\theta}$:

$$SC^*_{MP}(\underline{\theta},\overline{\theta})-SC^*_M(\overline{\theta})=\frac{(\overline{\theta}-\underline{\theta})(V+(\dfrac{1-\delta}{2})^2)^2}{8\overline{\theta}\underline{\theta}}>0$$

If $\Theta_{RH} = \left[\underline{\theta}, \overline{\theta}\right]$ with $\underline{\theta} > 0$ and $\overline{\theta} = \rho(a - \delta a)$:

$$SC^{*}_{MP}(\underline{\theta}, \overline{\theta}) = \frac{\overline{\theta}}{8} + \frac{(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)(V + (\frac{1-\delta}{2})^2)}{8\underline{\theta}}$$
$$SC^{*}_{M}(\underline{\theta}) = \frac{1}{8\underline{\theta}}\left(V + (\frac{1-\delta}{2})^2 + \underline{\theta}\right)^2$$

Then if if $\overline{\theta} > \underline{\theta}$:

$$SC^{*}_{MP}(\underline{\theta}, \overline{\theta}) - SC^{*}_{M}(\underline{\theta}) = \frac{\overline{\theta} - \underline{\theta}}{8} > 0$$

■

In the comparison between the single and the multi-channel monopoly broadcasters, we show that consumers obtain an higher surplus in the second case. Our results show that the chance of choosing among channels of different qualities is always beneficial for the viewers.

4 Competition among Single-Channel Broadcasters

In this section, we modify our set-up by considering competition among broadcasters. We present the case of a duopoly market where two single-channel platforms compete for viewers and advertisers. Without loss of generality, we assume that whereas one broadcaster provides the low-quality channel, the other broadcaster provides the high-quality one, and we set$i = L, H$.[16] For the remaining material, we maintain the same assumptions as in the multi-channel set-up. Notice that in contrast to the ongoing literature on vertically differentiated media, we consider an ex-ante uncovered market. This framework further complicates the model from an analytical point of view, as multiple equilibria arise. To overcome this issue, we restrict the analysis to a local equilibrium: we identify a technological range of qualities that allow a local equilibrium of maximal differentiation to exist. We strongly believe it is worthwhile to maintain an uncovered set-up because it better notes the effects of competition and because it fits the features of a broadcasting market well.

4.1 Viewers' and Advertisers' Shares

We identify two marginal consumers: the one indifferent between not accessing to any platform and accessing the low quality platform

$$\beta_{0L} = \frac{s_L + \delta a_L - V}{\theta_L} \tag{28}$$

[16] We relax this ex-ante assumption when we look at the choice of quality (stage 1).

and the one indifferent between the low quality platform and the high quality one

$$\beta_{LH} = \frac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L} \tag{29}$$

Given our distribution of the willingness to pay for quality β, the trivial case in which the low-quality platform always faces zero demand in the price game is automatically ruled out. Hence, we consider an ex-ante market structure where both firms are active (meaning that the individuals' demands for both platforms H and L are positive).

We do not impose any further condition on the configuration: namely, we consider an ex-ante uncovered duopoly structure.

Hence, the high quality platform's share on viewers side is

$$B_H = (1 - \beta_{LH}) = \left(1 - \frac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L}\right) \tag{30}$$

whereas the low quality platform's share is

$$B_L = (\beta_{LH} - \beta_{0L}) = \left(\frac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L} - \frac{s_L + \delta a_L - V}{\theta_L}\right) \tag{31}$$

The intensities of advertising for the two platforms are respectively:

$$a_H = 1 - F\left(\frac{r_H}{B_H}\right) \tag{32}$$

$$a_L = 1 - F\left(\frac{r_L}{B_L}\right) \tag{33}$$

Having defined the shares of viewers and of advertisers, for given prices, we solve the game backwards, from stage three, as previously described for the monopoly. Therefore we obtain:

$$r_H\left(s_H, s_L, a_H, a_L, \theta_H, \theta_L\right) = F^{-1}(1 - a_H)\left(\tfrac{\theta_H - \theta_L - (s_H - s_L) - \delta(a_H - a_L)}{\theta_H - \theta_L}\right) \tag{34}$$

$$r_L\left(s_H, s_L, a_H, a_L, \theta_H, \theta_L\right) = F^{-1}(1 - a_L)\left(\tfrac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L} - \tfrac{s_L + \delta a_L - V}{\theta_L}\right) \tag{35}$$

4.2 Subscription Fees and Advertising Intensities

According to the previous assumptions, each platform maximizes profits subject to a positivity constraint on advertising:

$$\begin{cases} \max_{a_i, s_i} \Pi_i = B_i(s_i + \rho_i) - K \\ s.t. a_i \geq 0 \end{cases}$$

for $i = H, L$.

Proposition 6 *For each platform $i = H, L$, if the profit maximizing advertising level is positive, then it is constant and it is determined by*

$$\rho'(a_i) = \delta$$

Proof. We consider first the maximization problem of the L platform. Under the assumption that $\frac{\partial B_L}{\partial a_L} = \delta \frac{\partial B_L}{\partial s_L}$ and $\frac{\partial B_H}{\partial a_L} = \delta \frac{\partial B_H}{\partial s_L}$, first order conditions are:

$$\frac{\partial \pi_L}{\partial s_L} = \frac{\partial B_L}{\partial s_L}(s_L + \rho(a_L)) + B_L = 0 \tag{36}$$

$$\frac{\partial \pi_L}{\partial a_L} = \delta \frac{\partial B_L}{\partial s_L}(s_L + \rho(a_L)) + B_L(\rho'(a_L)) \leqq 0 \tag{37}$$

If $a_L > 0$, optimality conditions rewrite as follows:

$$\frac{\partial B_L}{\partial s_L}(s_L + \rho(a_L)) = -B_L$$

$$\delta \frac{\partial B_L}{\partial s_L}(s_L + \rho(a_L)) + B_L(\rho\prime(a_L)) = 0$$

Hence, by substitution we get:

$$\rho'(a_L) = \delta \tag{38}$$

The same applies to the high quality platform, giving us:

$$\rho'(a_H) = \delta \tag{39}$$

■

Proposition 6 states that for both platforms, a fixed advertising space is the best option. In particular, the equilibrium intensity of the advertising depends exclusively on the nuisance parameter δ. If the aversion to ads is "too high", then it is optimal to set the advertising intensity equal to zero. Hence, the optimal advertising intensity considers just the negative externality of advertisers on viewers, suggesting that the two platforms only compete for individuals. Indeed, platforms act as "bottlenecks" between advertisers and individuals by offering sole access to their respective set of individuals.

Moreover, by considering the case of a uniform distribution of advertisers, we point out that:

Remark 7 *We consider the case of a uniform distribution of advertisers. The strategic choices of advertising intensity of the two platform are the same and depend just on the nuisance parameter* δ*:*

$$a_i^* = \frac{1-\delta}{2} \quad \text{for } i = H, L$$

if $\delta < 1$*. Otherwise, is zero.*

We can now compute the subscription fees, the advertising prices and the audience shares of the two platforms.

Proposition 8 *Platform* H *and platform* L *set the following equilibrium values for subscription fees, audience shares and advertising prices:*

$$s_H^* = \frac{(V - a^*\delta + 2\theta_H)(\theta_H - \theta_L) - 3\rho(a^*)\theta_H}{4\theta_H - \theta_L}$$
$$s_L^* = \frac{(2(V - a^*\delta) + \theta_L))(\theta_H - \theta_L) - 2\rho(a^*)\theta_H - \rho(a^*)\theta_L}{4\theta_H - \theta_L}$$

$$B_H^* = \frac{2\theta_H + V + \rho(a^*) - a^*\delta}{4\theta_H - \theta_L}$$
$$B_L^* = \frac{2\theta_H}{\theta_L}\frac{\frac{1}{2}\theta_L + V + \rho(a^*) - a^*\delta}{4\theta_H - \theta_L}$$

$$r_H^* = \frac{\rho(a^*)}{a^*(4\theta_H - \theta_L)}(V + 2\theta_H + \rho(a^*) - a^*\delta)$$
$$r_L^* = \frac{2\rho(a^*)}{a^*(4\theta_H - \theta_L)}(V + \frac{1}{2}\theta_L + \rho(a^*) - a^*\delta)$$

Proof. Given the results of Proposition 6, we compute equilibrium subscription fees for the two platforms from the second FOCs:

$$\frac{\partial B_H}{\partial s_H}(s_H+\rho(a_H))+B_H = \\ (-\frac{1}{\theta_H-\theta_L})(s_H+\rho(a_H))+\left(1-\frac{(s_H-s_L)+\delta(a_H-a_L)}{\theta_H-\theta_L}\right)=0 \tag{40}$$

$$\frac{\partial B_L}{\partial s_L}(s_L+\rho(a_L))+B_L = \\ (-\frac{1}{\theta_H-\theta_L}-\frac{1}{\theta_L})(s_L+\rho(a_L))+\left(\frac{(s_H-s_L)+\delta(a_H-a_L)}{\theta_H-\theta_L}-\frac{s_L+\delta a_L-V}{\theta_L}\right)=0 \tag{41}$$

Since at equilibrium the advertising intensity is the same, $a_i = a$ and $\rho(a_i) = \rho(a)$ for $i = H, L$:

$$s_H = \frac{\theta_H-\theta_L+s_L-\rho(a)}{2} \tag{42}$$

$$s_L = \frac{(V-a\delta)(\theta_H-\theta_L)-\rho(a)\theta_H+\theta_L s_H}{2\theta_H} \tag{43}$$

Then, if $\theta_H > \theta_L > 0$:

$$s_H^* = \frac{(V-a^*\delta+2\theta_H)(\theta_H-\theta_L)-3\rho(a^*)\theta_H}{4\theta_H-\theta_L} \tag{44}$$

$$s_L^* = \frac{(2(V-a^*\delta)+\theta_L))(\theta_H-\theta_L)-2\rho(a^*)\theta_H-\rho(a^*)\theta_L}{(4\theta_H-\theta_L)} \tag{45}$$

Shares become:

$$B_H^* = \frac{2\theta_H+V+\rho(a^*)-a^*\delta}{4\theta_H-\theta_L} \tag{46}$$

$$B_L^* = \frac{2\theta_H}{\theta_L}\frac{\frac{1}{2}\theta_L+V+\rho(a^*)-a^*\delta}{4\theta_H-\theta_L} \tag{47}$$

■

Differently from the multi-channel monopoly case, all the equilibrium values for each broadcaster depend upon both its own quality and the competitor's one. There is a strategic interdependence between the two broadcasters resulting in prices and shares depending on quality differentiation.

We consider the case of a uniform distribution of advertisers, to get a sharper intuition of our results. Equilibrium solutions of stage 2 rewrites as follows. Subscription fees:

$$s_H^* = \frac{(V+2\theta_H)(\theta_H-\theta_L)-\frac{1}{4}(1-\delta)(3\theta_H+\delta(5\theta_H-2\theta_L))}{4\theta_H-\theta_L}$$

$$s_L^* = 2\frac{(V+\frac{1}{2}\theta_L)(\theta_H-\theta_L)-\frac{1}{4}(1-\delta)(\theta_H+\frac{1}{2}\theta_L+3\delta(\theta_H-\frac{1}{2}\theta_L))}{(4\theta_H-\theta_L)}$$

Viewers' Shares:

$$B_H^* = \frac{2\theta_H + V + (\frac{1-\delta}{2})^2}{4\theta_H - \theta_L}$$
$$B_L^* = \frac{2\theta_H}{\theta_L}\frac{\frac{1}{2}\theta_L + V + (\frac{1-\delta}{2})^2}{4\theta_H - \theta_L}$$

Advertising prices

$$r_H^* = \frac{(\frac{1+\delta}{2})}{(4\theta_H - \theta_L)}\left(V + 2\theta_H + (\frac{1-\delta}{2})^2\right))$$
$$r_L^* = \frac{2(\frac{1+\delta}{2})}{(4\theta_H - \theta_L)}(V + \frac{1}{2}\theta_L + (\frac{1-\delta}{2})^2)$$

4.3 Qualities

We can now solve the initial stage of the game, namely the quality choice.

At the first stage, platforms' profits are respectively for H and L:

$$\pi_H^* = \frac{(2\theta_H + V + \rho(a^*) - a^*\delta)^2 (\theta_H - \theta_L)}{(4\theta_H - \theta_L)^2} - K$$
$$\pi_L^* = \left(\frac{4\theta_H}{\theta_L}\frac{(\frac{1}{2}\theta_L + V + \rho(a^*) - a^*\delta)^2(\theta_H - \theta_L)}{(4\theta_H - \theta_L)^2}\right) - K$$

Then FOCs with respect to qualities are:

$$\frac{\partial \pi_H}{\partial \theta_H} = \frac{(2\theta_H+Z)[4(\theta_H-\theta_L)+2\theta_H+Z](4\theta_H-\theta_L)-8(2\theta_H+Z)^2(\theta_H-\theta_L)}{(4\theta_H-\theta_L)^3} = 0 \tag{48}$$

$$\frac{\partial \pi_L}{\partial \theta_L} = 4\theta_H(\frac{(\frac{1}{2}\theta_L+Z)(4\theta_H-\theta_L)(\theta_H-\theta_L)(\frac{1}{2}\theta_L-Z)-\theta_L(\frac{1}{2}\theta_L+Z)^2(2\theta_H+\theta_L)}{(4\theta_H-\theta_L)^3\theta_L^2}) = 0 \tag{49}$$

with $Z = V + \rho(a^*) - a^*\delta$

Conditions (48) and (49) implicitly define the best replies in quality for the two platforms. Unfortunately, the simultaneous solution does not give us a unique outcome. To make our duopoly comparable with the multi-channel case, we decide to focus on an equilibrium with maximal differentiation in quality. Therefore, we restrict the technological range of quality (Θ) to a narrower set $\Theta^d = (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\overline{\theta}$. For $\theta_H, \theta_L \in \Theta^d$, we obtain the following result:

Proposition 9 *In the restricted range of qualities* $\Theta^d = (\underline{\theta}, \bar{\theta})$ *with* $\underline{\theta} > \frac{4}{7}\bar{\theta}$ *there is a unique local equilibrium of maximal differentiation, where subscription fees and audience shares are:*

$$s_H^* = \frac{(V - a^*\delta + 2\bar{\theta})(\bar{\theta} - \underline{\theta}) - 3\rho(a^*)\bar{\theta}}{4\bar{\theta} - \underline{\theta}}$$
$$s_L^* = \frac{(2(V - a^*\delta) + \underline{\theta})(\bar{\theta} - \underline{\theta}) - 2\rho(a^*)\bar{\theta} - \rho(a^*)\underline{\theta}}{(4\bar{\theta} - \underline{\theta})}$$

$$B_H^* = \frac{2\bar{\theta} + V + \rho(a^*) - a^*\delta}{4\bar{\theta} - \underline{\theta}}$$
$$B_L^* = \frac{2\bar{\theta}}{\underline{\theta}} \frac{\frac{1}{2}\underline{\theta} + V + \rho(a^*) - a^*\delta}{4\bar{\theta} - \underline{\theta}}$$

Proof. If $4\theta_H < 7\theta_L$, then we show that:

$$\frac{\partial \pi_H}{\partial \theta_H} > 0 \tag{50}$$
$$\frac{\partial \pi_L}{\partial \theta_L} < 0 \tag{51}$$

Hence, for every $\theta \in \Theta^D = (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\bar{\theta}$, (50) and (51) hold. Therefore:

$$\theta_H^* = \bar{\theta}$$
$$\theta_L^* = \underline{\theta}$$

■

Notice that if the market were uncovered ex-ante, we would have obtained fixed audience shares.[17] Instead, we show that the audience shares depend on the quality distance between the two platforms. Hence, we can use our results on audience shares to highlight the effects of our assumption of an uncovered market. Furthermore, in our analysis the levels of quality are fixed at maximum differentiation. However, our setting also allows us to model an endogenous decision on quality levels, though such a model is not analytically tractable. Nevertheless, our results in the local equilibrium may give a suggestion on how these quality levels would change if the decisions regarding the quality were endogenous.

[17] See Weeds (2013).

4.4 Viewers' Surplus

We now address the welfare analysis from the point of view of the viewers. Viewers' surplus in the uncovered duopoly is:

$$SC_D(\underline{\theta},\bar{\theta}) = \int_0^{\beta_{0L}} (u_0)\,d\beta + \int_{\beta_{0L}}^{\beta_{LH}} (u_L)\,d\beta + \int_{\beta_{LH}}^{1} (u_H)\,d\beta \tag{52}$$

At the local equilibrium, we obtain:

$$SC_D^*(\underline{\theta},\bar{\theta}) = \frac{1}{2}\frac{\bar{\theta}}{\underline{\theta}\left(4\bar{\theta}-\underline{\theta}\right)^2}\left(\left(4\bar{\theta}+5\underline{\theta}\right)\left(\bar{\theta}\underline{\theta}+Z^2\right)+2\underline{\theta}\left(8\bar{\theta}+\underline{\theta}\right)Z\right) \tag{53}$$

with $Z = V + \rho(a^*) - a^*\delta$.

5 The Welfare Effects of Competition

Viewers' surplus is an important element to be considered when we analyze the effect of potential competition. In this perspective we first compare our duopoly with the multi-channel monopoly case described in the first section. In this comparison we pay particular attention to the difference between viewers' surpluses and we also consider how prices and audiences change according to the degree of competition.

Proposition 10 *If both the duopoly and the multi-channel monopoly configuration show a situation of maximum differentiation, viewers are better off with more competition (duopoly). That is:*

$$SC_D^*(\underline{\theta},\bar{\theta}) - SC_{MP}^*(\underline{\theta},\bar{\theta}) > 0$$

Proof. Recall equilibrium viewers' surplus in duopoly (ex-ante uncovered) with maximal differentiation (with $\Theta^d = (\underline{\theta},\bar{\theta})$ such that $\underline{\theta} > \frac{4}{7}\bar{\theta}$) from equation (53):

$$SC_D^*(\underline{\theta},\bar{\theta}) = \frac{1}{2}\frac{\bar{\theta}}{\underline{\theta}\left(4\bar{\theta}-\underline{\theta}\right)^2}\left(\left(4\bar{\theta}+5\underline{\theta}\right)\left(\bar{\theta}\underline{\theta}+Z^2\right)+2\underline{\theta}\left(8\bar{\theta}+\underline{\theta}\right)Z\right)$$

with $Z = V + \rho(a^*) - a^*\delta$, and equilibrium viewers' surplus in the multichannel monopoly from equation (27):

$$SC_{MP}^*(\underline{\theta},\bar{\theta}) = \frac{1}{8\underline{\theta}}(\underline{\theta}\bar{\theta} + (V+\rho(a^*)-a^*\delta)^2 + 2\underline{\theta}V + \rho(a^*) - a^*\delta)$$

If we compare them we get:

$$
\begin{aligned}
& SC_D^*(\underline{\theta},\bar{\theta}) - SC_{MP}^*(\underline{\theta},\bar{\theta}) \\
&= \frac{1}{8\left(\underline{\theta}-4\overline{\theta}\right)^2}\Big((28\overline{\theta}-\underline{\theta})\left(Z^2+\overline{\theta}\underline{\theta}\right)+2Z(16\overline{\theta}^2-\underline{\theta}^2)+24\overline{\theta}\underline{\theta}Z\Big)
\end{aligned}
$$

with $Z = V + \rho(a^*) - a^*\delta$. The above expression is for sure positive, provided that $\rho(a^*) - a^*\delta > 0$. Notice that this is the case if we consider a uniform distribution of advertisers. Namely, in the uniform case we have $\rho(a^*) - a^*\delta = \left(\frac{1-\delta}{2}\right)^2 > 0$, which give us:

$$SC_D^*(\underline{\theta},\bar{\theta}) - SC_{MP}^*(\underline{\theta},\bar{\theta}) > 0$$

■

As shown in Table 3, this result is driven by lower prices in the duopoly case, provided that $\rho(a^*) - a^*\delta > 0$ (as in the uniform case). In addition, there is a better market coverage by the two competing firms, as emerges from the shares' comparison.[18]

Table 3: DUOPOLY vs MULTI-CHANNEL MONOPOLY

Viewers' Surplus	$SC_D^* > SC_{MP}^*$
Channels' Quality Levels	$\theta_H^{D*} = \theta_H^{MP*}$ $\theta_L^{D*} = \theta_L^{MP*}$
Viewers' Fees	$s_H^{*D} < s_H^{*MP}$ $s_L^{*D} < s_L^{*MP}$
Viewers' Market Shares	$B_H^{*D} > B_H^{MP}$ $B_L^{*D} > B_L^{MP}$
Advertisers' Market Shares	$a_i^{D*} = a_i^{MP*}$

[18] We compare a duopoly of single-channel broadcasters with a multi-channel monopoly broadcaster. We concentrate on a local equilibrium where both market configurations exhibit maximal differentiation in quality. Hence, we must impose some restrictions on the tecnological range Θ, namely $\Theta^d = (\underline{\theta},\bar{\theta})$ such that $\underline{\theta} > \frac{4}{7}\bar{\theta}$.

Finally we make a last comparison between our duopoly (uncovered) and an ex-ante covered duopoly. If we consider the restricted range $\Theta^d = (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\overline{\theta}$, both configurations show maximal differentiation but different subscription fees and audience shares.

Proposition 11 *In the duopoly case, if both the ex-ante covered and the uncovered configuration lead to a situation of maximum differentiation, viewers are better off in the uncovered duopoly.*

Proof. If the market is ex-ante covered we just need one marginal individual β_{LH}. We compute viewers' surplus in the ex-ante covered case using equilibrium values:

$$\beta^*_{LHDcovered} = \frac{1}{3}$$

$$B^*_{HDcovered} = \frac{2}{3}$$
$$B^*_{LDcovered} = \frac{1}{3}$$

$$s^*_{HDcovered} = \frac{2}{3}\left(\overline{\theta} - \underline{\theta}\right) - \rho(a^*)$$
$$s^*_{LDcovered} = \frac{1}{3}\left(\overline{\theta} - \underline{\theta}\right) - \rho(a^*)$$

$$SC^*_{Dcovered}(\underline{\theta}, \bar{\theta}) = \frac{-2\overline{\theta} + 11\underline{\theta}}{18} + V + \rho(a^*) - a^*\delta \tag{54}$$

We compare this surplus with the one from equation (53), under the constraint $\Theta^d = (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\overline{\theta}$:

$$SC^*_D(\underline{\theta}, \bar{\theta}) - SC^*_{Dcovered}(\underline{\theta}, \bar{\theta}) > 0$$

provided that $\rho(a^*) - a^*\delta > 0$ (which is true in the uniform case).

Since we have considered the covered and the uncovered duopoly as two different market configurations, we have to check if this distinction still holds in equilibrium. In equilibrium, the duopoly broadcasting market is ex-post uncovered if: [19]

$$\begin{gathered}\beta_{0L} > 0 \\ \overline{\theta} - \underline{\theta} > \frac{2\overline{\theta} + \underline{\theta}}{\underline{\theta}}(V + \rho(a^*) - a^*\delta)\end{gathered} \tag{55}$$

[19] Notice that the market is covered ex-post if condition (55) is not satisfied. However, we omit this case from our analysis, since a comparison between two covered market structures is meaningless.

■

If this were the case, we can provide some more intuitions by looking at fees and audience shares.When quality differentiation is sufficiently high, the covered duopoly shows higher market shares but also higher subscription fees on both channels compared to the uncovered scenario. Higher prices explain the distance between covered and uncovered surpluses. Indeed, the possibility of catching extra viewers, as it happens in the uncovered market, disciplines the behavior of platform in duopoly making, consumers' surplus higher.

It is trivial to show that if:

$$\overline{\theta} - \underline{\theta} > \underbrace{\frac{3}{2}(V + \rho(a^*) - a^*\delta)}_{A} \tag{56}$$

then $B^*_{Hcovered} > B^*_H$ and $s^*_{icovered} > s^*_i$ for $i = H, L$.

Analogously if

$$\overline{\theta} - \underline{\theta} > \underbrace{6\frac{\overline{\theta}}{\underline{\theta}}(V + \rho(a^*) - a^*\delta)}_{B} \tag{57}$$

then $B^*_{Lcovered} > B^*_L$.

Recall from condition (55), that the market is uncovered (ex-post) if

$$\overline{\theta} - \underline{\theta} > \underbrace{\frac{2\overline{\theta} + \underline{\theta}}{\underline{\theta}}(V + \rho(a^*) - a^*\delta)}_{C}$$

It is possible to show that $A < C < B$. If $\overline{\theta} - \underline{\theta} > B$ then $s^*_{icovered} > s^*_i$ and $B^*_{icovered} > B^*_i$ for $i = H, L$. The covered duopoly has higher prices and audiences on both channels. If instead $C < \overline{\theta} - \underline{\theta} < B$ then $s^*_{icovered} > s^*_i$ and $B^*_{Hcovered} > B^*_H$ but $B^*_{Lcovered} < B^*_L$: prices are still higher but now the uncovered has a higher share on the low quality channel. Finally if $\overline{\theta} - \underline{\theta} < C$ the uncovered market becomes covered. However, as already mentioned, a comparison between two covered market structures is meaningless. Given that, when we considered a comparison between an uncovered ($\overline{\theta} - \underline{\theta} > C$) and a covered structure, it must be the case that the covered one privileges the high quality channels and sets higher subscription fees on both channels.

6 Conclusions

In this paper, we perform a welfare analysis in a setting of vertically differentiated two-sided broadcasters where competition prevails on one side of the market, namely, among the viewers. A broadcaster acts as a "bottleneck" between advertisers and viewers by offering sole access to its audience. We provide a full characterization of the equilibria with respect to the advertising, subscription fees, market shares and qualities for the monopoly with a single-channel platform, a multi-channel monopoly and a duopoly. In our welfare analysis, we focus on the viewers' side and calculate the consumers' surplus for each market structure.

It important to stress that in contrast to the ongoing literature on vertically differentiated media, we consider an ex-ante uncovered market for the duopoly case. This framework further complicates the model from an analytical point of view, as multiple equilibria arise. To overcome this issue, we identify a technological range of qualities that allows a local equilibrium of maximal differentiation to exist. Nevertheless, we strongly believe it is worthwhile to maintain an uncovered set-up to better note the effects of competition on the audience and the prices.

We remark that the equilibrium quality also depends on the cost structure used in this model. Under the assumption of fixed costs, the monopoly profit function is convex in quality. One might expect this shape to strictly depend on the assumption of K, i.e., a fixed cost of quality. However, in a single-side framework the standard model of vertical differentiation is solved with a quadratic cost of quality that induces concavity in the profit function. However, in a two-sided setting the issue of the concavity of the profit function is more complex. As expected, even in the two-sided approach, the linear cost of quality does not resolve the problem of convexity of the profit function. More surprisingly, even an increasing marginal cost of quality does not guarantee a well-shaped monopoly profit function. For instance, the quadratic cost of quality (see Weeds (2013)) does not make the monopoly profit function concave with respect to the quality without *ad hoc* assumptions on the derivatives. One possible resolution would be to have implicit quality cost functions (see Anderson (2007); however, this change should preclude us from providing a close solution to the model. Therefore, we introduce a simplest cost function and a technological range that bounds the levels of quality.

Our results show that the chance of choosing among channels of different qualities is always beneficial for the viewers. In the comparison between single-channel and multi-channel monopoly broadcasters, this result is mainly driven by two forces: the possibility of choosing among different qualities and the combination of greater audience coverage and a pricing effect.

In addition, we prove that competition is beneficial for the audience. The audience surplus is larger in the duopoly configuration than in the monopoly setting when both circumstances provide high-quality and low-quality channels. Indeed, on both types of channels the subscription fees are lower and the shares of viewers are larger. This result suggests that the ownership in broadcasting markets matters. In this respect, our model supports the existing regulation practice of setting limits on the ownership of TV channels to induce a more fragmented market structure.

Finally, we highlight the comparison between a covered and uncovered duopoly. In the case of an uncovered market, a chance of catching extra viewers disciplines the platforms' behavior; in a duopoly, this discipline renders consumers' surpluses higher. From the policy makers' point of view, this result is crucial in the broadcasting sector, where the convergence between television and the internet continuously opens up new market segments.

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7 Appendix

7.1 Multi-product Monopoly with Different Costs of Quality (Quality Stage)

Linear Costs [20]

Profits at stage 1 are:

$$\pi_{MP} = \frac{\theta_H}{4} + \frac{(V + \rho(a^*) - \delta a^*)(2\theta_L + V + \rho(a^*) - \delta a^*)}{4\theta_L} - \gamma\theta_H - \gamma\theta_L \tag{58}$$

Looking at first order conditions we get:

$$\frac{\partial \pi_{MP}}{\partial \theta_H} = \frac{1}{4} - \gamma = 0 \tag{59}$$

$$\frac{\partial \pi_{MP}}{\partial \theta_L} = -\frac{1}{4\theta_L^2}\left(V + \rho(a^*) - \delta a^*\right)^2 - \gamma < 0 \tag{60}$$

[20] Both linear and quadratic costs are assumed to be separable. See Section 2.

Optimal qualities are:

$$\theta_H^* = \overline{\theta} \text{ if } \gamma < \frac{1}{4} \tag{61}$$

$$\theta_H^* = \underline{\theta} \text{ if } \gamma > \frac{1}{4} \tag{62}$$

$$\theta_L^* = \underline{\theta} \tag{63}$$

The degree of differentiation depends on the cost parameter γ.

- If $\gamma < \frac{1}{4}$ the platform chooses to maximally differentiate the two channels.
- If $\gamma > \frac{1}{4}$ the platform chooses to duplicate the minimum quality

  In the first case profits become:

$$\pi_{MP}^* = \frac{\overline{\theta}}{4} + \frac{(V - \delta a^* + \rho(a^*))(2\underline{\theta} + V - \delta a^* + \rho(a^*))}{4\underline{\theta}} - \gamma(\overline{\theta} + \underline{\theta}) \tag{64}$$

In the uniform case equilibrium values are:

$$a_H^* = a_L^* = a^* = \frac{1-\delta}{2} \tag{65}$$

$$s_H^* = \frac{\overline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2} \tag{66}$$

$$s_L^* = \frac{\underline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2} \tag{67}$$

$$B_H^* = \frac{1}{2} \tag{68}$$

$$B_L^* = \frac{1}{2} - \frac{\underline{\theta} - V - (\frac{1-\delta}{2})^2}{2\underline{\theta}} \tag{69}$$

$$\pi_{MP}^* = \frac{\overline{\theta}}{4} + \frac{(V + (\frac{1-\delta}{2})^2)(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)}{4\underline{\theta}} - \gamma(\overline{\theta} + \underline{\theta}) \tag{70}$$

Quadratic Costs

Profits at stage 1 are:

$$\pi_{MP} = \frac{\theta_H}{4} + \frac{(V + \rho(a^*) - \delta a^*)(2\theta_L + V + \rho(a^*) - \delta a^*)}{4\theta_L} - \frac{1}{2}\gamma\theta_H^2 - \frac{1}{2}\gamma\theta_L^2 \tag{71}$$

Looking at first order conditions we get

$$\frac{\partial \pi_{MP}}{\partial \theta_H} = \frac{1}{4} - \gamma\theta_H = 0 \tag{72}$$

$$\frac{\partial \pi_{MP}}{\partial \theta_L} = -\frac{1}{4\theta_L^2}\left(V + \rho(a^*) - \delta a^*\right)^2 - \gamma\theta_L < 0 \tag{73}$$

Optimal qualities are:

$$\theta_H^* = \frac{1}{4\gamma} \tag{74}$$

$$\theta_L^* = \underline{\theta} \tag{75}$$

The degree of differentiation depends on the dimension of the technological constraint with respect to the cost parameter γ.

- If $\underline{\theta} < \frac{1}{4\gamma}$ the platform chooses a quality above the minimum.
- If $\underline{\theta} < \frac{1}{4\gamma} < \overline{\theta}$ the platform chooses a quality above the minimum but below the maximum.
- If $\overline{\theta} < \frac{1}{4\gamma}$ the platform chooses to reach the upper bound of the range $\overline{\theta}$.

Hence,with $\underline{\theta} < \overline{\theta} < \frac{1}{4\gamma}$, we get a result of maximal differentiation and profits become:

$$\pi_{MP}^* = \frac{\overline{\theta}}{4} + \frac{(V - \delta a^* + \rho(a^*))(2\underline{\theta} + V - \delta a^* + \rho(a^*))}{4\underline{\theta}} - \frac{1}{2}\gamma(\overline{\theta}^2 + \underline{\theta}^2) \tag{76}$$

In the uniform case equilibrium values are:

$$a_H^* = a_L^* = a^* = \frac{1-\delta}{2} \tag{77}$$

$$s_H^* = \frac{\overline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2} \tag{78}$$

$$s_L^* = \frac{\underline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2} \tag{79}$$

$$B_H^* = \frac{1}{2} \tag{80}$$

$$B_L^* = \frac{1}{2} - \frac{\underline{\theta} - V - (\frac{1-\delta}{2})^2}{2\underline{\theta}} \tag{81}$$

$$\pi_{MP}^* = \frac{\overline{\theta}}{4} + \frac{(V + (\frac{1-\delta}{2})^2)(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)}{4\underline{\theta}} - \frac{1}{2}\gamma(\overline{\theta}^2 + \underline{\theta}^2) \tag{82}$$

7.2 Monopoly (Single-Product)

7.2.1 Monopoly (Single Product): Viewers' and Advertisers' Shares

By considering the individual indifferent between accessing the monopoly platform or not accessing at all, we obtain the demand function by viewers/readers.[21]

$$\beta_{0M} = \frac{s_M + \delta a_M - V}{\theta_M} \tag{83}$$

Since individuals are uniformly distributed between 0 and 1, the demand for the monopoly platform is simply given by the fraction of population with a taste for quality greater than β_{0M} :

$$B_M = (1 - \beta_{0M}) = \left(\frac{V + \theta_M - s_M - \delta a_M}{\theta_M}\right) \tag{84}$$

The amount of advertising for the platform becomes:

$$a_M = 1 - F\left(\frac{r_M}{B_M}\right) \tag{85}$$

Having defined the demand function of viewers and advertisers, for given prices r_M and s_M, we solve the game backwards, from stage three. Therefore by simultaneously solving the equations (84) and (85) we get:

$$r_M\left(s_M, a_M, \theta_M\right) = F^{-1}(1 - a_M)(\frac{V + \theta_M - s_M - \delta a_M}{\theta_M}) \tag{86}$$

This equation describes how the advertising charge reacts to changes in subscribers' price, advertising and quality.

[21]This section summarizes the results for a single-channel monopoly case and it builds on the model of Battaggion and Drufuca (2014).

We present results either for a monopoly choosing the mimimum quality and for a monopoly choosing the maximum quality.

7.2.2 Monopoly (Single Product): Subscription Fee and Advertising Intensity

According to the above assumptions, the platform maximizes profits, subject to a positivity constraint on advertising level.

$$\begin{cases} \max_{a_H, s_H} \Pi_M = B_M(s_M + \rho_M) - K \\ s.t. a_M \geq 0 \end{cases} \tag{87}$$

First order conditions with respect to the advertising spaces a_M and subscription fee s_M are respectively:

$$\frac{\partial \Pi_M}{\partial a_M} = \frac{\partial B_M}{\partial a_M} s_M + r_M + a_M \frac{\partial r_M}{\partial a_M} \leq 0 \tag{88}$$

and

$$\frac{\partial \Pi_M}{\partial s_M} = B_M + \frac{\partial B_M}{\partial s_M} s_M + a_M \frac{\partial r_M}{\partial s_M} = 0 \tag{89}$$

Then, according to the literature, we define the advertising revenues per viewer as:

$$\rho(a_i) = \frac{a_i r_i}{B_i} = \frac{a_i F^{-1}(1 - a_i) B_i}{B_i} = a_i F^{-1}(1 - a_i) \tag{90}$$

We assume $\rho(a_i)$ to be concave in the interval $a \in [0, 1]$. Given that $\rho(a_i) = 0$ for $a_i = 0$ and $a_i = 1$, the function is single-peaked.

Using the definition (90) for the monopoly platform we can rewrite optimality conditions, proving the following Proposition.

Proposition 12 *The optimal advertising level for the monopoly single-channel broadcaster is:*

$$\rho'(a_M) = \delta$$

Proof. Given (90) for the monopoly platform

$$\rho(a_M) = \frac{a_M r_M}{B_M} = \frac{a_M F^{-1}(1 - a_M) B_M}{B_M} = a_M F^{-1}(1 - a_M) \tag{91}$$

we have:

$$r_M = \frac{B_M \rho(a_M)}{a_M} \tag{92}$$

Therefore optimality conditions (88) and (89) rewrite into (93) and (94):

$$s_M \frac{\partial B_M}{\partial a_M} + r_M + a_M \left[\frac{\left(B_M \rho(a_M) + \frac{\partial B_M}{\partial a_M} \rho(a_M)\right) a_M - B_M \rho(a_M)}{a_M^2} \right] \leq 0 \tag{93}$$

$$B_M + s_M \frac{\partial B_M}{\partial s_H} + a_M \frac{\partial r_M}{\partial s_M} = 0 \tag{94}$$

By easy calculation, (93) and (94) become respectively:

$$\frac{\partial B_M}{\partial a_M} (s_M + \rho(a_M)) + B_M \rho(a_M) \leq 0 \tag{95}$$

$$\frac{\partial B_M}{\partial s_H} (s_M + \rho(a_M)) + B_M = 0 \tag{96}$$

Given that $\frac{\partial B_M}{\partial a_M} = -\frac{\delta}{\theta_M}$ and $\frac{\partial B_M}{\partial s_M} = -\frac{1}{\theta_M}$, we get:

$$\frac{\partial B_M}{\partial a_M} = \delta \frac{\partial B_M}{\partial s_M} \tag{97}$$

Therefore, plugging in (95) and (96), we get the following system:

$$\begin{cases} \delta \frac{\partial B_M}{\partial s_M} (s_M + \rho(a_M)) + B_M \rho'(a_M) \leq 0 \\ \frac{\partial B_M}{\partial s_H} (s_M + \rho(a_M)) + B_M = 0 \end{cases} \tag{98}$$

Finally, if $a_M > 0$ the above inequality is satisfied with equality. Therefore, given that $\rho(a_M)$ is single-peaked, a_M is uniquely determined by the following condition:

$$\rho'(a_M) = \delta.$$

■

We can now solve for the equilibrium values, as stated in the following proposition.

Proposition 13 *With $\rho(a_M)$ concave, we obtain the equilibrium price s_M^* and demand B_M^* as function of quality, revenues per viewer and advertising level.*

Proof. By plugging the expression for B_M in the optimality condition (96) we obtain:

$$s_M^* = \frac{V + \theta_M - \rho(a_M^*) - \delta a_M^*}{2} \tag{99}$$

Then,

$$B_M^* = \frac{V + \theta_M + \rho(a_M^*) - \delta a_M^*}{2\theta_M} \tag{100}$$

■

Proposition 13 shows the result of profit neutrality. In fact, an increase in revenues on the advertisers side are counterbalanced by a decrease on the subscription fees.

7.2.3 Monopoly (Single Product): Platform's quality

In order to solve the quality stage, we maximize the monopoly profit, $\Pi_M\left(s_M^*, a_M^*, r_M^*, \theta_M\right)$, with respect to the quality, θ_M. We obtain the following FOC, subject to $\theta_M \geq 0$:

$$\begin{gathered} \frac{\partial \Pi_M\left(s_M^*, a_M^*, r_M^*, \theta_M\right)}{\partial \theta_M} = \\ \frac{\left(V+\theta_M+\rho(a_M^*)-\delta a_M^*\right)\left(\theta_M-\rho(a_M^*)+\delta a_M^*\right)}{4\theta_M^2} = 0 \end{gathered} \tag{101}$$

Unfortunately, in this general framework we cannot calculate the equilibrium value of θ_M^*.

By calculating the second order conditions we show the convexity of the profit function:

$$\frac{\partial^2 \Pi_M}{\partial \theta_M^2} = \frac{\left(\rho\left(a_M^*\right)-\delta a_M^*\right)^2}{2\theta_M^3} \geq 0 \tag{102}$$

Given convexity, the monopoly platform will reach one of the boundaries, choosing $\underline{\theta}$ or $\bar{\theta}$. Hence we describe two possible local equilibria, each of them characterized by a specific configuration of the technological range.

Proposition 14 *In equilibrium, under the technological constraint* $\Theta_{RL} = \left[\underline{\theta}, \bar{\theta}\right]$ *with* $\underline{\theta} = \rho\left(a_M^*\right) - \delta a_M^*$ *, the monopoly platform chooses the maximum quality. Differently, under the technological constraint* $\Theta_{RH} = \left[\underline{\theta}, \bar{\theta}\right]$ *with* $\underline{\theta} > 0$ *and* $\bar{\theta} = \rho\left(a_M^*\right) - \delta a_M^*$ *, the monopoly platform chooses the maximum quality.*

Proof. In the first case we restrict ourselves on the increasing slope of the profit function. By comparing monopoly profit functions in $\underline{\theta}$ and $\bar{\theta}$, respectively:

$$\Pi_M^*\left(\underline{\theta}\right) = \frac{\left(\underline{\theta}+\rho\left(a_M^*\right)-\delta a_M^*\right)^2}{4\underline{\theta}} - K$$

$$\Pi_M^*\left(\bar{\theta}\right) = \frac{\left(\bar{\theta}+\rho\left(a_M^*\right)-\delta a_M^*\right)^2}{4\bar{\theta}} - K$$

we get:

$$\Pi_M^*\left(\bar{\theta}\right) - \Pi_M^*\left(\underline{\theta}\right) > 0$$

For $\theta \in \Theta_{RL}$ profit are convex and increasing in quality. Therefore to maximize profit the monopoly platform sets $\theta_M^* = \bar{\theta}$.

In the second case, we restrict ourselves on the decreasing slope of the profit function. By comparing monopoly profit functions in $\underline{\theta}$ and $\bar{\theta}$, respectively we get:

$$\Pi_M^*\left(\bar{\theta}\right) - \Pi_M^*\left(\underline{\theta}\right) < 0$$

For $\theta \in \Theta_{RH}$ profit are convex and decreasing in quality. Therefore to maximize profit the monopoly platform sets $\theta_M^* = \underline{\theta}$. ■

Considering the uniform case, we can suggest some interesting insights. By easy calculation, in the uniform case with $\rho\left(a_M\right) = a_M\left(1 - a_M\right)$, we obtain:

$$a_M^* = \frac{1-\delta}{2} \tag{103}$$

$$s_M^* = \frac{V + \theta_M - \delta a_M^* - \rho(a_M^*)}{2} = \frac{V + \theta_M - \left(\frac{1-\delta}{2}\right)\left(\frac{1+3\delta}{2}\right)}{2} \tag{104}$$

$$B_M^* = \frac{1}{2\theta_M}\left[V + \theta_M + \left(\frac{1-\delta}{2}\right)\left(\frac{1-\delta}{2}\right)\right] \tag{105}$$

According to the equilibrium solutions of stage 3 and stage 2, the profit function - in the uniform case - becomes:

$$\Pi_M^* = B_M^*(s_M^* + \rho_M^*) - K = \frac{1}{4\theta_M}(V + \theta_M + \left(\frac{1-\delta}{2}\right)^2)^2 - K \tag{106}$$

Given our result on quality, if we consider the case of Θ_{RL}, we obtain equilibrium values for subscription fees and viewers' demand:

$$s_M^* = \frac{V + \bar{\theta} - \left(\frac{1-\delta}{2}\right)\left(\frac{1+3\delta}{2}\right)}{2} \tag{107}$$

$$B_M^* = \frac{1}{2\bar{\theta}}\left[V + \bar{\theta} + \left(\frac{1-\delta}{2}\right)\left(\frac{1-\delta}{2}\right)\right] \tag{108}$$

$$\Pi_M^* = \frac{1}{4\bar{\theta}}(V + \bar{\theta} + \left(\frac{1-\delta}{2}\right)^2)^2 - K$$

For the case of technological range Θ_{RH} equilibrium results are unchanged but for quality.

7.2.4 Monopoly (Single Product): Viewers' surplus

Viewers' surplus is:

$$
\begin{aligned}
SC_M &= \int_0^{\beta_I} (u_0)\, d\beta + \int_{\beta_I}^{1} (u_M)\, d\beta \\
&= \frac{1}{2}\left(1-\beta_I^2\right)(\theta_M) + (1-\beta_I)(V - s_M - \delta a_M)
\end{aligned}
$$

Substituting equilibrium values for β_I , s_M , a_M and θ_M, we get:

$$
SC_M(\overline{\theta}) = \frac{1}{8\overline{\theta}}\left(V + \rho(a^*) - \delta a^* + \overline{\theta}\right)^2 \tag{109}
$$

if Θ_{RL} and

$$
SC_M(\overline{\theta}) = \frac{1}{8\underline{\theta}}\left(V + \rho(a^*) - \delta a^* + \underline{\theta}\right)^2 \tag{110}
$$

if $\Theta_{RH.}$

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