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Abstract

Entrants often need to make considerable sunk investments with highly uncertain returns. The option to exit if returns are low reduces investment risks and stimulates innovation. We examine the interaction between exit policy and up-front investment by entrants, finding an inverted U-shaped relationship between innovation and exit value. Consumer welfare is shaped by a trade-off between encouraging firms to stay in the market through higher exit barriers, and stimulating investment through a more permissive exit policy. As future returns become more uncertain, consumer welfare maximization requires lower exit barriers. These insights are applied to optimal merger policy and bankruptcy law.

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1 Introduction

Industrial economists have traditionally centred their research agenda on competition, innovation and market structure around the concept of market entry. Less emphasis has been devoted to industrial exit, even though barriers to exit have been debated in policy circles on both sides of the Atlantic for many years (e.g., Ezekiel, 1992; Frank, 1988). Market entry and exit are two sides of the same coin.[1] Both contribute to ensuring the benefits of competition and innovation. The threat of entry disciplines market power by incentivising incumbents to keep prices low and offer what consumers demand. Market exit, instead, is a crucial instrument to sanction unprofitable business ideas, thereby spurring the industry life cycle through creative destruction (Schumpeter, 1942).

While the policy debate to date has primarily focused on barriers to entry and their impact on competition, effective competition, and thus a well-grounded industrial policy, is also shaped by exit. Barriers to exit weaken the selection mechanism that relocates demand and resources across and within industries when market conditions change. They may also affect competition, innovation, and ultimately economic growth.

In this paper, we consider how the exit policy can stimulate or deter demand-enhancing (e.g., quality-improving) investments when investments are sunk and demand is uncertain. In our baseline framework, there are two firms. One is an incumbent with committed investments, while the other is a challenger, who must decide whether or not to undertake an investment before learning of its demand. Then, upon privately observing the demand realization, the challenger decides whether to leave the market and enjoy an exit value (which, as we explain below, is affected by the regulatory regime in place) or remain active and compete with the incumbent. Finally, firms in the marketplace compete for customers by setting quantities (Bayes-Cournot competition) based on committed investment levels.

This framework is inspired by the growing policy and strategic influence of many infrastructure sectors, such as the wireless communication sector, where entrants must sunk significant investments with long and uncertain payback periods. Risk aversion, capital market failures or regulatory uncertainty can mean that private investment decisions result in infrastructure provision below the socially optimal level.[2] In response to such concerns, investment in some infrastructure sectors is supported by explicit subsidies.[3] However, such interventions, which can influence exit costs, may be costly to consumers and governments, and can also limit the benefits of rivalry between firms to win customers.

Within the framework outlined above, we characterize the challenger's exit strategy and

[1] Indeed, "most guidelines link exit barriers to entry barriers, as exit costs can deter entry if firms can anticipate them before entering"(OECD, 2019).

[2] For instance, Gruber (2019) concludes that meeting the European Commission's aims in the Digital Agenda for Europe and the European Gigabit Society would require about €384 billion of investment in wireless and broadband technology between 2019 and 2025. However, such investments are made when there is uncertainty both about the scale of consumer demand for new technologies and about the proportion of value that will be captured by infrastructure providers. He then identifies an investment gap of around €254 billion.

[3] For instance, in the energy sector, network firms typically own regulated asset bases that receive guaranteed returns, while low-carbon generators often receive public support through top-up payments or guaranteed prices: see, e.g., Roques and Finon (2017).

study how the exit value affects the market outcome. We show that exit has a selection effect with strategic implications. The challenger chooses to stay in the market and thus competes with the incumbent only when it observes favourable demand states. Hence, when exit becomes more likely, as implied by a more attractive exit value, the incumbent is less aggressive, since it expects the challenger's output to be higher, and quantities are strategic substitutes.

Building on these insights, we then examine the challenger's incentive to invest, for given exit value. We find that the return of the investment is positive and determined by two intuitive forces. First, when the challenger invests, it exits less often compared to the case of no investment. Second, conditional on remaining in the market, the challenger has a larger market share when it invests since the investment spurs demand. More interestingly, the relationship between the challenger's exit value and its incentive to invest is non-monotone. In particular, we find that this relationship is inverted U-shaped — i.e., an increase in the exit value tends to stimulate investment when the exit value is not too large, reducing the incentive to invest when it is large enough. Hence, we find that the investment return is maximized for an intermediate exit value, which in turn is increasing with demand volatility, market size, and the degree of substitutability between the challenger's and the incumbent's products. When the exit value is large, further increases in that value reduce the investment return since they make exit more likely and the return on investment only materializes when the challenger remains in the market. In contrast, when the exit value is small, an increase in that value increases the set of circumstances (i.e., demand realizations) for which the investment pays off and results in a larger market share due to the selection effect discussed above.

We also consider the potential implications for consumer welfare. We first show that the investment benefits consumers and identify an under-investment problem. Next, we characterize the optimal exit policy that a regulator whose objective is to maximize consumer surplus would choose. This exercise hinges on the idea that regulators can influence the challenger's exit option by designing policies that, for example, impact its labour related exit costs (e.g., costs related to employees' contractual rights such as staff redundancy costs and insurance benefits), bankruptcy rules (e.g., filing and litigation fees), environmental regulations (e.g., remediation costs), etc. Our main finding is that, for intermediate values of the investment costs — i.e., when the exit value affects the challenger's incentives to invest[4] — the regulator always sets a positive exit value determined by the challenger's (binding) incentive compatibility constraint. That is, in this range of parameters, the regulator's first-order concern is always to solve the under-investment problem. The intuition is as follows: for given exit value, by increasing consumers' willingness to pay, the investment creates value and when the firm is indifferent between making or not the investment, this value is fully appropriated by consumers, who must therefore be better off with the investment than without it. The range of parameters in which this happens

[4]If the investment cost is sufficiently low, so that there is no under-investment, the regulator optimally sets the exit value at zero. The reason is simple: conditional on the investment taking place, consumers always benefit from rivalry, which is achieved minimizing the challenger's incentives to leave the industry. By contrast, when the investment cost is so high that the challenger never invests irrespective of its exit value, the regulator sets a positive exit value when products are sufficiently homogeneous and demand volatility is low. This is because the loss from monopoly becomes a second-order concern when the challenger's products are inferior, as it happens when it does not invest and horizontal differentiation is negligible.

expands with demand uncertainty.

We then examine instances in which the exit value depends on whether the challenger has invested or not. We develop two applications where this can happen (as, e.g., in Schary, 1991). First, we allow for the possibility of a merger to monopoly, in which the merger profitability depends on whether the investment takes place or not — i.e., the challenger's exit value coincides with the takeover price offered by the incumbent upon observing its investment decision. Second, we consider the case in which the challenger is able to recoup a fraction of its investment cost when it exits, which is determined by the prevailing bankruptcy law.

When the exit value is determined by the takeover price, the regulator faces the following trade-off. On the one hand, a restrictive merger policy, which prohibits the challenger's acquisition by the incumbent, strengthens competition in the product market, which benefits consumers since it induces lower prices and greater product variety. On the other hand, a lenient merger policy, which allows any merger to occur, fosters investment incentives, which in turn increases the products' quality and/or quantity available to consumers.[5] We show that, even though, absent horizontal differentiation between products, any acquisition in equilibrium is a *killer merger* (i.e., the incumbent always shuts down the acquired firm's product), a lenient merger policy maximizes consumer surplus in industries which require relatively more expensive investments. Interestingly, the regulator can improve over a fully lenient merger policy by adopting a transaction price-contingent policy.

Similarly, consumers benefit from efficient bankruptcy rules that minimize rent dissipation and enable the challenger to recoup a relatively large share of the ex-ante investment when it decides to quit, thereby enhancing its incentive to invest. In this light, our approach can help to understand the wide disparities in the pace of infrastructure investment between countries with similar income levels, due to differences in merger policy and bankruptcy law.[6]

Our results remain qualitatively unchanged when considering multiple incumbents: in this case, the optimal policy still requires a positive exit value to secure the investment by the challenger when the market is sufficiently concentrated or demand is sufficiently volatile. We also consider the possibility of leapfrogging — i.e., a technology such that, upon investment, the challenger's product features higher quality than the incumbent's one. In this case, consumers are more likely to benefit from a policy mandating a positive exit value the larger the impact of the investment on the challenger's quality. Our findings remain true qualitatively also when considering a technology in which the investment itself has an uncertain, rather than deterministic, impact on demand, and allowing for a continuum of investment levels.

The paper is organized as follows. After clarifying our contribution to several strands of existing literature, in Section 2 we describe the baseline two-firm model and analyse the effects of exit policy on investment and consumer welfare. The exit value is endogenised in Section 3.

[5] Of course, a lenient merger policy is effective in stimulating investments if and only if antitrust authorities can commit in advance to their policy, as otherwise they would have incentives to implement a strict merger policy after the challenger sunk its investment.

[6] For instance, GSMA (2020) expects 48% of mobile connections in North America to be 5G in 2025, 47% in Greater China, and 34% in Europe. Similarly, it expects almost $300 billion of mobile network operator investment in their networks in the US between 2020 and 2025, compared to around $170 billion in Europe.

In Section 4, we present some extensions and robustness checks of the main results. Section 5 concludes. Proofs of the main results are in the Appendix. Additional proofs are contained in the online Appendix.

1.1 Related literature

Our paper is related to several streams of the existing literature on exit in oligopoly games, innovation and merger policy.

Exit in oligopoly. The first rigorous analysis of exit in oligopoly games is due to Telser (1965) who studies, in a linear and symmetric Cournot model, how equilibrium outputs and price change when a group of firms is removed altogether from the market, an exit decision that he then rationalizes with a merger. Ghemawat and Nalebuff (1985) extend Telser's framework by examining the exit decisions of oligopolists with asymmetric market shares. They focus on how firms' relative size affects the order of exit and show that survivability is inversely related to size — i.e., smaller firms have, ceteris paribus, lower incentives to reduce capacity in declining industries than their larger competitors. Fudenberg and Tirole (1986) were the first to examine exit under incomplete information. In their framework, each firm is uncertain about its rival's costs. By assuming a positive probability that no exit will be required, they prove the existence of a unique equilibrium in which the most efficient firm outlasts its rival. In contrast to our model, these papers do not focus on investment incentives and treat the exit value as exogenous.

By combining irreversible investment under demand uncertainty with strategic interactions, our paper is also related to the literature on real options (e.g., Weeds, 2002) and timing games with private information (e.g., Hopenhayn and Squintani, 2011), which focuses on investment delay in oligopoly and preemption incentives.[7] In contrast to these papers, the dynamic elements of our model are only related to the timing of the investment. Yet, the innovative angle of our model is to make the exit value endogenous and determined, or at least influenced, by a regulator whose objective function is consumer surplus. Unlike the previous literature, our focus is on studying how the option to exit, partly determined by bankruptcy laws, the market for corporate control and merger policy, affects competition and consumer welfare.

Startup acquisitions. Our work is thus closer to the recent literature on 'killer acquisitions' and, more broadly, startup acquisitions.[8] Cunningham et al. (2021) show how pharmaceutical firms may have an incentive to merge in order to discontinue innovations and pre-empt competition. However, Letina et al. (2020) show that a restrictive acquisition policy reduces the variety of research approaches pursued by the firms and thereby the probability of discovering innovations, and also leads to strategic duplication of the entrant's innovation by the incumbent. Yet, Fumagalli et al. (2020) argue that merger policy need not be lenient towards acquisitions of

[7] Related contributions study entry games under entrant's private information: see, e.g., Jovanovic (1981) for an earlier work, and Bergemann and Valimaki (2002) and Kolb (2015) for continuous time models with learning.

[8] A less closely related strand of the literature examines the effects of mergers between market incumbents on the incentives to invest of the merged entity: see, e.g., Bourreau and Jullien (2018), Federico et al. (2017, 2018), Motta and Tarantino (2021), Denicolò and Polo (2018, 2021).

potential competitors to take advantage of their pro-competitive effects on project development, as the same purpose is achieved by a strict merger policy that pushes the incumbent towards the acquisition of potential competitors lacking the financial resources to develop their projects independently, or, equivalently, by blocking takeovers whose acquisition price is above a certain threshold. Also in our model the optimal merger policy turns out to be based on the proposed takeover price, though we find that in some cases imposing a floor rather than a cap on the takeover price is optimal to foster investment incentives.

Existing studies focusing on digital platform markets also call for a rather strict merger policy. Specifically, Katz (2021) shows that incumbents' acquisitions of emerging or potential competitors should be subject to heightened antitrust scrutiny when competition is *for* (rather than *in*) the market, and allowing these acquisitions has mixed effect on the potential entrants' incentives to innovate. Similarly, Kamepalli et al. (2020) argue that acquisitions of entrant firms by an incumbent can deter innovation and entry in the presence of strong network externalities and customers' switching costs.[9] Differently from our work, these papers do not consider how the optimal exit policy (and, therefore, merger control policy) is impacted by demand uncertainty.

Merger policy. The literature on startup acquisitions and, in turn, our paper, is related to recent works on optimal merger policy which accounts for industry dynamics aspects. These papers focus on the impact of merger policy on firms' entry and R&D investment decisions.[10] Mermelstein et al. (2020) analyze how antitrust policy can affect firms' investment behaviour where there are scale economies, identifying a trade-off between internal growth, which is supported by a restrictive merger policy, and external growth, which is supported by a permissive merger policy. In this setting, they find that the optimal antitrust policy for maximizing aggregate value is more restrictive than the optimal static policy. By contrast, Hollenbeck (2020), studying a dynamic oligopoly model where firms endogenously engage in investment, entry, exit and mergers, finds that the prospect of a buyout creates a powerful incentive for firms to preemptively enter the industry and invest to make themselves an attractive merger partner.[11]

All these papers consider finitely or infinitely repeated games, at the expense of analytical tractability, whereas other contributions resort to simpler models and analytically derive the equilibrium, as we do in this paper. Gilbert and Katz (2021) argue that policies that focus solely on a proposed merger's ex-post welfare effects can induce an entrant to choose an inefficient 'direction' for its pre-merger investment, either because doing so maximizes the profits of a merger that would be approved regardless of the direction of its efforts, or because the nature

[9] Relatedly, Bryan and Hovenkamp (2020), focusing on the startups that produce inputs for competing incumbents, find that startups who can choose what kind of technology to invent are biased toward those that improve the leader's technology rather than those that help the less efficient incumbent to catch up. On the effect of start-up acquisitions on the direction of innovation, see also Dijk et al. (2021) and Motta and Shelegia (2021).

[10] For an earlier work on a dynamic oligopoly model with firms' entry/exit, investment, and merger decisions, see Gowrisankaran (1999), who however does not explicitly investigate the optimal dynamic merger policy. This is instead the focus of Nocke and Whinston (2010), though in a model which abstracts from considering entry and innovation.

[11] A similar model (but with stochastically alternating moves and a finite horizon) is analysed and estimated with data from the Hard Disk Drive industry by Igami and Uetake (2020). Their counterfactual simulations suggest that the currently adopted rule-of-thumb policy, which stops mergers when three or fewer firms exist, constitutes an approximately optimal policy given the dynamic trade-offs at play.

of the approval process itself distorts incentives with respect to the direction of pre-merger innovation. By contrast, and more in line with our model and findings, Mason and Weeds (2013) analyze how the possibility of future merger raises the firms' expected value of entry: by facilitating exit in case of low profitability and thus raising the value of entry, a more lenient merger policy may prove optimal as it stimulates entry.[12] Unlike our paper, they do not consider demand uncertainty or asymmetric information. This is a crucial difference since we find that optimal merger policy is influenced by the demand volatility and the selection effect which plays a central role in our model is driven by asymmetric information.

Other contributions explicitly focus on the welfare effects of horizontal mergers and on the optimal merger policy under private information about firms' costs (Banal-Estanol, 2007) or merger-specific cost efficiencies (Amir et al., 2009; Hamada, 2012) — i.e., as in our paper, they consider a Bayesian-Cournot game. However, these papers focus on the static market power vs cost efficiencies trade-off, and overlook exit and innovation concerns.

2 The baseline model

Consider a market in which two rivals (each denoted by $i = 0, 1$) compete by setting quantities. Firm 1 is an incumbent already committed to invest and compete in the market, while firm 0 is an uncommitted competitor (the challenger), which could accept an exit value K (exogenous for the moment), which is common knowledge and, normalizing to zero fixed costs of staying in the market, is assumed to be positive without loss of generality.[13]

Following the literature on asymmetric information in oligopoly (see, e.g., Vives, 1999) we focus on a linear-quadratic framework, which typically allows to obtain closed-form solutions in Bayesian games.[14] Firm 1 faces an (inverse) demand function

$$p_1(x_1, x_0) \triangleq \max\{0, \mu - x_1 - bx_0\}, \tag{1}$$

while firm 0's (inverse) demand function is

$$p_0(x_0, x_1) \triangleq \max\{0, \mu I + \theta - x_0 - bx_1\}, \tag{2}$$

where θ is a zero-mean random intercept shifter, which is uniformly distributed over the support $[-\sigma, +\sigma]$ and, in the spirit of Fudenberg and Tirole (1986), it is firm 0's private information. This assumption is meant to capture in the simplest possible way instances in which a firm entering a market with a new product receives private signals on how consumers value this product. As in every war of attrition game, private information is key also in our setting since it creates a

[12] Other contributions dealing with an optimal merger policy which takes into account firms' incentives to enter, but abstract from firms' innovation efforts, include Jaunaux et al. (2017) and Caradonna et al. (2020).

[13] In Section 4.1 we extend the model considering N symmetric incumbents. While considering multiple challengers would considerably complicate the analysis, owned to coordination problems at the investment stage, we expect our main qualitative results to be robust also in that direction.

[14] While the main features of our equilibrium characterization remain true with a more general demand function, assuming linear demand greatly simplifies the consumer surplus and welfare analysis.

selection effect that enables the challenger to signal its strength to the incumbent through its exit decision. The variable $I \in \{0, 1\}$ represents a binary investment decision, taking value 1 if the challenger invests and 0 otherwise. Investment shifts the demand function upward — i.e., it increases consumers' willingness to pay for firm 0's product — at a cost of $\psi I \geq 0$.[15]

The parameter $\mu > 0$ represents a common demand intercept — i.e., if firm 0 invests, its inverse demand function is ex-ante identical to that faced by its rival. Hence, under this specification, assuming that the incumbent does not invest allows us examine an unbiased competition model where the two rivals are ex-ante identical provided that $I = 1$, while the incumbent has an ex-ante competitive advantage if $I = 0$.[16] Finally, $b \in (0, 1]$ is a measure of product substitutability. Thus, the products of firms 0 and 1 are both horizontally and vertically differentiated. While the degree of horizontal differentiation is exogenous (measured by b), vertical differentiation is endogenous (as it depends on I) and uncertain (as it depends on the realization of θ).

The timing of the game, summarized in Figure 1, is as follows.

$t = 1$ Firm 0 decides whether to undertake the investment — i.e., chooses $I \in \{0, 1\}$.

$t = 2$ The demand state θ realizes and only firm 0 observes it. Conditional on the realized demand state, firm 0 decides whether to exit the market and receive the exit value K, or to remain active in the market and compete with the incumbent.

$t = 3$ Firm 1 learns the investment and exit decisions of firm 0. Firms set outputs simultaneously, which determine the prices for the two products, and payoffs materialize.

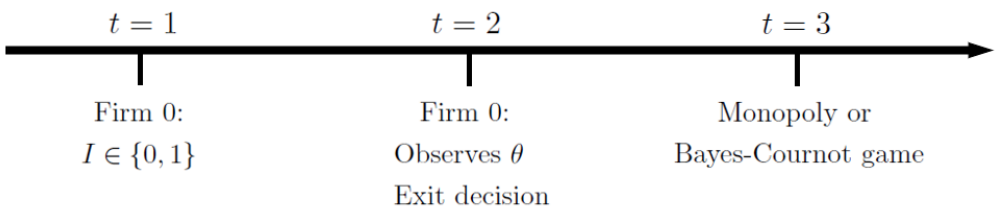

Figure 1: Timing of the game.

The game is intended to capture real-world market dynamics in infrastructure sectors, where firms, particular newcomers to the market, need to make significant investments in advance of having a high degree of confidence in future demand. This means that, as market information is realized, they may wish to exit or reduce their commitment to the market over time. The

[15]Notice that, unlike in other papers on start-up acquisitions (e.g., Dijk et al., 2021; Fumagalli et al., 2021; Letina et al., 2020), in our model the challenger's investment has a deterministic impact on demand (even though the actual demand is still uncertain at the investment stage). Uncertain investment returns are considered in Section 4.3. In Section 4.4 we show that results are robust to the introduction of a continuous investment decision.

[16]Section 4.2 examines how our results change if we allow for leapfrogging — i.e., the case in which, when the challenger invests, it sells, on average, a superior product compared to the incumbent's one. In Section 4.5, instead, we discuss what would change if the incumbent can protect its dominant position with a demand-enhancing investment.

hypothesis of quantity competition is in line with the approach taken in the bulk of the literature on horizontal mergers (e.g., Mermelstein et al., 2020; Nocke and Whinston, 2010, 2013) and captures the long run dynamics of industries in which capacity constraints determine firms' pricing behaviour (Kreps and Scheinkman, 1983).

We assume, initially, that the exit value K is the same irrespective of whether firm 0 has invested or not. For example, one could imagine that K is negatively affected by labour related exit costs (e.g., costs related to employees' contractual rights such as staff redundancy costs and insurance benefits), bankruptcy rules (e.g., filing and litigation fees), regulatory exit requirements (e.g., remediation costs due to environmental regulations), etc. In Section 3 we extend the model by making K explicitly dependent of I.

Since firm 0 is privately informed about θ and the game is sequential, the solution concept is Perfect Bayesian Equilibrium (PBE). In the remainder we impose the following technical requirements.[17]

A1 The support of θ is neither too dispersed nor too narrow: $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, with $\overline{\sigma} > \underline{\sigma} > 0$.

A2 The exit value K is not too large: $K \leq \overline{K}$.

These restrictions rule out the uninteresting cases in which firm 0 never exits if it invests, or always exits if it does not invest, as well as the case in which the incumbent does not produce when the rival is in the market.[18]

In addition, for simplicity we normalize marginal production costs to zero and make the standard assumption that firms are risk-neutral (i.e., maximize their expected profits). Finally, as standard tie-breaking conditions, we assume that the challenger invests and stays in the market when it is indifferent in its first-stage and second-stage decisions, respectively.

2.1 Equilibrium analysis

In this section we characterize the PBE of the game. Since actions are sequential and firm 0's investment decision is observable, we proceed backward and solve first the quantity setting (sub)game under the hypothesis that firm 0 is active (otherwise the incumbent is a monopolist and the game has a straightforward solution, as we explain below).

Consider a candidate equilibrium in which firm 1 sets $x_1^\star(I, K)$ upon observing the challenger's investment decision I and knowing its exit value K. Moreover, suppose that firm 0 adopts a cut-off strategy such that it leaves the market if and only if $\theta < \theta^\star(I, K)$; otherwise, it remains active and competes with the incumbent.

[17] The expressions for the thresholds defined in Assumptions A1 and A2 are in the Appendix.

[18] In particular, while $\sigma \geq \underline{\sigma}$ guarantees that firm 0 may decide to take the exit value even if it has invested, $K \leq \overline{K}$ guarantees that for some demand states it decides to remain active in the market even if it has not invested. Finally, $\sigma \leq \overline{\sigma}$ ensures that the incumbent's output is positive under all circumstances. Notably, the restriction $K \leq \overline{K}$ is immaterial to the welfare results as the challenger's incentive to invest as well as consumer surplus for any given investment level are decreasing in K for $K > \overline{K}$, thereby a regulator who maximizes consumer surplus would never choose an exit value $K \geq \overline{K}$. Thus, such restriction is imposed just to simplify the exposition and focus on the most interesting region of parameters.

For given investment I in the first stage, firm 0's maximization problem is

$$\max_{x_0 \geq 0} \left(\mu I + \theta - x_0 - b x_1^\star (I, K)\right) x_0,$$

whose first-order condition yields

$$x_0^\star (\theta, I, K) = \frac{\mu I + \theta - b x_1^\star (I, K)}{2}. \tag{3}$$

As intuition suggests, $x_0^\star (\cdot)$ is increasing in μ, I and θ, and decreasing in the rival's output.

The incumbent's maximization problem is

$$\max_{x_1 \geq 0} \left(\mu - x_1 - b x_0^e (I, K)\right) x_1.$$

Since the realization of θ is firm 0's private information, conditional on exit not taking place, the incumbent must form an expectation $x_0^e (I, K)$ on the rival's output which has to be consistent with equilibrium behaviour — i.e., given the cut-off $\theta^\star (I, K)$, the incumbent expects firm 0 to produce

$$x_0^e (I, K) \triangleq \mathrm{E}\left[x_0^\star (\theta, I, K) \,|\theta \geq \theta^\star (I, K)\right].$$

The first-order condition of firm 1's problem immediately yields

$$x_1^\star (I, K) = x^M - \frac{b}{2} x_0^e (I, K), \tag{4}$$

where by $x^M \triangleq \frac{\mu}{2}$ hereafter we define the monopoly output set by firm 1 when it is the only market actor. As expected, $x_1^\star (\cdot)$ is increasing in μ and decreasing in firm 0's expected output.

The following lemma characterizes the equilibrium of the quantity-setting subgame when firm 0 does not exit — i.e., for all demand states such that $\theta \geq \theta^\star (I, K)$.

Lemma 1. *For given investment I made by firm 0 in the first stage, when $\theta \geq \theta^\star(I, K)$ equilibrium outputs are positive and such that, other things being equal, $x_1^\star (\cdot)$ falls with σ and $\theta^\star (\cdot)$, while $x_0^\star (\cdot)$ rises with θ, σ and $\theta^\star (\cdot)$.*

The reason why $x_1^\star (\cdot)$ is decreasing in σ and $\theta^\star (\cdot)$ is as follows. Firm 0 is active when $\theta \geq \theta^\star (\cdot)$, and from the incumbent's perspective its expected output is

$$x_0^e (I, K) = \frac{\mu I + \mathrm{E}\left[\theta | \theta \geq \theta^\star (\cdot)\right] - b x_1^\star (\cdot)}{2}.$$

Hence, as the support of θ widens (i.e., σ expands) and/or exit becomes more likely (i.e., $\theta^\star (\cdot)$ increases), the incumbent expects the rival to compete more aggressively when it remains active in the market, and thus its output is expected to be larger. As a result, since quantities are strategic substitutes, the incumbent's best reply is to reduce its output, which, in turn, leads firm 0's output to expand further, and so on.

Note that I affects outputs through two different channels. First, other things being equal, investment has a direct positive effect on firm 0's output because it increases the intercept of

its inverse demand function, and thus reduces firm 1's output because quantities are strategic substitutes. Second, I also affects the cut-off $\theta^{\star}(\cdot)$. We will show below that this cut-off is decreasing in I, so that through this channel I increases $x_1^{\star}(\cdot)$ and reduces $x_0^{\star}(\cdot)$. When I increases, firm 0 remains active for lower values of θ, which means that the incumbent anticipates a lower expected output by the rival, and thus expands its own quantity, thereby reducing $x_0^{\star}(\cdot)$. The net balance between these two opposing forces can be assessed only after having characterized $\theta^{\star}(\cdot)$ and the difference $\theta^{\star}(1,K) - \theta^{\star}(0,K)$, which is our next task.

Conditional on firm 0 being active — i.e., for every $\theta \geq \theta^{\star}(\cdot)$ — firm 0's profit is

$$\pi_0(\theta, I, K) = x_0^{\star}(\theta, I, K)^2,$$

which is increasing in θ. Hence, the threshold $\theta^{\star}(\cdot)$ is determined by the indifference condition

$$\pi_0(\theta^{\star}, I, K) = K, \tag{5}$$

We can thus state the following.

Proposition 1. *For given investment I made by firm 0 in the first stage, firm 0 exits the market if and only if $\theta < \theta^{\star}(I,K)$, with $\theta^{\star}(I,K)$ being the unique solution of (5) in $[-\sigma, \sigma]$. The function $\theta^{\star}(I,K)$ is such that $\theta^{\star}(1,K) < \theta^{\star}(0,K)$ for all K. Moreover, it is increasing in K, decreasing (increasing) in μ if $I = 1$ ($I = 0$) and decreasing in σ. The effect of b is ambiguous, as $\theta^{\star}(\cdot)$ may be inverted U-shaped in b.*

Firm 0 quits the market when it observes a low demand realization and remains active otherwise. Exit is relatively less profitable when firm 0 invests, since in this case it gains a stronger competitive position vis-à-vis the incumbent. Similarly, a larger market size (as captured by a higher μ) makes firm 0 less likely to exit when it invests and more likely to exit when it does not invest. Indeed, when firm 0 does not invest, the incumbent will produce more as μ grows large, which reduces the market price, thereby marginalizing firm 0. This is because when $I = 0$, the larger μ, the more vertically differentiated the two firms are. The effect of σ on exit depends on the fact that profit functions are quadratic and that firms' profit functions are convex in θ — i.e., they prefer to face demand with greater dispersion. Hence, as demand becomes more uncertain, firm 0's expected profit increases, which makes exit relatively less profitable. Finally, the effect of b is ambiguous for the following reason. First, when b increases, product market competition is fiercer and thus, other things being equal, firm 0 has a stronger incentive to exit. Second, a higher b also means that the incumbent's reaction function will be more responsive to its rival's expected output. Essentially, for relatively large values of b, when firm 1 expects firm 0 to be sufficiently aggressive it will react by lowering its output to a large extent, thereby inducing firm 0 to exit less often, and vice versa. When σ is small, the first effect dominates because firm 0's expected demand is low; by contrast, when σ is large the second effect dominates, at least for relatively large values of b. Hence, there can be an inverted U-shaped relationship between $\theta^{\star}(\cdot)$ and b.

Having characterized $\theta^{\star}(\cdot)$, we can rank firms' outputs and equilibrium prices with and

without the investment. Letting $p_0^\star(\theta, I, K)$ and $p_1^\star(\theta, I, K)$ denote the equilibrium prices, we can state the following.

Lemma 2. *The investment reduces firm 1's output and expands firm 0's and aggregate output. Moreover,* $p_0^\star(\theta, 1, K) > p_0^\star(\theta, 0, K)$ *and* $p_1^\star(\theta, 1, K) \leq p_1^\star(\theta, 0, K)$.

Hence, the direct effect of investment on firms' outputs discussed above always dominates the indirect effect. This implies that aggregate output in equilibrium increases when the investment is undertaken. As a consequence, the market clearing price for the products sold by the incumbent falls as well, whereas firm 0 sells its products at a larger price when it invests, as the investment increases consumers' willingness to pay for its products.

We are now in the position of characterizing firm 0's investment decision. Firm 0's expected profit (without the investment cost) is

$$\pi_0^\star(I, K) \triangleq \int_{-\sigma}^{\sigma} \max\left\{x_0^\star(\theta, I, K)^2, K\right\} \frac{d\theta}{2\sigma} = \underbrace{\int_{-\sigma}^{\theta^\star(I,K)} K \frac{d\theta}{2\sigma}}_{\text{Exit value}} + \underbrace{\int_{\theta^\star(I,K)}^{\sigma} x_0^\star(\theta, I, K)^2 \frac{d\theta}{2\sigma}}_{\text{Market value}}.$$

Let $\Delta\pi_0(K) \triangleq \pi_0^\star(1, K) - \pi_0^\star(0, K)$ be the value of the investment for firm 0. Since $\theta^\star(1, K) < \theta^\star(0, K)$, we obtain the following useful decomposition

$$\Delta\pi_0(K) = \underbrace{\int_{\theta^\star(1,K)}^{\theta^\star(0,K)} \left[x_0^\star(\theta, 1, K)^2 - K\right] \frac{d\theta}{2\sigma}}_{\text{Participation effect } (+)} + \underbrace{\int_{\theta^\star(0,K)}^{\sigma} \left[x_0^\star(\theta, 1, K)^2 - x_0^\star(\theta, 0, K)^2\right] \frac{d\theta}{2\sigma}}_{\text{Rivalry effect } (+)}. \tag{6}$$

This expression shows that the value of the investment for firm 0 can be decomposed into two intuitive terms, both of which are positive. First, when firm 0 invests, there are more states of nature in which the exit value is lower than the market value. In these states of nature, firm 0 gains $x_0^\star(\theta, 1, K)^2 \geq K$. Second, conditional on the states of nature in which there is a duopoly irrespective of I (i.e., $\theta \geq \theta^\star(0, K)$), firm 0's output and profits are higher when it undertakes the investment: $x_0^\star(\theta, 1, K) \geq x_0^\star(\theta, 0, K)$. Hence, we can state the following.

Proposition 2. *The value of the investment for firm 0 is positive — i.e.,* $\Delta\pi_0(K) > 0$ *for any* $K \in \left[0, \overline{K}\right]$. *Hence,* $I^\star = 1$ *if and only if* $\psi \leq \Delta\pi_0(K)$ *and* $I^\star = 0$ *otherwise. The difference* $\Delta\pi_0(K)$ *is single peaked with respect to* K *and features a maximum at*

$$K^\star \triangleq \left(\frac{b^2(2\sigma + \mu(1-b))}{8(4-b^2)}\right)^2, \tag{7}$$

with $0 < K^\star < \overline{K}$. *The value* $K^\star$ *is increasing in* σ, μ *and* b.

This proposition shows that firm 0's incentive to invest is inverted U-shaped with respect to K. Since $\partial x_0^\star(\theta, \cdot)/\partial\theta^\star(\cdot)$ and $\partial\theta^\star(\cdot)/\partial K$ are independent of I, the derivative of $\Delta\pi_0(K)$ with

respect to K is

$$\frac{\partial \Delta\pi_0(K)}{\partial K} = \underbrace{-\int_{\theta^\star(1,K)}^{\theta^\star(0,K)} \frac{d\theta}{2\sigma}}_{-} + \int_{\theta^\star(0,K)}^{\sigma} \underbrace{\frac{\partial x_0^\star(\cdot)}{\partial \theta^\star(\cdot)} \frac{\partial \theta^\star(\cdot)}{\partial K}}_{+} \underbrace{[x_0^\star(\theta,1,K) - x_0^\star(\theta,0,K)]}_{+} \frac{d\theta}{\sigma}.$$

There are two effects at play. First, an increase in K makes staying in the market more costly, so that firm 0 is more likely to stay if $I = 1$. Second, when K increases, exit becomes more likely (i.e., $\partial\theta^\star(\cdot)/\partial K \geq 0$), which means that when firm 0 remains active it must have observed a (relatively) high realization of θ and, hence, its output is relatively high (i.e., $\partial x_0^\star(\cdot)/\partial\theta^\star(\cdot) > 0$). This output expansion leads to a greater increase in profits when firm 0 invests. In simpler terms, on the one hand, as K increases, the cost of staying in the market increases, which reduces the incentive to invest; on the other hand, when K goes up, only entrants with an attractive product (i.e., with a high θ) remain in the market. Due to this *selection effect*, the incumbent will respond by reducing its output. The incentive to invest increases for larger K, because investment has strategic value: it leads firm 1 to reduce its output more. The balance between these two forces determines $K^\star$.

Importantly, $K^\star$ is increasing in σ: the more dispersed demand, the stronger the second, strategic effect described above. A similar argument explains why $K^\star$ is increasing in μ as well. The effect of b is also straightforward: as competition intensifies, firm 1's reaction function becomes more responsive to firm 0's expected output, which again reinforces the positive strategic effect of K on the incentive to invest.

2.2 Consumer surplus analysis

In this section we examine consumer surplus. To start with, note that the demand functions (1) and (2) can be derived from a representative consumer with preferences described by the following utility function[19]

$$U(x_0, x_1, \theta) \triangleq (\mu I + \theta) x_0 + \mu x_1 - \frac{1}{2}\sum_{i=0,1} x_i^2 - b x_0 x_1 - \sum_{i=0,1} p_i x_i. \tag{8}$$

Substituting the inverse demand functions $p_i(\cdot)$, with $i = 0, 1$, back into (8), we obtain an expression for consumer surplus that depends only on firms' outputs — i.e.,

$$U(\cdot) = \frac{1}{2}\sum_{i=0,1} x_i^2 + b x_0 x_1.$$

[19]Note that we do not consider any spillover benefits of investment — e.g., to consumers of firms in related markets which use the infrastructure provided — though these could be significant. Accounting for such spillovers would further increase the social desirability of the investment.

Hence, evaluating the above expression at the equilibrium values and taking the expectation with respect to θ, we have

$$CS(I,K) \triangleq \Pr\left[\theta < \theta^\star(I,K)\right] CS^M + \int_{\theta^\star(I,K)}^{\sigma} \underbrace{\left[\frac{1}{2}\sum_{i=0,1} x_i^\star(\cdot)^2 + b x_0^\star(\cdot)\, x_1^\star(\cdot)\right]}_{\triangleq CS^D(\theta,I,K)} \frac{d\theta}{2\sigma},$$

where the consumer surplus in case of monopoly is simply $CS^M \triangleq \frac{\mu^2}{8}$, and $CS^D(\theta, I, K)$ denotes consumer surplus in the duopoly case where firms 0 and 1 compete.

Define by $\Delta CS(K) \triangleq CS(1,K) - CS(0,K)$ the social value of the investment. Since $\theta^\star(1,K) < \theta^\star(0,K)$, we can rewrite $\Delta CS(K)$ as

$$\Delta CS(K) = \underbrace{\int_{\theta^\star(1,K)}^{\theta^\star(0,K)} \left(CS^D(\theta,1,K) - CS^M\right) \frac{d\theta}{2\sigma}}_{\text{Switch to duopoly } (+)} + \underbrace{\int_{\theta^\star(0,K)}^{\sigma} \left(CS^D(\theta,1,K) - CS^D(\theta,0,K)\right) \frac{d\theta}{2\sigma}}_{\text{Investment effect in duopoly } (+)}. \tag{9}$$

The social value of the investment can be decomposed into two terms, which are closely related to those described in equation (6). The first term captures the idea that when firm 0 invests, it is more likely to remain active in the market to the benefit of consumers. Specifically, in the states of demand in which there is a duopoly conditional on firm 0 having invested ($\theta \geq \theta^\star(1,K)$) and a monopoly otherwise ($\theta \leq \theta^\star(0,K)$), consumers are on average better off if $I^\star = 1$, because with $I^\star = 0$ they would face a monopolistic incumbent — i.e., the first term in (9) is positive. Second, in the states of nature in which firm 0 remains active regardless of whether it has invested, the investment also benefits consumers because it increases the challenger's quality — i.e., $CS^D(\theta,1,K) - CS^D(\theta,0,K) > 0$ for all $\theta \geq \theta^\star(0,K)$. In sum, the investment always benefits consumers. We can thus state the following.

Proposition 3. *Consumers always benefit from the investment — i.e., $\Delta CS(K) \geq 0$ for any $K \in \left[0, \overline{K}\right]$. Hence, when $\psi \leq \Delta\pi_0(K)$ firm 0's equilibrium behaviour is aligned with consumers' preferences. By contrast, when $\psi > \Delta\pi_0(K)$ there is an under-investment problem from the perspective of consumers — i.e., they would benefit from the investment, but firm 0 has no incentive to pay the (relatively high) investment cost ψ.*

Therefore, a regulator whose objective function is consumer surplus maximization needs to incentivize firm 0 to invest when ψ is too high. Assuming that the regulator cannot control the investment cost, a possible policy instrument to achieve its goal is to implement policies that affect the exit value K in a way to increase the value of the investment $\Delta\pi_0(K)$ above the cost ψ.

In the remainder of this section we first study the relationship between consumer surplus and the exit value, and then we characterize the ex-ante optimal exit policy assuming that the regulator can commit at the outset of the game (i.e., before firm 0 invests) to a given value of K.[20]

[20] In this section we analyse the optimal policy from a consumer welfare maximization standpoint. While this is

2.3 Consumer welfare and the value of exit

Recall that $K^\star < \overline{K}$ is the value of K that maximizes $\Delta\pi_0(K)$, and let

$$K^{\star\star}(I) \triangleq \underset{K\in[0,\overline{K}]}{\arg\max} CS(I,K)$$

denote the exit value that maximizes consumer surplus conditional on a given investment I. We can state the following.

Lemma 3. *The function $K^{\star\star}(I)$ exhibits the following features: (i) $K^{\star\star}(1) = 0$; (ii) there are two thresholds $b_0^\star \in (0,1)$ and $\sigma_0^\star \in (\underline{\sigma}, \overline{\sigma})$ such that $K^{\star\star}(0) \in (0, K^\star)$ if and only if $b > b_0^\star$ and $\sigma < \sigma_0^\star$, and $K^{\star\star}(0) = 0$ otherwise.*

Maximizing consumer surplus when the investment takes place ($I = 1$) is equivalent to maximizing the probability of duopoly, which involves reducing the value of exit to zero. By contrast, when firm 0 does not invest, consumer surplus maximization may require exit, and hence a positive exit value $K^{\star\star}(0) > 0$. To understand these results, it is useful to differentiate $CS(I,K)$ with respect to K. We have:

$$\frac{\partial CS(I,K)}{\partial K} = \frac{1}{2\sigma}\underbrace{\frac{\partial\theta^\star(\cdot)}{\partial K}}_{(+)}\Bigg\{\underbrace{CS^M - CS^D(\theta^\star(I,K),I,K)}_{\text{Switch to monopoly (?)}}+$$

$$+\int_{\theta^\star(I,K)}^{\sigma}\underbrace{\frac{\partial x_1^\star(\cdot)}{\partial\theta^\star(\cdot)}\left[x_1^\star(\cdot)+bx_0^\star(\cdot)\right]}_{\text{Strategic effect }(-)}d\theta+\int_{\theta^\star(I,K)}^{\sigma}\underbrace{\frac{\partial x_0^\star(\cdot)}{\partial\theta^\star(\cdot)}\left[x_0^\star(\cdot)+bx_1^\star(\cdot)\right]}_{\text{Output enhancing effect }(+)}d\theta\Bigg\}.$$

Three forces shape the effect of K on consumer surplus. First, as K grows larger, firm 0 is more likely to exit — i.e., $\theta^\star(\cdot)$ increases — thereby inducing a monopoly by firm 1. Interestingly, the effect on consumers of a switch to monopoly is ambiguous. On the one hand, the loss of competition makes consumers worse off, as the monopolist faces no competitive constraint when making its quantity-setting decision. On the other hand, when $\theta^\star(\cdot)$ is low, firm 0 sells an inferior product and, therefore, the duopoly scenario involves consumers purchasing a product for which they obtain less value (albeit at a lower price). This second effect dominates when K is close to zero as in this case $\theta^\star(\cdot)$ is negative. Instead, the first effect is greater and the second smaller when $I = 1$. That is, the effect on consumer welfare associated to the change in market structure is more likely to be negative (and/or less positive) when firm 0 invests.

Second, as K increases, provided $\theta \geq \theta^\star(\cdot)$, firm 1 expects tougher competition and, accordingly, reduces its output: a strategic effect that softens competition to the detriment of consumers. Third, since firm 1 reduces output in response to a higher K, firm 0's output expands because quantities are strategic substitutes. The sum of the second and third effects is

consistent with antitrust law in many countries (including US and EU), it is also worth observing that studying total welfare in this model with exogenous exit value is uninteresting. The reason is that total welfare maximization would require to set K arbitrarily large so to increase firm 0's payoff ad libitum, while consumer surplus and firm 1's profit would correspond to the monopoly outcome. We shall thus consider the total welfare maximizing policy in Section 3 where we endogenise the exit value with applications to merger policy and bankruptcy law.

negative — i.e., the strategic effect dominates the output expansion effect. The main reason is that while firm 0 can adjust its output to demand uncertainty perfectly, because it observes θ, the incumbent can only adjust its output imperfectly. Hence, an increase of $\theta^{\star}(\cdot)$ reduces $x_1^{\star}(\cdot)$ more than it increases $x_0^{\star}(\cdot)$.[21] This is true irrespective of firm 0's investment decision.

When $I = 1$, the sum of the last two effects dominates the first effect even for $K = 0$. This is intuitive, since the cost of exit for consumers is larger when firm 0 invests and its product is ex-ante at par with that of the incumbent. When $I = 0$, instead, there are parameter values for which the first effect is positive and larger than the sum of the other two when $K = 0$. This is because, when firm 0 does not invest, the incumbent's product is ex-ante more highly valued than that of its rival. In this case, maximizing consumer surplus is not necessarily equivalent to maximizing rivalry.

In particular, it is optimal to set a positive exit value if demand is not too uncertain (σ small) and products are sufficiently close substitutes (b high).

2.4 Optimal ex-ante exit policy

We can now turn to characterize the optimal exit policy — i.e., the value of K that maximizes consumer surplus accounting for the fact that the challenger's incentive to invest depends on the exit value chosen by the regulator. We structure the analysis by looking at three different regions of parameters.

First, when $\psi > \Delta\pi_0(K^{\star})$ the regulator's problem has a trivial solution: there exists no value of K at which firm 0 is willing to invest. Hence, the ex-ante optimal exit value is simply $K^{\star\star}(0)$, which maximizes $CS(0, K)$ as discussed in Lemma 3 above and is positive for σ sufficiently small and b large. Second, when $\psi \leq \Delta\pi_0(0)$ the regulator's problem has a trivial solution too: in this case, firm 0 always invests irrespective of K. Hence, the regulator simply sets $K^{\star\star}(1) = 0$. Third, when $\psi \in \Psi \triangleq (\Delta\pi_0(0), \Delta\pi_0(K^{\star})]$ the analysis becomes more interesting because the investment can be induced but only if the regulator designs an incentive compatible policy. For this parameter range, we proceed in two steps. First, we consider the choice of K assuming that the regulator wants to implement $I = 1$. Second, we consider whether the optimal policy is indeed to implement $I = 1$ or, instead, to maximize consumer surplus conditional on $I = 0$.

If the regulator wants to encourage the investment, it must solve the following constrained maximization problem

$$\begin{cases} \max_{K \in [0, \overline{K}]} CS(1, K) \\ \text{s.t. } \psi \leq \Delta\pi_0(K) \end{cases} \tag{10}$$

Essentially, the regulator maximizes consumer surplus given $I = 1$ subject to firm 0's incentive compatibility constraint.

From the previous analysis, we know that the function $\Delta\pi_0(K)$ is strictly concave and

[21] Indeed, it can be checked that, for all $b \in (0, 1]$,

$$\frac{\partial x_0^{\star}}{\partial \theta^{\star}(\cdot)} = \frac{b^2}{4(4 - b^2)} < \frac{b}{2(4 - b^2)} = \left| \frac{\partial x_1^{\star}}{\partial \theta^{\star}(\cdot)} \right|.$$

features a maximum at $K^\star > 0$. Hence, defining (with a slight abuse of notation) $\widehat{K} \triangleq \Delta\pi_0^{-1}(\psi)$ as the smaller root of $\Delta\pi_0(K) = \psi$, we can show the following.

Lemma 4. *For every $\psi \in \Psi$, the solution of (10) is $\widehat{K} \in [0, K^\star]$, which is increasing in ψ.*

Consumer surplus given $I = 1$ is decreasing in K. Hence, the solution of (10) must be equal to the lowest value K that makes the firm indifferent between investing and not — i.e., the lowest solution of $\psi = \Delta\pi_0(K)$. The intuitive reason why the regulator wants to enforce an exit policy less lenient than the one that maximizes the firm's incentive to invest is that consumers, in contrast to firm 0, benefit from a higher probability of duopoly when $I = 1$.

Hence, when the investment cost is neither too large nor too small, the regulator faces a potential trade-off. It can either choose $\widehat{K}$, thereby promoting investment but reducing consumer surplus below its first-best level $CS(1, 0)$, or it may give up the investment and set $K^{\star\star}(0)$ so as to maximize consumer surplus without the investment. As a result, the optimal exit policy is $\widehat{K}$ if and only if

$$CS(1, \widehat{K}) \geq CS(0, K^{\star\star}(0)).$$

We can show the following.

Proposition 4. *For all $\psi \in \Psi$, the optimal exit policy is $K^R = \widehat{K}$. This region of parameters expands as μ, σ and b grow large.*

This result shows that the regulator always prefers to secure the investment by setting an exit policy more lenient than the one which maximizes ex-post consumer surplus — i.e., than the one it would choose if the investment was already sunk by the challenger. The intuition is as follows: for given K, by increasing consumers' willingness to pay, the investment creates value — i.e., it increases social welfare — and when the firm is indifferent between making or not making the investment, this value is fully appropriated by consumers, who must therefore be better off with the investment than without. The region of parameters in which this is true expands when demand is more volatile or dispersed (σ large), when products are closer substitutes (b large) and when consumers are more willing to pay for both firms' products (μ high).

Summing up, if the regulator could credibly commit to an exit value, it would set:

$$K^R = \begin{cases} 0 & \text{if } \psi \leq \Delta\pi_0(0) \\ \widehat{K} & \text{if } \psi \in (\Delta\pi_0(0), \Delta\pi_0(K^\star)] \\ K^{\star\star}(0) & \text{if } \psi > \Delta\pi_0(K^\star) \end{cases},$$

which is non-monotone in ψ because, in the region of parameters where $K^{\star\star}(0) > 0$, $\widehat{K}$ is larger than $K^{\star\star}(0)$ if and only if ψ is sufficiently high. Hence, while the optimal ex-ante policy is always more lenient than the one that would maximize consumer welfare with $I = 1$, since $\widehat{K} \geq K^{\star\star}(1) = 0$, it is not necessarily more lenient than what a regulator who is unable to induce investment would choose. The relationship between K^R and ψ is illustrated in Figure 2.

Indeed, this policy is optimal ex ante — i.e., before firm 0's investment decision is made. Yet, it can be successfully implemented only under the assumption that the regulator can credibly

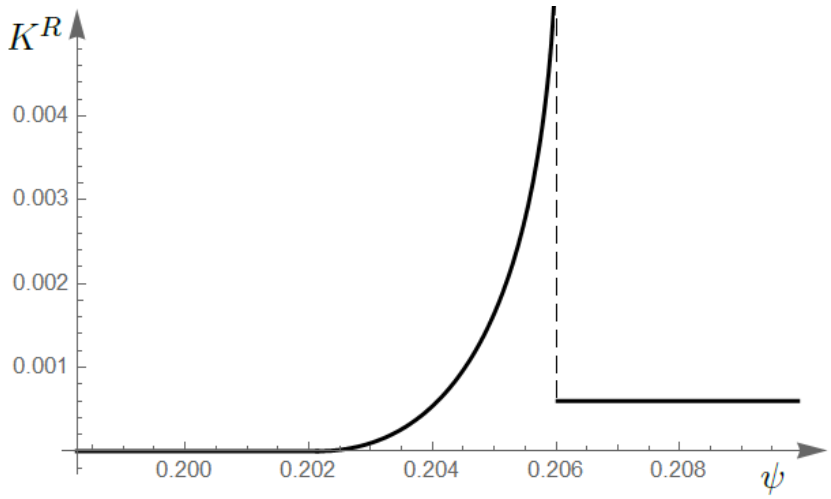

Figure 2: Relationship between K^R and ψ (for $\mu = b = \sigma = 1$).

commit to an exit value or to a policy that influences it. Without such commitment, the regulator faces an obvious time-inconsistency problem. While it may have an incentive to announce $\widehat{K}$, once the investment has been undertaken it will renege on its initial commitment and optimally set $K^{\star\star}(1) = 0$ to maximize the likelihood of duopoly. This dynamic will be anticipated by a rational firm 0, who will then refuse to invest, thereby reducing consumer surplus. A similar time inconsistency problem has been often identified in the previous literature on optimal dynamic merger policy — see, e.g., Mason and Weeds (2013) and Mermelstein et al. (2020), among many others.

3 Exit policy at work

Hitherto, we have assumed that the exit value K is unresponsive to whether firm 0 has invested or not. However, it is often the case that the investment decision affects the exit value, so that K is a function of I. Following Schary (1991), in this section we propose two alternative avenues for endogenising the exit value. The first approach determines it through a merger — i.e., following the investment stage, the incumbent has the option of acquiring firm 0 and the offer made depends on whether it has invested or not. The regulator can affect the exit value by prohibiting or allowing the merger. In the second approach, we endogenise the exit value by positing that when firm 0 decides to exit the market it is able to recoup a fraction of the investment costs sunk in the first stage of the game. In this case, the regulator can expand the exit value by increasing the fraction of these costs that the challenger can recoup — e.g., by adopting a more efficient bankruptcy procedure that dissipates fewer rents.

For simplicity, throughout this section we assume $b = 1$. We keep assuming **A1**, whereas we drop **A2**, as the exit value is now endogenous.

3.1 Merger policy

In this section we determine firm 0's exit value by considering the possibility of a merger — i.e., K now represents firm 0's option value of selling its assets to firm 1. To start with, we consider two merger policy regimes:

- *Strict merger policy* ($y = s$): in this regime, a merger to monopoly is forbidden.
- *Lenient merger policy* ($y = l$): in this regime, firm 1 is allowed to purchase firm 0 without conditions.

The timing of the game is as follows:

$t = 1$ Firm 0 decides whether to invest.

$t = 2$ The state θ realizes and is observed only by firm 0. Firm 1 makes a take-it-or-leave-it offer K to firm 0, who decides whether to accept the offer and sell its assets to the incumbent or compete with it.

$t = 3$ Active firms sell their products in the market and payoffs materialize.

This timing, which is in line with the literature on start-up acquisitions (e.g., Dijk et al., 2021; Letina et al., 2020), is adequate to model acquisitions in environments in which new market entrants have developed their products before they are bought, as it is often the case in hi-tech sectors.

The underlying informational assumption is that, if the merger takes place, firm 1 learns θ before choosing quantities. Hence, upon acquiring firm 0, the incumbent optimally sells

$$(x_0(\theta, I), x_1(\theta, I)) \triangleq \begin{cases} (0, \frac{\mu}{2}) & \text{if } \theta \leq \mu(1-I) \\ (\frac{1}{2}(\mu I + \theta), 0) & \text{if } \theta > \mu(1-I) \end{cases}$$

and its profit is given by $\pi^M(\theta, I) = x^M(\theta, I)^2$, with $x^M(\theta, I) \triangleq x_0(\theta, I) + x_1(\theta, I)$. This is because, absent product differentiation, the merged entity finds it optimal to produce only the highest quality good.

We shall describe the equilibrium outcome under each merger policy regime and compare firm 0's incentive to invest and consumer surplus across the two regimes.

3.1.1 Strict merger policy

When a merger to monopoly is forbidden, the challenger has no exit value — i.e., $K = 0$ — and firm 0 quits if and only if $\theta < \theta^\star(I, 0)$, which is characterized as in the baseline model.

Let $\Delta\pi_0(s)$ be the increase in firm 0's profit due to the investment when $y = s$, which corresponds to equation (6) given in the baseline model for $K = 0$. Then, the the baseline analysis immediately yields the following result.

Lemma 5. *With a strict merger policy, firm* 0 *invests if and only if* $\psi \leq \Delta\pi_0(s)$*, with* $\Delta\pi_0(s)$ *being always positive and increasing in* σ*.*

Notice that, in our environment, a strict merger policy is equivalent from a consumer welfare point of view to a policy based on the so called 'failing firm defence' (e.g., Bouckaert and Kort, 2014; Persson, 2005; Vasconcelos, 2013) — i.e., a rule according to which a merger is approved only if the target would file for bankruptcy and leave the market absent the transaction. This is because under this policy the incumbent would optimally offer $K = 0$ and, anticipating this, the challenger's investment decision is as under a strict merger policy.

3.1.2 Lenient merger policy

The analysis of the quantity setting game when firm 0 rejects firm 1's offer is as in the baseline model. Hence, for given investment I, firm 0 accepts an offer K and the merger occurs if and only if $\theta \leq \theta^\star(I, K)$, with $\theta^\star(I, K)$ being decreasing in I and increasing in K. In this case, letting

$$\widetilde{K}(I) \triangleq \left(\frac{\sigma + (3 + I)\mu}{12}\right)^2,$$

be the level of K that solves

$$\mu = \mu I + \theta^\star(I, K) \quad \Rightarrow \quad \theta^\star(I, K) = \mu(1 - I),$$

we can distinguish between the following regions of parameters:

- For every $K \leq \widetilde{K}(I)$ we have $\theta^\star(I, K) \leq \mu(1 - I)$. In this region, every accepted offer results in a *killer acquisition* (Cunningham et al., 2021): the merged entity will always sell product 1 only.
- For every $K > \widetilde{K}(I)$ we have $\theta^\star(I, K) > \mu(1 - I)$. In this region, while for $\theta \leq \mu(1 - I)$ we have a *killer acquisition*, for $\theta \in (\mu(1 - I), \theta^\star(I, K))$ the merged entity will replace its own product with that developed by firm 0.

By contrast, for all $\theta \geq \theta^\star(I, K)$, firm 0 rejects the offer, no merger takes place, and both firms compete in the market, so that the equilibrium outputs are as in the baseline model. Note that $x_1^\star(\cdot)$ is a function of K even when the offer is rejected because the value of K affects what firm 1 learns about θ when the offer is rejected. Specifically, firm 1 learns that $\theta > \theta^\star(I, K)$, where $\theta^\star(\cdot)$ is increasing in K. Thus, firm 1 anticipates that firm 0's output is likely to be large, so that firm 1's output when the offer is rejected is decreasing in K, which entails firm 0's output in such circumstances to be increasing in the offer K (even when the offer is rejected and the actual exit value is zero).

Thus, firm 1's expected profit when making an offer K is

$$\pi_1^\star(I, K) \triangleq \int_{-\sigma}^{\theta^\star(I,K)} \left[\pi^M(\theta) - K\right] \frac{d\theta}{2\sigma} + \int_{\theta^\star(I,K)}^{\sigma} \pi_1^\star(\theta, I, K) \frac{d\theta}{2\sigma},$$

where, as before, $\pi^M(\theta) \triangleq x^M(\theta)^2$ and

$$\pi_1^\star(\theta, I, K) \triangleq (\mu - x_1^\star(I, K) - x_0^\star(\theta, I, K))\, x_1^\star(I, K).$$

Differentiating the expected profit with respect to K, we have

$$\frac{\partial \pi_1^\star(I,K)}{\partial K} \underbrace{- \int_{-\sigma}^{\theta^\star(I,K)} \frac{d\theta}{2\sigma}}_{\text{Acquisition cost } (-)} + \underbrace{\frac{\Delta\pi_1(I,K)}{2\sigma} \frac{\partial \theta^\star(I,K)}{\partial K}}_{\text{Market-power gain } (+)} + \int_{\theta^\star(I,K)}^{\sigma} \underbrace{\frac{\partial \pi_1^\star(\theta,I,K)}{\partial K}}_{\text{Learning effect } (-)} \frac{d\theta}{2\sigma},$$

where

$$\Delta\pi_1(I,K) \triangleq \pi^M(\theta^\star(I,K)) - \pi_1^\star(\theta^\star(I,K), I, K) > 0,$$

is the profit differential that firm 1 gains when the merger occurs and $\theta = \theta^\star(I,K)$.

The derivative of $\pi_1^\star(I,K)$ with respect to K is shaped by three effects. First, for given acceptance probability, increasing the offer K is costly and reduces the incumbent's profit in case the merger occurs. Second, when the offer is accepted (which is more likely for a larger K), firm 1 becomes a monopolist and therefore gains the difference between the monopoly and the duopoly profit $\Delta\pi_1(I,K)$. Third, a higher K reduces firm 1's output in duopoly because, as explained above, when firm 0 rejects a high offer, firm 1 learns that θ is large, which means that firm 1's output would be low. This *learning effect* hinges on the assumption that the incumbent does not know the challenger's type, which de facto grants a rent to the informed party as in all adverse selection models — e.g., Akerlof (1970)'s lemons.

Let $K^e(I)$ denote the equilibrium offer made by the acquirer to the target. We can show the following.

Proposition 5. *If $I = 0$, the incumbent never makes an acceptable offer to firm 0 — i.e., $K^e(0) = 0$. If $I = 1$, the optimal offer $K^e(1)$ is positive and U-shaped with respect to σ. Moreover, $K^e(1) < \widetilde{K}(1)$ — i.e., all accepted offers result in a killer acquisition.*

In order to gain intuition, notice that firm 1's incentive to merge with the rival is determined by the following forces. First, the incumbent would like to acquire the rival to soften competition and consolidate its monopoly power, and this incentive is more pronounced when firm 0 has undertaken the investment, since it is more likely that it stays in the market and more aggressive when it produces. Second, since firm 0 sells a superior product than the incumbent's one when θ is relatively large, a merger empowers the acquirer to sell whichever product attracts a higher willingness to pay — i.e., with some probability, increasing in I, it increases its monopoly profit. Third, as explained before, by increasing the offer made to firm 0, the incumbent will reduce its output when the offer is rejected. Because of this last effect, firm 1 has an incentive to make a low offer in order to commit to compete aggressively when the offer is rejected.

The reason why firm 1 never acquires a 'zombie' rival — i.e., a firm that has not invested — is that, in this case, both the first and the second effect are relatively weak. Hence, the dominating force is the third effect. Hence, a non-investing firm will never be acquired by the incumbent even if this would result in monopolization. By contrast, the incumbent has an incentive to

acquire a rival that has undertaken the investment. In this case the first two effects strengthen, which may make the merger profitable.

The reason why the offer $K^e(1)$ is U-shaped with respect to σ is as follows. Even though the strengthening of the learning effect as σ grows larger calls for a lower offer, when σ increases further firm 1 has to make a higher offer for two reasons: first, firm 0 is more likely to reject the offer (as $\theta^\star(1, K)$ is decreasing in σ); second, the difference between the monopoly profit and its profit in duopoly becomes larger — i.e., for σ large, firm 0's output is large in duopoly, thereby reducing firm 1's output and profit.

The reason why when a merger takes place it always results in a killer acquisition hinges on asymmetric information. First, because of its private information, the challenger always gets a rent when it is acquired: the equilibrium transaction price is always too high compared to the minimum price that a challenger would be willing to accept under symmetric information (which is pinned down by its type contingent participation constraint). Second, there is also a learning effect in place that magnifies this rent: since the reservation utility of the challenger is increasing in θ when it rejects an offer, the incumbent revises its beliefs about firm 0's output upward, and reduces its output expecting strong competition from the challenger. Therefore, other things being equal, this effect induces the challenger to refuse some offers that above the price that it would be willing to accept if the incumbent was informed about θ at the quantity-setting stage, thereby magnifying the rent that the incumbent needs to pay to undertake a non-killer acquisition. As a result, the incumbent never finds it optimal to offer a takeover price so large to be acceptable by a challenger selling a superior product, as this would imply paying (in expectation) a large rent.

We can now study firm 0's incentive to invest with an endogenous K. Let

$$\pi_0^\star(I, y = l) \triangleq \underbrace{\int_{-\sigma}^{\theta^\star(I,K^e(I))} K^e(I)\,\frac{d\theta}{2\sigma}}_{\text{The target accepts the offer}} + \underbrace{\int_{\theta^\star(I,K^e(I))}^{\sigma} x_0^\star(\theta, I, K^e(I))^2\,\frac{d\theta}{2\sigma}}_{\text{The target rejects the offer}},$$

be firm 0's expected profit (without the investment cost) under lenient merger control for given investment I. Moreover, let

$$\Delta\pi_0(l) \triangleq \pi_0^\star(1, y = l) - \pi_0^\star(0, y = l) \geq 0,$$

be firm 0's increase in profit due to the investment. We can then show the following.

Lemma 6. *With a lenient merger policy, firm* 0 *invests if and only if* $\psi \leq \Delta\pi_0(l)$*, with* $\Delta\pi_0(l)$ *being always positive and increasing in* σ*.*

A more dispersed demand strengthens the strategic (learning) effect discussed above, which in turn magnifies the challenger's incentive to invest and secure an information rent.

3.1.3 Optimal merger policy with a consumer surplus standard

We can finally analyse the impact of a merger policy $y \in \{s, l\}$ on investment and consumer surplus. Comparing $\Delta\pi_0(s)$ with $\Delta\pi_0(l)$ we can show the following.

Lemma 7. *$\Delta\pi_0(l) > \Delta\pi_0(s)$ for every admissible σ.*

A lenient merger policy always stimulates the investment. The reason is as follows. Anticipating that it will obtain a positive exit value if and only if $I = 1$ and $y = l$, firm 0 is more willing to invest under a lenient policy. This *innovation for buyout effect* is well known in the literature (see, e.g., Cabral, 2021): the anticipation of being bought by established incumbents may foster start-ups' innovation activities. Yet, in our model a positive exit value also triggers the strategic effect when the merger does not occur, thereby the challenger benefits from a lenient policy even when it rejects the incumbent's offer and competes in the market.

Turning to consumer surplus, recalling that we denoted by $CS(I, K)$ the (expected) consumer surplus for given I and K, we obtain the following results.

- For every $\psi \leq \Delta\pi_0(s)$, firm 0 always invests, irrespective of the merger policy. In this case, the optimal policy is the strict one — i.e., $y^\star = s$.

This is because a strict policy achieves the first-best $CS(1, 0)$ — i.e., firm 0 invests irrespective of the merger policy but a strict merger policy maximizes rivalry.

- For every $\psi \in (\Delta\pi_0(s), \Delta\pi_0(l)]$, firm 0 invests if and only if the regulator adopts a lenient merger policy. In this case, the optimal policy is lenient — i.e., $y^\star = l$.

This is because with a lenient merger policy consumer surplus is $CS(1, K^e(1))$, whereas consumer surplus under a strict policy is $CS(0, 0)$, and, as in the baseline model, $CS(1, K^e(1)) > CS(0, 0)$.

- For every $\psi > \Delta\pi_0(l)$, firm 0 does not invest, regardless of the merger policy. In this case, the merger policy is neutral from a consumer welfare viewpoint.

This is because the incumbent is not willing to acquire firm 0, and thus consumer surplus is always $CS(0, 0)$ irrespective of the merger policy.

Therefore, we can state the following proposition.

Proposition 6. *A lenient merger policy maximizes consumer surplus whenever $\psi \geq \Delta\pi_0(s)$.*

A lenient merger policy is optimal from the consumers' standpoint when it triggers the investment — i.e., when the investment would not take place under a strict merger policy. Hence, a sufficient condition for a lenient merger policy not to harm consumers is $\psi \geq \Delta\pi_0(s)$, and for $\psi \in (\Delta\pi_0(s), \Delta\pi_0(l)]$ a lenient merger policy strictly increases consumer surplus because it solves the under-investment problem. Then, even though the challenger may be acquired by the incumbent, it is actually less likely to leave the market than under a strict policy given that:

(i) its products are of higher quality, and *(ii)* the strategic effect discussed above reduces the probability of merger — i.e., $\theta^\star(1, K^e(1)) < \theta^\star(0,0)$ (see the Appendix). Thus, in our model the innovation for buyout effect increases the chances of a more competitive market structure. Moreover, conditionally on the challenger remaining in the market, its higher product quality unambiguously benefits consumers.

3.1.4 Optimal merger policy with a total welfare standard

In the foregoing analysis we have assumed that the merger policy is set to maximize consumer welfare — i.e., that, in line with the US and EU antitrust law, regulators and Antitrust Authorities adopt a consumer surplus standard. In what follows, we characterize the optimal merger policy from a total welfare maximization standpoint (as it is the one used, e.g., in Canada).

When considering a total welfare standard, the optimal merger policy combines the effects on consumer surplus and the effect on aggregate profits. Hence:

Proposition 7. *The merger policy $y^\star$ that maximizes total welfare has the following features:*

- *if $\psi \leq \Delta\pi_0(s)$, then $y^\star = l$;*
- *if $\psi \in (\Delta\pi_0(s), \Delta\pi_0(l)]$, there exists a threshold $\psi^\star$ such that $y^\star = l$ if and only if $\psi \leq \psi^\star$, with $\psi^\star > \Delta\pi_0(s)$ for all σ and $\psi^\star = \Delta\pi_0(l)$ for σ large enough.*
- *if $\psi > \Delta\pi_0(l)$, then $y^\star = \{s, l\}$ — i.e., the merger policy is total welfare neutral.*

The intuition is as follows. Clearly, when $\psi > \Delta\pi_0(l)$ the investment is so costly that the challenger never undertakes it irrespective of the merger policy. As shown above, in this case, the incumbent never proposes a merger, and therefore total welfare is the same irrespective of the merger policy. By contrast, when $\psi < \Delta\pi_0(s)$ the investment always takes place irrespective of the merger policy. In this region of parameters, in contrast to the case of a consumer surplus standard, total welfare maximization mandates a lenient policy. The reason is that, because of asymmetric information, a strict merger policy results in an excessive dissipation of profits which is, by and large, driven by the fact that the uninformed incumbent cannot condition its output on the challenger's type. This inefficiency is solved with the merger.

Finally, when $\psi \in (\Delta\pi_0(s), \Delta\pi_0(l)]$ the challenger invests with a lenient merger policy and does not with a strict policy. In this region of parameters, total welfare maximization mandates a lenient policy less often compared to the policy that maximizes consumer surplus — i.e., $y^\star = l$ if and only if $\psi \leq \psi^\star$ — simply because a regulator that cares about total welfare weights the investment cost while a regulator that only cares about consumer surplus does not internalize this cost. However, for σ sufficiently large $\psi^\star = \Delta\pi_0(l)$ as, conditional on the challenger remaining in the market, the expected quality of its product is higher. Hence, for large uncertainty, total welfare maximization mandates a lenient merger policy in a larger set of parameters compared to a consumer welfare standard.

3.1.5 Transaction-based merger policy

Up until now we assumed that the regulator can only commit to reject or to approve any merger proposal. Here we consider a more general policy space. Specifically, we assume that the regulator commits to an approval rule that depends on the proposed acquisition price — i.e., the decision to approve or reject a merger depends on the takeover price K. For example, the merger policy may allow any merger whose takeover price is lower than a given threshold, or mandate a minimum takeover price.

As noticed by Fumagalli et al. (2020), in policy discussions it is often mentioned that the price of the transaction should be seen as a signal that the merger is likely anti-competitive. The idea is that the incumbent may be willing to share part of its profits with the challenger in order to protect its market power. However, in our model, two contrasting forces are in place. On the one hand, a too high takeover price may induce a merger too often compared to what consumers would like, thereby undermining competition. On the other hand, since the main source of welfare loss is the under-investment problem, the regulator may want to guarantee a minimum takeover price in order to stimulate the challenger's investment (provided that the incumbent is willing to offer such price).

The trade off between these two forces is non-obvious and depends on the challenger's incentive to invest in the lenient and strict merger policy regimes examined above.

Proposition 8. *Let K^P be the unique solution of $\psi = \pi_0^\star(1, K) - \pi_0^\star(0, 0)$, with K^P being increasing in ψ and positive for $\psi > \Delta\pi_0(s)$. Then, the takeover price contingent optimal policy improves consumer surplus relative to the uncontingent policies examined before and has the following features:*

- *for $\psi \leq \Delta\pi_0(s)$, it is optimal not to approve any merger (i.e., a strict policy is still optimal);*
- *for $\psi \in (\Delta\pi_0(s), \Delta\pi_0(l)]$, it is optimal to approve every merger with takeover price $K \leq K^P$, with $K^P \leq K^e(1)$. In equilibrium, the challenger invests, the incumbent optimally offers K^P and the merger takes place with positive probability;*
- *for $\psi > \Delta\pi_0(l)$, it is optimal to approve every merger with takeover price $K \geq K^P$, with $K^P > K^e(1)$. Moreover, there exists a threshold $\overline{\psi} > \Delta\pi_0(l)$ such that:*
  - *for $\psi \in (\Delta\pi_0(l), \overline{\psi}]$, the challenger invests, the incumbent optimally offers K^P and the merger takes place with positive probability;*
  - *for $\psi > \overline{\psi}$, the incumbent is not willing to offer K^P even if $I = 1$. Hence, the challenger does not invest, the incumbent offers $K = 0$ and the merger never takes place.*

This proposition shows that the regulator can improve consumer surplus by tailoring the merger policy to the takeover price. By doing so, it can: (i) control more accurately the level of rivalry when the merger occurs too often compared to what consumers would like; and (ii)

boost the challenger's incentive to invest when the incumbent's takeover offer under a lenient policy would be too low compared to what would secure the investment.

When the investment cost is sufficiently small, the challenger always invests regardless of the takeover price and therefore a strict merger policy is always optimal since allowing a merger would only strengthen market power. Things become more interesting in the region of parameters in which the investment does not take place with a strict merger policy but it takes place with a lenient merger policy. In this case, a strict merger policy is clearly suboptimal, but also a 'plain vanilla' lenient policy may be improved upon because, without restrictions on the takeover price, the incumbent would acquire too often the challenger compared to what consumers would like. Hence, by capping the takeover price, the regulator is able to still secure the investment but protect at the same time rivalry. Finally, in the region of parameters where the investment does not take place even with a lenient merger policy, the regulator has a clear incentive to accept only mergers that guarantee a minimum takeover price to stimulate the challenger's investment. Clearly, while this policy strictly increases consumer surplus when ψ is not too large, so that the incumbent still benefits from the merger and optimally offers $K = K^P$, when ψ is extremely large, the floor K^P is so high to induce the incumbent giving up the merger. Anticipating that it will never be acquired, the challenger will not invest and the merger policy is consumer welfare neutral as in the unconditional merger policy analysis.

To conclude, it is worth pointing out that this transaction price based policy, by guaranteeing to the challenger the lowest exit value at which it is willing to invest, is the best merger policy that can be implemented by a regulator facing the incumbent's and the challenger's incentive compatibility constraint in the takeover price offer and investment decision, respectively.

3.2 Exit option as the investment liquidation value

An alternative way of endogenising K is to think of it as being firm 0's liquidation value. Hence, also in this case, firm 0's exit value depends on whether firm 0 has invested or not. In particular, K can be endogenised by assuming, for simplicity, that it is a fraction of the investment cost ψI — i.e., $K(I) \triangleq \alpha\psi I$, with $\alpha \in [0,1]$ being the share of the ex-ante investment that firm 0 is able to cash back when it decides to quit. For example, the value of α is inversely related to the degree of specificity of the investment — i.e., a higher α means that firms exiting a market have (relatively) better chances to redeploy the investment in other production activities — and/or depend on the efficiency of the liquidation process under the prevailingly bankruptcy law.

In what follows, we replace **A2** by assuming that ψ cannot be too large (see the Appendix). While outputs are the same as in the baseline model, with $K(I) = \alpha\psi I$, it is easy to verify that there exists a threshold $\theta^\star(I,\alpha) \in (-\sigma,\sigma)$ increasing in α and I, such that firm 0 quits if and only if $\theta < \theta^\star(I,\alpha)$. Let $\Delta\pi_0(\alpha) \triangleq \pi_0^\star(1,\alpha) - \pi_0^\star(0)$ denote firm 0's profit increase due to the investment as function of α, where its expected profit conditional on $I = 1$, $\pi_0^\star(1,\alpha)$, is increasing in α, and its expected profit conditional on $I = 0$, $\pi_0^\star(0)$, is independent of α (see the Appendix). The following is true.

Lemma 8. *$I^\star = 1$ if and only if $\psi \leq \Delta\pi_0(\alpha)$, this threshold being increasing in α.*

As intuition suggests, the investment is profitable when its cost is not too large and more likely when the share of this investment cost that firm 0 can recoup when it decides to exit is greater.

Denoting consumer surplus by $CS(I, \alpha)$, it can be shown (see the Appendix) that $CS(1, \alpha)$ is strictly decreasing in α, for the very same reason why consumer welfare is decreasing in K when $I = 1$ in the baseline model: namely, when investment is sunk, consumer welfare increases with the probability of duopoly. By contrast, consumer surplus when $I = 0$ is independent of α. Then, we can show the following welfare result.

Proposition 9. *The optimal policy is* $\alpha^\star = \Delta\pi_0^{-1}(\psi) > 0$*, which is increasing in* ψ*.*

In line with the results of the baseline model, an efficient bankruptcy law, which induces the challenger to invest, maximizes consumer surplus. The fact that $\alpha^\star$ increases with ψ implies that industries in which investments are considerably costly require a relatively more lenient liquidation policy to secure the investment.

Finally, it can be argued that, as long as it is socially optimal to induce the challenger to invest, the value of α that maximizes total welfare must be at least equal to $\alpha^\star$. This is because the total welfare maximizing bankruptcy law also internalizes the positive effect of a larger α on the challenger's expected profit.

4 Extensions and robustness

In this section we extend the framework developed above in several natural dimensions. For illustrative purposes, throughout we keep assuming $b = 1$. All proofs of the results of this section can be found in the online Appendix.

4.1 Multiple incumbents

Consider the case in which there are $N > 1$ symmetric incumbents in the market, each denoted by $i = 1, .., N$. The (inverse) demand functions are

$$p_i(\cdot) \triangleq \max\left\{0, \mu - \sum_{j=0}^{N} x_j\right\}, \quad \forall i = 1, .., N,$$

and

$$p_0(\cdot) \triangleq \max\left\{0, \mu I + \theta - \sum_{j=0}^{N} x_j\right\}.$$

Analogously to the baseline model, we impose a lower and upper bound on σ and an upper bound on $\overline{K}$ so that in equilibrium firm 0's exit decision is a random variable with a non degenerate distribution and all the incumbents have positive production in equilibrium (see the online Appendix). The rest of the assumptions and the timing of the game are as in the baseline model.

Once again, we characterize a cut-off equilibrium such that firm 0 takes the exit value K if and only if $\theta < \theta_N^\star(I, K)$, with this threshold being increasing in N for all $I = 0, 1$, and such that the difference

$$\Delta\theta_N^\star \triangleq \theta_N^\star(0, K) - \theta_N^\star(1, K) \geq 0,$$

is increasing in N. A higher N implies tougher competition in the product market, making the option to exit relatively more appealing to firm 0. Hence, for given investment, $\theta_N^\star(I, K)$ increases with N. Yet, when the market becomes relatively more competitive, exit becomes relatively more likely when firm 0 has not invested compared to the case in which it has invested. The reason is that the investment shields firm 0 against competition by the incumbents.

We can now turn to examine firm 0's incentive to invest. As in the baseline model, the value of the investment for the challenger is always positive — i.e., $\Delta\pi_0(K, N) \geq 0$ for all N. That is, for sufficiently low investment cost $\psi \leq \Delta\pi_0(K, N)$, which is a function of K, firm 0 invests. Moreover, we can prove the following.

Lemma 9. *The function $\Delta\pi_0(K, N)$ is strictly concave in K and features a maximum at $K_N^\star > 0$ when: (a) $N \leq 4$ or (b) $N \geq 5$ and $\sigma > \sigma_N^\star$, where $\sigma_N^\star$ is increasing in N and $K_N^\star$ is increasing in σ and μ. The effect of N on $K_N^\star$ is ambiguous: it is increasing for σ sufficiently large, and decreasing otherwise. By contrast, for $N \geq 5$ and $\sigma \leq \sigma_N^\star$, the function $\Delta\pi_0(K, N)$ is maximized at $K_N^\star = 0$.*

The number of incumbents N impacts both the region of parameters in which $\Delta\pi_0(K, N)$ has an interior maximum and the value $K_N^\star$ that maximizes this function.

To begin with, we explain why, with N incumbents, the return on the investment $\Delta\pi_0(K, N)$ is always decreasing in K when N is large enough and σ is sufficiently small. Other things being equal, when the market becomes less concentrated the profit that firm 0 earns when it produces falls since it faces tougher competition. This means that the strategic effect described in the baseline model weakens as N increases. In this case, being able to cash the exit value relatively more often becomes the dominating force. The negative effect of an increase in N on the investment return is stronger when σ is low: as discussed in the baseline model, for relatively low values of σ, firm 0's output and profit tends to drop because, conditional on remaining active in the market, its demand falls, thereby making the incumbents more aggressive.

Next, we explain why $K_N^\star$ is increasing in N when σ is large. To gain intuition, let us consider

$$\frac{\partial^2 \Delta\pi_0(K, N)}{\partial K \partial N} = -\underbrace{\frac{1}{2\sigma}\frac{\partial \Delta\theta_N^\star}{\partial N}}_{(+)} - \underbrace{\frac{1}{\sigma}\frac{\partial x_0^\star(\cdot)}{\partial \theta_N^\star(\cdot)}\frac{\partial \theta_N^\star(\cdot)}{\partial K}\left[x_0^\star\left(\theta_N^\star(0, K), 1, K\right) - K\right]\frac{\partial \theta_N^\star(0, K)}{\partial N}}_{(+)} +$$

$$+\frac{1}{\sigma}\int_{\theta_N^\star(0,K)}^{\sigma} \underbrace{\frac{\partial}{\partial N}\left[\frac{\partial x_0^\star(\cdot)}{\partial \theta_N^\star(\cdot)}\frac{\partial \theta_N^\star(\cdot)}{\partial K}\left(x_0^\star(\theta, 1, K) - x_0^\star(\theta, 0, K)\right)\right]}_{(+)} d\theta,$$

with $x_0^\star(\cdot)$ and $x_1^\star(\cdot)$ being the equilibrium outputs of the challenger and each incumbent.

Varying the number of incumbents impacts the relationship between the investment return and the exit value through three different channels. First, since $\Delta\theta_N^\star$ is increasing in N, when the market becomes less concentrated there are more states of nature in which firm 0 exits when it has not invested and remains active when it has invested. Hence, a higher N tends to increase the cost of staying in the market that the challenger pays when it has invested and faces a higher exit value K. Second, since $\theta_N^\star(0,K)$ is increasing in N, as the market becomes less concentrated, there are also fewer states of nature in which an increase in K generates a strategic effect that limits the incumbents' output. Finally, when N increases, the strategic nature of the investment as a means to shield firm 0 against competition by the incumbents increases, thereby reinforcing the positive output enhancing effect that a higher exit value generates on the investment value.

When σ is large, the strategic effect created by a higher exit value is strong since, ceteris paribus, $\mathrm{E}[\theta|\theta \geq \theta_N^\star(\cdot)]$ rises with σ. Hence, the third effect dominates because the challenger is more willing to invest when the increase of the aggregate output of its rivals, induced by a higher N, is compensated by a strong strategic effect, which in turn requires a high exit value (as explained before). By contrast, when σ is small, the strategic effect is weak, therefore the first and second effects gain weight, so that $K_N^\star$ falls with N.

We can now examine the relationship between consumer surplus and K. With N incumbents, expected consumer surplus is

$$CS_N(I,K) \triangleq \underbrace{\int_{-\sigma}^{\theta_N^\star(I,K)} CS^N \frac{d\theta}{2\sigma}}_{\text{Symmetric oligopoly with } N \text{ firms}} + \underbrace{\int_{\theta_N^\star(I,K)}^{\sigma} CS^{N+1}(\theta,I,K)\frac{d\theta}{2\sigma}}_{\text{Asymmetric oligopoly with } N+1 \text{ firms}} .$$

Defining, as before, $\Delta CS_N(K) \triangleq CS_N(1,K) - CS_N(0,K)$, we have that $\Delta CS_N(K) > 0$ for every K and consumers face the same under-investment problem discussed before when $\psi > \Delta\pi_0(K,N)$. In these instances, a regulator may step in and design an exit policy that fosters the investment. The relationship between K and consumer surplus is qualitatively as in the baseline model: in brief, while $CS_N(1,K)$ is always decreasing in K, $CS_N(0,K)$ may have an interior maximum. As in the baseline model, the most interesting case is that in which $\psi \in \Psi_N \triangleq (\Delta\pi_0(0,N), \Delta\pi_0(K_N^\star,N)]$. Yet, with $N \geq 5$ incumbents we also need $\sigma > \sigma_N^\star$ to guarantee that this region of parameters is non-empty.

Proposition 10. *For any $\psi \in \Psi_N$, the optimal policy always features an exit value equal to $\widehat{K}_N$ such that $\psi = \Delta\pi_0^{-1}(\widehat{K}_N, N)$.*

The qualitative result of this proposition is in the spirit of the baseline model: within the region of parameters where the investment cost is neither too low nor excessively large, the optimal policy features the minimum exit value compatible with firm 0's incentive constraint, and the region of parameters in which this happens expands as demand uncertainty rises.

4.2 Leapfrogging

In the baseline model we assumed that, conditional on the investment being undertaken, the quality of the challenger's product is (in expected terms) the same as that of the incumbent. In

this section we allow for leapfrogging — i.e., we examine the case in which, when the challenger invests, it sells, on average, a superior product compared to the incumbent's one. To this purpose, consider the following system of demand functions

$$p_0(x_0, x_1) \triangleq \max\left\{0, \nu I + \theta - \sum_{i=0,1} x_i\right\},$$

$$p_1(x_1, x_0) \triangleq \max\left\{0, \mu - \sum_{i=0,1} x_i\right\},$$

where $\nu > \mu$ to guarantee leapfrogging.[22]

Let again $\theta^\star(I, K)$ be the equilibrium cut-off defining the challenger's exit strategy. Following the same steps as in the baseline model we obtain that the value of the investment for firm 0 is positive — i.e., $\Delta\pi_0(K) > 0$ for any K. Hence, $I^\star = 1$ if and only if $\psi \leq \Delta\pi_0(K)$ and $I^\star = 0$ otherwise. The difference $\Delta\pi_0(K)$ is single peaked with respect to K and features a maximum $K^\star$, which is increasing in ν and decreasing in μ.

Thus, the equilibrium characterization with leapfrogging has the same qualitative features as the baseline model. The new insights are related to the comparative statics with respect to ν. First, as expected, a higher ν increases firm 0's output and reduces firm 1's output. Second, a higher ν makes exit less likely, conditional on $I = 1$. Third, the extent of leapfrogging, as measured by the difference $\nu - \mu$, increases the value of K that maximizes the challenger's incentive to invest.

Turning to the analysis of consumer welfare, it is easy to show that with leapfrogging consumers a fortiori benefit from the investment, since the challenger supplies, in expected terms, a superior good compared to the incumbent's one. Yet, when $\psi > \Delta\pi_0(K)$ there is an under-investment problem, which the regulator can solve by setting any $K \geq \Delta\pi_0^{-1}(\psi)$ as long as $\psi \leq \Delta\pi_0(K^\star)$.

Proposition 11. *For $\psi \in (\Delta\pi_0(0), \Delta\pi_0(K^\star)]$ consumer surplus is maximized when $I = 1$, which requires the regulator to set a positive exit value $\widehat{K} = \Delta\pi_0^{-1}(\psi)$. This region of parameters expands as ν and σ increase, and shrinks as μ increases.*

The novel result here is that the region of parameters in which the optimal policy mandates a positive exit value expands when the leapfrogging phenomenon becomes more relevant — i.e., when $\nu - \mu$ increases. The intuition is as follows. When, conditional on $I = 1$, the challenger supplies a product that is on average better than that of the incumbent, it has stronger incentives to undertake the investment, thereby the regulator can optimally secure the investment for a larger range of its cost ψ. Mutatis mutandis, the same logic implies that, with leapfrogging, a lenient merger policy and a soft bankruptcy law are even more likely to foster investment and benefit consumers.

[22] Once again, we impose a lower and upper bound on σ and an upper bound on K. Moreover, we posit that the ratio ν/μ is not too large, which ensures that the region of parameters defined by these restrictions is non-empty. The rest of the assumptions are the same as in the baseline model.

4.3 Uncertain investment return

So far we assumed an additive relationship between the parameter θ capturing demand uncertainty and the investment I — i.e., the investment may substitute adverse demand shocks. We now examine the case in which these two variables are complements — i.e., the investment has an uncertain return measured by θ. To this purpose, suppose that firm 0's (inverse) demand function is

$$p_0(x_0, x_1) \triangleq \max\left\{0, \nu + \theta I - \sum_{i=0,1} x_i\right\},$$

where θ distributes uniformly over the support $[0, \sigma]$ and, once again, it is firm 0's private information.[23] The rest of the game, including the incumbent's demand, is as in the baseline model.

When $I = 1$, firm 0 exits the market if and only if $\theta < \theta^\star(K)$ which, as intuition suggests, is decreasing in ν and increasing in μ. In what follows, we focus on the most interesting region of parameters where $\theta^\star(K) \in [0, \sigma]$ and the incumbent is always active. This region of parameters is not empty as long as $\nu < \frac{\mu}{2}$, which implies that the challenger always exits the market when it does not invest.

Once again, the value of the investment for firm 0 is positive — i.e., $\Delta\pi_0(K) > 0$. Hence, $I^\star = 1$ if and only if $\psi \leq \Delta\pi_0(K)$ and $I^\star = 0$ otherwise. Moreover, the difference $\Delta\pi_0(K)$ is single peaked with respect to K and features a maximum at $K^\star$ increasing in ν and decreasing in μ.

Turning to the optimal policy, it is easy to show once again that the investment always benefits consumers and that there is an under-investment problem when $\psi > \Delta\pi_0(K)$. Notice that the role of investment in the parameter configuration at hand is particularly relevant because in addition to increase the quality of the challenger's product, it also guarantees rivalry (since the challenger always exits when it does not invest). Hence, assuming that the investment is feasible — i.e., $\psi \leq \Delta\pi_0(K^\star)$ — and denoting by $K^{\star\star}$ the exit value which maximizes consumer surplus conditional on $I = 1$, the optimal policy has the following features (see Figure 3 for an illustration).

Proposition 12. *For $\psi \in (\Delta\pi_0(0), \Delta\pi_0(K^\star)]$ the optimal policy always requires a positive exit value. However, in contrast to the baseline model, when θ and I are complements, consumer surplus (conditional on $I = 1$) is single peaked with respect to K when σ is sufficiently low. In this case, the optimal exit policy may even require an exit value too large compared to the value that maximizes the challenger's incentive to invest (i.e., $K^{\star\star} > K^\star$).*

The interesting result of this section is that, conditional on $I = 1$, maximizing consumer surplus and maximizing rivalry are not necessarily equivalent. The reason is as follows: when σ is sufficiently small, the challenger's investment has a low expected return — i.e., the expected quality of its product is lower than that of the incumbent. In these instances, increasing the exit

[23] By assuming that θ is positive we focus on a freely disposable investment — i.e., the challenger can always decide to disregard the investment or, equivalently, to sell the pre-investment product when the draw is unfavourable (i.e., $\theta < 0$).

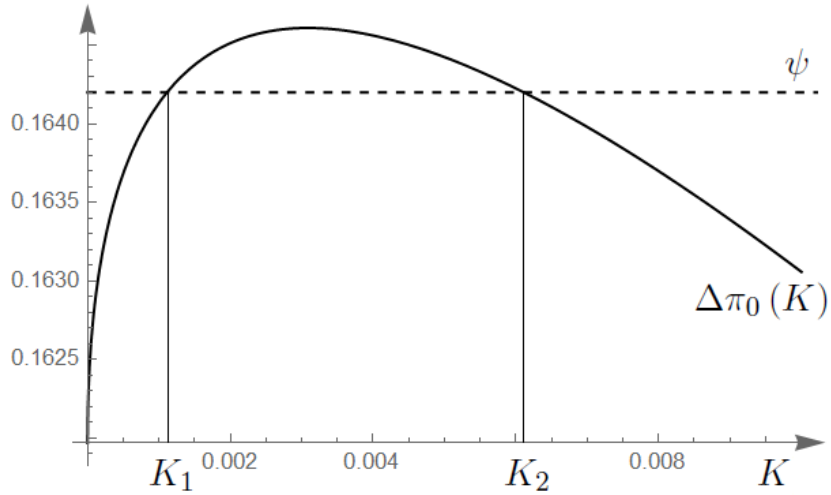

Figure 3: The optimal policy is $K^R = K_1$ for all $K^{\star\star} \in [0, K_1]$; $K^R = K^{\star\star}$ for all $K^{\star\star} \in (K_1, K_2)$; $K^R = K_2$ for all $K^{\star\star} \in [K_2, \overline{K}]$.

value above zero even when $I = 1$ allows the regulator to mitigate the strategic effect described in the baseline model and induce the incumbent to expand production, thereby benefiting consumers since, in this case, its product has the best quality. Therefore, the trade-off faced by the regulator may go in the opposite direction than in the baseline model. While setting a positive exit value, whereby sacrificing rivalry, was the (social) cost that the regulator had to incur to secure investment in the baseline model, with complementarities between I and θ, securing the investment may require a too low exit value compared to the level mandated by consumer surplus maximization (see Figure 3).[24] Hence, when σ is sufficiently small, the social cost of securing the investment is excessive rivalry. Of course, when σ is sufficiently large, the logic of the model and its results are similar to the case of leapfrogging, where the challenger's product is superior (in expected terms) to that of the incumbent.

4.4 Continuous investment technology

To simplify exposition, up until now we considered a binary investment level — i.e., $I \in \{0, 1\}$. In this section, we show that the insights of the baseline model carry over to a more general investment technology. Specifically, suppose that the challenger can set any investment level $I \geq 0$ incurring a cost $\Psi(I) \triangleq \psi I^2$.[25]

The quantity setting subgames and the challenger's exit decision are as in the base model — i.e., for any given exit value, the challenger is less likely to exit the market when it has sunk more demand-enhancing investments. Hence, the only substantial change materializes at the

[24] Thus, when σ is sufficiently low, the regulator does not face the time inconsistency problem highlighted in the baseline model. In these instances, it may actually have an incentive to announce a positive exit value such that the challenger invests and subsequently renege on this announcement by offering an even higher exit value to secure an expansion of the incumbent's superior production.

[25] The demand functions are specified as in the baseline model. We keep assuming **A1** and **A2** and impose a lower bound on ψ to ensure that firm 0 exits the market with positive probability (see the online Appendix for details).

investment stage, where the challenger maximizes its expected profit with respect to I. In the online Appendix we show that the following holds.

Proposition 13. *With a continuous investment technology, the challenger chooses an investment level that is an inverted U-shaped function of the exit value, with its unique maximizer being denoted by $K^\star > 0$. Moreover, expected consumer surplus is always maximized at a positive exit value $K^{\star\star}$, with $0 < K^{\star\star} < K^\star$.*

The selection effect is at play also with a continuous investment technology, and it implies that investment is maximized at a positive exit value $K^\star > 0$, this value being increasing in σ and decreasing in ψ. Similarly, the regulator always optimally commits to a positive exit value. As in the baseline model, this result is driven by the investment-enhancing role played by a positive (yet, not too large) exit value, which benefits consumers. Finally, once again $K^{\star\star} < K^\star$ — i.e., providing the challenger the maximal incentive to invest is suboptimal as this would require a too large exit value, thereby resulting in excessive exit.

4.5 Investment by the incumbent

Suppose now that also the incumbent can invest to increase the intercept of its (inverse) demand function, and assume that investments take place simultaneously at $t = 1$. There are two intuitive forces that shape the incumbent's incentive to invest. First, as well known in the literature, the incumbent has an incentive to invest in order to make exit by the challenger more likely (see, e.g., Rey and Tirole, 2007). Second, other things being equal, the incumbent will want to invest in order to face higher demand and expand its output. These two effects point in the direction of making exit more likely, and therefore also less profitable for the challenger to invest.

Therefore, from a social welfare point of view, increasing the exit value has two effects. First, it softens the negative spillover of the incumbent's investment on the challenger's incentive to invest. Second, it makes exit more likely, thereby reducing rivalry. Notice, however, that the loss from monopoly is lower when the incumbent invests because it sells a better product and its output expands. Therefore, compared to the case in which the incumbent does not invest, the socially optimal exit value is likely to be larger when the incumbent is allowed to invest too.

5 Conclusions

We have shown that, in environments characterized by uncertain demand and asymmetric information, the ability of firms to exit a market has a non-monotone effect on their investment decisions: entrants invest more when exit values increase provided that they are not too high, and vice versa. The incentive to invest is thus maximized at an intermediate exit value, which is increasing in the size of the market, the degree of competition and, critically, the uncertainty of demand. From a consumer welfare viewpoint, we have identified a trade-off between encouraging more firms to stay in the market by reducing the profitability of exit and stimulating ex-ante investment, and therefore more vigorous rivalry between firms, through a more permissive exit

policy. As uncertainty about future returns increases and products become closer substitutes, an exit policy intended to protect consumers should more likely be somewhat permissive, because of the benefits to consumers from stimulating investment.

Interestingly, with lack of commitment, depending on the specific features of the industry — e.g., demand uncertainty and product substitutability — regulators may face a time inconsistency problem: they will tend to hold up firms by choosing (post investment) a strict exit policy that maximizes rivalry, thereby dissipating the ex-ante return from the investment. This suggests that promoting commitment power on the regulators side is key to protect ex-ante consumer surplus, and the extent to which such commitment is desirable depends on the industry characteristics.

These findings have direct implications for merger policy and bankruptcy law. In particular, industries in which investments are considerably costly require relatively lenient merger and liquidation policies to secure demand-enhancing investments. In a context in which competition agencies,[26] political leaders[27] and pundits[28] are advocating for stricter merger policy, our contribution may be seen as 'reactionary'. It is not. Our contention is that merger policy is a form of exit policy that is bound to have significant incentives effects in industries where material investment decisions are made subject to considerable demand uncertainty. In such industries, an overly strict merger policy will reduce business dynamism to the ultimate detriment of consumers.

[26] Australian Competition and Consumer Commission, Bundeskartellamt and UK Competition and Markets Authority, "Joint statement on merger control enforcement", 20 April 2021, available at `https://www.gov.uk/government/publications/joint-statement-by-the-competition-and-markets-authority-bundeskartellamt-and-australian-competition-and-consumer-commission-on-merger-control/joint-statement-on-merger-control-enforcement`.

[27] "Senator Klobuchar introduces sweeping bill to promote competition and improve antitrust enforcement", 4 February 2021, available at `https://www.klobuchar.senate.gov/public/index.cfm/2021/2/senator-klobuchar-introduces-sweeping-bill-to-promote-competition-and-improve-antitrust-enforcement`.

[28] Professor Tommaso Valletti in Asher Schechter, "Mergers are bad for innovation", 29 September 2017, available at `https://promarket.org/2017/09/29/mergers-bad-innovation/?amp`.

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Appendix

Technical assumptions. The parametric restrictions imposed in **A1** and **A2** are

$$\underline{\sigma} \triangleq \frac{2}{2+b}\mu \leq \sigma \leq \overline{\sigma} \triangleq \frac{16 - b(4+2b-b^2)}{b(8-b^2)}\mu,$$

and

$$K \leq \overline{K} \triangleq \left(\frac{2\sigma - b\mu}{4-b^2}\right)^2,$$

respectively.

Proof of Lemma 1. Equilibrium outputs are determined by the system of best-response functions (3) and (4), whose solution yields

$$x_1^\star(I,K) = x^M - b\frac{\mu(2I-b) + \theta^\star(I,K) + \sigma}{2(4-b^2)},$$

and

$$x_0^\star(\theta, I, K) = \frac{\mu I + \theta}{2} - b\frac{2\mu(2-bI) - b(\sigma + \theta^\star(I,K))}{4(4-b^2)}.$$

Comparative statics results follow from simple inspection of these expressions. □

Proof of Proposition 1. The value $\theta^\star(I,K)$ is obtained from the indifference condition

$$\underbrace{\frac{\mu I + \theta^\star}{2} - b\frac{2\mu(2-bI) - b(\sigma+\theta^\star)}{4(4-b^2)}}_{x_0^\star(\theta,I,K)} = \sqrt{K},$$

yielding

$$\theta^\star(I,K) \triangleq \frac{4\left(4-b^2\right)\sqrt{K} - \left(4\mu(2I-b) + b^2\sigma\right)}{8-b^2} \in (-\sigma, \sigma),$$

Differentiating $\theta^\star(I,K)$ with respect to b yields that it is increasing in b if and only if

$$\sqrt{K} < \frac{1}{2}\left(\frac{1}{4}\mu\left(\frac{8}{b} + b\right) - \mu I - \sigma\right),$$

where this threshold is lower than $\overline{K}$ for σ large enough. Moreover, we have:

$$\frac{\partial^2\theta^\star(\cdot)}{\partial b^2} = -\frac{8(4(8+3b^2)\sqrt{K} - 24b\mu - b^3\mu + 16(\sigma+\mu I) + 6b^2(\sigma+\mu I))}{(8-b^2)^3},$$

which is negative for all admissible K if and only if

$$\sigma > \sigma(I) \triangleq \left(\frac{b(24+b^2)}{16+6b^2} - I\right)\mu,$$

with $\sigma(1) < \underline{\sigma}$ and $\sigma(0) < \frac{4+b^2}{4b}\mu$. Hence, $\theta^\star(\cdot)$ is always inverted U-shaped when it is non-monotone in b. The remaining comparative statics results are straightforward. □

Proof of Lemma 2. Substituting $\theta^\star(I,K)$ into the equilibrium outputs, we have

$$x_0^\star(\theta,I,K)=\frac{\theta}{2}+\frac{b^2(2\sqrt{K}+\sigma)+8\mu I-4b\mu}{2(8-b^2)},$$

with $x_0^\star(\theta,1,K)>x_0^\star(\theta,0,K)$ and

$$x_1^\star(I,K)=\frac{4\mu-b(2\sqrt{K}+\sigma+\mu I)}{8-b^2},$$

with $x_1^\star(1,K)<x_1^\star(0,K)$.

Aggregate output is

$$x_0^\star(\theta,I,K)+x_1^\star(I,K)=\frac{\theta}{2}+\frac{8\mu-b(2(2-b)\sqrt{K}+4\mu+(2-b)\sigma)+2(4-b)\mu I}{2(8-b^2)},$$

increasing in I.

Finally, equilibrium market clearing prices are

$$p_0^\star(\theta,I,K)=\frac{\theta}{2}+\frac{b^2(2\sqrt{K}+\sigma)-4b\mu+8\mu I}{2(8-b^2)},$$

with $p_0^\star(\theta,1,K)>p_0^\star(\theta,0,K)$, and

$$p_1^\star(\theta,I,K)=\frac{8\mu+b(2(2-b^2)\sqrt{K}+2b\mu+(2-b^2)\sigma-6\mu I)}{2(8-b^2)}-\frac{b}{2}\theta,$$

with $p_1^\star(\theta,1,K)<p_1^\star(\theta,0,K)$. □

Proof of Proposition 2. Manipulating $\Delta\pi_0(K)$, we have

$$\Delta\pi_0(K)=\frac{16\mu((4-3b(2-b))\mu^2+12(1-b)\mu\sigma+12\sigma^2+3b^2\sqrt{K}(\mu(1-b)+2\sigma)-12(4-b^2)K)}{3(8-b^2)^3\sigma},$$

which is clearly positive by (6).

Differentiating with respect to K, and solving the first-order condition, we immediately obtain $K^\star$ in (7), which is indeed a maximum since $\Delta\pi_0(K)$ is globally concave — i.e.,

$$\frac{\partial^2\Delta\pi_0(\cdot)}{\partial K}=-\frac{4b^2\mu(\mu(1-b)+2\sigma)}{(8-b^2)^3K^{3/2}\sigma}<0.$$

Moreover,

$$K^\star<\overline{K}\quad\Leftrightarrow\quad\sigma>b\mu\frac{8+(1-b)b}{2(8-b^2)},$$

this threshold being lower than $\underline{\sigma}$. Differentiating $K^\star$ with respect to b, we have

$$\frac{\partial K^\star}{\partial b}>0\quad\Leftrightarrow\quad\sigma>-\frac{4\,(2-3b)+b^3}{16}\mu,$$

this threshold being lower than $\underline{\sigma}$. The other comparative statics results are straightforward and

omitted for brevity. □

Proof of Proposition 3. We show that both terms in the decomposition (9) are strictly positive. As for the first term, integrating with respect to θ, we have

$$\int_{\theta^\star(1,K)}^{\theta^\star(0,K)} \left(CS^D(\theta,1,K) - CS^M\right)\frac{d\theta}{2\sigma} = \frac{\mu}{6\sigma(8-b^2)^3}\big(12(64-44b^2+5b^4)K+ \\ + 24((2-b)(8+16b+(2-b)b^2)\mu - b^2(6-b^2)\sigma)\sqrt{K}+ \\ + (64+3b(32+4b-b^3))\mu^2\sigma + 12b\sigma(b\sigma - 2b(4+b)\mu)\big)$$

which is positive for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ since, for all admissible values of σ, this difference is clearly increasing in K and positive when evaluated at $K = 0$.

As for the second term, we have that the difference

$$CS^D(\theta,1,K) - CS^D(\theta,0,K) = \frac{(4-b^2)\mu\theta}{2(8-b^2)} + \frac{\mu(16\mu + b(8\mu - b(2(2+b^2)\sqrt{K} + (7-4b)\mu + (2+b^2)\sigma)))}{2(8-b^2)^2}$$

is clearly increasing in θ, and

$$CS^D(\theta^\star(0,K),1,K) - CS^D(\theta^\star(0,K),0,K) = \frac{2(32-b^2\left(18-b^2\right))\sqrt{K} + (4-b)(4+7b)\mu - 6b^2\sigma}{2(8-b^2)^2}\mu > 0,$$

for all admissible values of σ, which suffices to show that also the second integral in (9) is strictly positive.

Hence, while consumers always benefit from firm 0's investment, firm 0 finds it optimal to invest if and only if $\psi \leq \Delta\pi_0(K)$. □

Proof of Lemma 3. For $I = 1$, the first-order condition with respect to K, $\frac{\partial CS(1,K)}{\partial K} = 0$, admits the following solutions:

$$\sqrt{K} = \frac{2b\left(2b\left(12-5b^2\right)\sigma + (b-2)(3b^3-4b^2+4b+32)\mu\right) \pm b\left(8-b^2\right)\sqrt{\Delta_1}}{2(4-b^2)\left(b^4-44b^2+64\right)},$$

with

$$\Delta_1 \triangleq 4(b^2+4)(3b^2-4)\sigma^2 - 8(b+1)(b^2+4)(2-b)^2\mu\sigma + ((b^3-4b^2+16b+48)b+48)(2-b)^2\mu^2,$$

being positive if and only if

$$\sigma \leq \frac{(4-b^2)\sqrt{(4+b^2)(64-44b^2+b^4)} - 2(2-b)^2(1+b)(4+b^2)}{2(4+b^2)(4-3b^2)}\mu.$$

Under this condition the smallest solution of the first-order condition is always negative and can therefore be neglected. Moreover, the largest solution is positive if and only if

$$\sigma \leq \frac{b^3-6b^2+20b-8}{8+6b^2}\mu < \underline{\sigma},$$

which implies that $CS(1,K)$ admits no stationary points in $[0,\overline{K}]$. Moreover, since

$$\left.\frac{\partial CS(1,K)}{\partial K}\right|_{K=\overline{K}} > 0 \quad \Leftrightarrow \quad \sigma < \frac{2b(2+b)(2-(5-b)b)+\sqrt{768-640b^2+256b^4-24b^6+b^8}}{64-56b^2+6b^4}b\mu < \underline{\sigma},$$

we can conclude that $CS(1,K)$ is decreasing in K for all $\sigma \in [\underline{\sigma},\overline{\sigma}]$ — i.e., $K^{\star\star}(1)=0$.

Next, suppose that $I=0$. The first-order condition $\frac{\partial CS(0,K)}{\partial K}=0$ admits solutions

$$\sqrt{K} = \frac{6b^5\mu + 24b^3\mu - 20b^4\sigma + 48b^2\sigma - 128b\mu \pm \left(8-b^2\right)b\sqrt{\Delta_0}}{2\left(256-240b^2+48b^4-b^6\right)},$$

with

$$\Delta_0 \triangleq \left(b^6+24b^4-112b^2+256\right)\mu^2 - 8\left(b^2+4\right)b^3\mu\sigma + 4\left(3b^4+8b^2-16\right)\sigma^2$$

being positive if and only if

$$\sigma \leq \hat{\sigma}_0 \triangleq \frac{(4-b^2)\sqrt{(4+b^2)(64-44b^2+b^4)}-2b^3(4+b^2)}{2(4+b^2)(4-3b^2)}\mu.$$

However, under this condition the smallest root of the first-order condition is always negative and can therefore be neglected. Denoting by K_0 the largest solution of the first-order condition, it can be easily shown that

$$K_0 \geq 0 \quad \Leftrightarrow \quad \sigma \leq \sigma_0^\star \triangleq \frac{b(20+b^2)}{8+6b^2}\mu < \hat{\sigma}_0,$$

and

$$K_0 < K^\star \quad \Leftrightarrow \quad (4+b^2)(64-44b^2+b^4)\sigma^2 - \frac{b(8+(1-b)b)(4+b^2)(64-44b^2+b^4)}{8-b^2}\mu\sigma +$$

$$+\frac{(64-44b^2+b^4)(1024+320b-320b^2-44b^3+40b^4-11b^5-2b^6+b^7)}{4(8-b^2)^2}b\mu^2 > 0,$$

which is satisfied for all $\sigma \in [\underline{\sigma},\overline{\sigma}]$. Moreover,

$$\left.\frac{\partial CS(0,K)}{\partial K}\right|_{K=\overline{K}} = -\frac{8\sigma + b((4+b^2)\mu - 6b\sigma)}{8(8-b^2)\sigma} < 0 \quad \forall \sigma \in [\underline{\sigma},\overline{\sigma}].$$

Hence, since the objective function is continuous and everywhere differentiable with respect to K in the interval $[0,\overline{K}]$, we can conclude that $K^{\star\star}(0)=K_0$ if and only if $K_0>0$, and $K^{\star\star}(0)=0$ otherwise. Specifically, we have:

- for $b < b_0^\star \simeq 0.37$: $\overline{\sigma} > \hat{\sigma}_0 > \underline{\sigma} > \sigma_0^\star \Rightarrow K^{\star\star}(0)=0$ for all $\sigma \in [\underline{\sigma},\overline{\sigma}]$;
- for $b > b_0^\star$: $\min(\overline{\sigma},\hat{\sigma}) > \sigma_0^\star > \underline{\sigma} \Rightarrow K^{\star\star}(0)=K_0$ for $\sigma \in [\underline{\sigma},\sigma_0^\star]$ and $K^{\star\star}(0)=0$ for $\sigma \in (\sigma_0^\star,\overline{\sigma}]$. □

Proof of Lemma 4. From Proposition 2 we know that $\Delta\pi_0(K)$ is positive and strictly increasing in K for all $K \in [0,K^\star]$. It then immediately follows that $\widehat{K} \in [0,K^\star]$ — i.e., the smaller root of $\psi = \Delta\pi_0(K)$ — is strictly increasing in ψ. □

Proof of Proposition 4. Since $CS(1, K)$ is a decreasing function of K, a sufficient condition in order for $K^R = \widehat{K}$ to be the optimal policy for all $\psi \in (\Delta\pi_0(0), \Delta\pi_0(K^\star)]$ is

$$CS(1, K^\star) > CS(0, K^{\star\star}(0)).$$

To begin with, consider the case in which $K^{\star\star}(0) = 0$. After simple manipulations, we obtain

$$CS(1, K^\star) > CS(0, K^{\star\star}(0)) \quad \Leftrightarrow \quad F(\sigma) \triangleq \beta_0\mu^3 + \beta_1\mu^2\sigma + \beta_2\mu\sigma^2 + \beta_3\sigma_3 > 0,$$

with

$$\begin{aligned}
\beta_0 &\triangleq 262144 - 475136b^2 + 172032b^3 + 123904b^4 - 75264b^5 - 1792b^6 + \\
&\quad + 13152b^7 - 2116b^8 - 956b^9 + 179b^{10} - 17b^{11} - 3b^{12} + b^{13} > 0, \\
\beta_1 &\triangleq 6(131072 - 139264b^2 + 57344b^3 + 43008b^4 - 24064b^5 - 7008b^6 + \\
&\quad + 3456b^7 + 452b^8 - 184b^9 + 35b^{10} + 2b^{11} - b^{12}) > 0, \\
\beta_2 &\triangleq 12(65536 - 69632b^2 + 21248b^4 + 1280b^5 - 2624b^6 - 32b^7 + 124b^8 - 52b^9 - b^{10} + b^{11}) > 0, \\
\beta_3 &\triangleq -8b^4(768 + 160b^2 - 68b^4 + b^6) < 0.
\end{aligned}$$

Notice that $\beta_3 < 0$ implies that $F''(\sigma)$ is a decreasing function. Moreover, it can be easily verified that $F''(\overline{\sigma}) > 0$, which implies that $F(\cdot)$ is strictly convex. Then, it can be seen that $F(\cdot)$ has no stationary points in the interval $[\underline{\sigma}, \overline{\sigma}]$. Finally, we check that $F(\underline{\sigma}) > 0$ and $F(\overline{\sigma}) > 0$,[29] which suffices to prove $CS(1, K^\star) > CS(0, 0)$.

Next consider the case in which $K^{\star\star}(0) = K_0 \in (0, K^\star)$. As in this case the expression of consumer surplus for $I = 0$ is much more cumbersome, we show the result resorting to a 3D plot: see Figure 4.

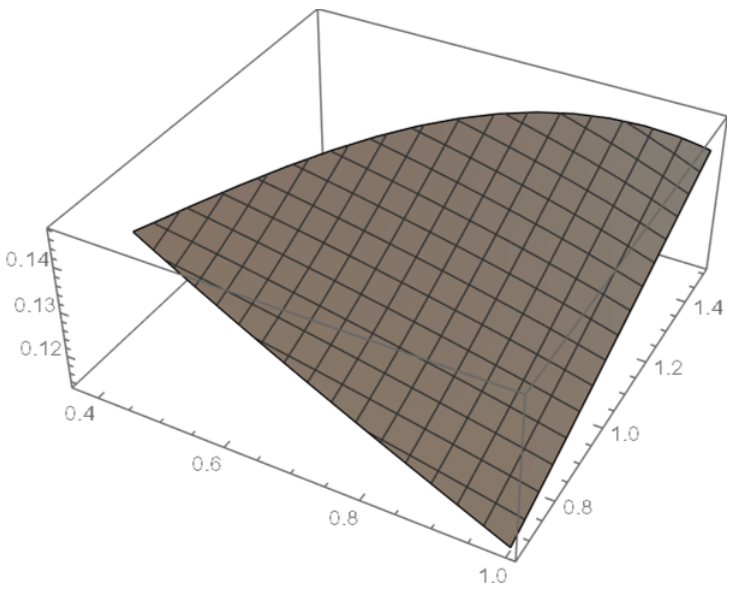

Figure 4: Difference $CS(1, K^\star) - CS(0, K_0) > 0$ as function of $b \in [b_0^\star, 1]$ and $\sigma \in [\underline{\sigma}, \sigma_0^\star]$.

[29] All these results can be established plotting the relevant functions against of $b \in (0, 1)$ (as μ is just a multiplicative term in all relevant expressions, which can be normalized to one without loss of generality). Details are available upon request.

Finally, we compute

$$\Delta\pi_0(K^\star) - \Delta\pi_0(0) = \frac{b^4\mu(\mu(1-b)+2\sigma)^2}{(8-b^2)^3(4-b^2)\sigma}.$$

This difference is clearly increasing in μ. Moreover, it is increasing in σ for all $\sigma > \frac{(1-b)\mu}{2}$, and in b for all

$$\sigma > \frac{96b + 4b^2 - 16b^3 + 2b^4 - b^5 - 64}{4(32 - b^2\,(2+b^2))},$$

these thresholds being lower than $\underline{\sigma}$. □

Proof of Lemma 5. As a strict merger policy implies that the challenger's exit value is equal to zero, by the baseline analysis, we have that the challenger invests if and only if

$$\psi \leq \Delta\pi_0\,(s) = \Delta\pi_0(0) = \frac{16\mu(\mu^2+12\sigma^2)}{1029\sigma},$$

with $\Delta\pi_0(0)$ given by (6) for $K=0$, which we proved positive and increasing in σ. □

Proof of Proposition 5. To begin with, notice that

$$\theta^\star(I,K) \leq \sigma \quad \Leftrightarrow \quad K \leq \overline{K}(I) \triangleq \frac{(2\sigma+\mu(2I-1))^2}{9},$$

with

$$\widetilde{K}(I) < \overline{K}(I) \quad \Leftrightarrow \quad \sigma > \mu(1-I),$$

which is always satisfied when $I=1$ and holds also when $I=0$ for $\sigma > \mu$. Recall that, in this region of parameters, the acquisition need not be a *killer acquisition* even for $I=0$ — i.e., the region of parameters such that $K \in (\widetilde{K}(0), \overline{K}(0))$ and $\theta > \mu$ is non-empty. Of course, firm 1 never finds it optimal to make an offer larger than $\overline{K}(I)$, as an offer $\overline{K}(I)$ is always accepted.

Assume that $K \in (\widetilde{K}(I), \overline{K}(I))$ and that this region is non-empty. It can be immediately checked that

$$\frac{\partial\pi_1^\star(\cdot)}{\partial K} = 0 \quad \Leftrightarrow \quad \frac{(\mu(I-4)+\sigma)(\mu(23I+20)+23\sigma)}{\sqrt{K}} - 1476\sqrt{K} + 8\mu(85I+31) - 692\sigma = 0,$$

which has no real solutions. Moreover,

$$\left.\frac{\partial\pi_1^\star(\cdot)}{\partial K}\right|_{K=\widetilde{K}(I)} = \frac{1}{28}\left(\frac{\mu}{\sigma}\left(\frac{84\mu}{\mu(I+3)+\sigma} + 17I - 37\right) - 11\right) < 0$$

for all $I=0,1$ and $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. Therefore, $\pi_1^\star(I,K)$ is always decreasing in K for all $K \geq \widetilde{K}(I)$. As it can be immediately checked that the function $\pi_1^\star(\cdot)$ is continuous and has continuous first derivative at $K = \widetilde{K}(I)$, we can conclude that firm 1's optimal offer is always strictly lower than $\widetilde{K}(I)$.

For $K \leq \widetilde{K}(I)$, instead, the first-order condition $\frac{\partial\pi_1^\star(\cdot)}{\partial K} = 0$ yields

$$\sqrt{K_{1,2}^\star(I)} = \frac{1}{954}\left(2\mu(94I-5) - 155\sigma \pm 7\sqrt{\mu^2\,(916I^2-544I+187) - 4\mu\sigma(200I+101) + 685\sigma^2}\right).$$

It can be immediately checked that the smaller solution, $K_1^\star(I)$, is negative for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ when $I = 0$, and also when $I = 1$ for $\sigma > \frac{9}{10}\mu$. Yet, when it is positive, it is a minimum point.[30] By contrast, for $I = 1$ and at the larger solution of the first-order condition, $K = K_2^\star(1)$, the second order condition is always satisfied.[31] Moreover, $K_2^\star(I)$ is always smaller than $\widetilde{K}(I)$, and it is positive if and only if $I = 1$, for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. Since $\pi_1^\star(\cdot)$ is decreasing at $K = \widetilde{K}(I)$ and

$$\pi_1^\star(0,0) = \frac{-30\mu^3 + 467\mu^2\sigma - 136\mu\sigma^2 + 16\sigma^3}{1372\sigma} > \pi_1^\star(0, \widetilde{K}(0)) = \frac{-9\mu^3 + 81\mu^2\sigma - 35\mu\sigma^2 + 3\sigma^3}{288\sigma},$$

we can conclude that, for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, firm 1's optimal offers are $K^e(1) = K_2^\star(1)$ and $K^e(0) = 0$.

Finally, we compute

$$\frac{\partial K^e(1)}{\partial \sigma} = \frac{1}{954}\left(\frac{7(685\sigma - 602\mu)}{\sqrt{559\mu^2 - 1204\mu\sigma + 685\sigma^2}} - 155\right) > 0 \quad \Leftrightarrow \quad \sigma > \frac{1204 + 31\sqrt{215}}{1370}\mu \in (\underline{\sigma}, \overline{\sigma}),$$

which shows that $K^e(1)$ is U-shaped in σ. □

Proof of Lemma 6. The proof of this result follows the same lines as in the baseline model and is omitted for brevity. □

Proof of Lemma 7. It is easy to find

$$\Delta\pi_0(l) > \Delta\pi_0(s) \quad \Leftrightarrow \quad 686(-3925\mu^2 + 30510\mu\sigma - 6281\sigma^2)\sqrt{559\mu^2 - 1204\mu\sigma + 685\sigma^2} +$$

$$-61907996\mu^3 + 576619459\mu^2\sigma - 606299632\mu\sigma^2 + 142183010\sigma^3 > 0,$$

which is satisfied for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. □

Proof of Proposition 6. It holds

$$\theta^\star(0,0) = \frac{4\mu - \sigma}{7} > \theta^\star(1, K^e(1)) = \frac{2\sqrt{559\mu^2 - 1204\mu\sigma + 685\sigma^2} - 40\mu - 67\sigma}{159}.$$

Next, using the formula for consumer surplus it can be shown that the difference $CS(1, K^e(1)) - CS(0,0)$ is positive for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. The rest of the proof is in the text. □

Proof of Proposition 7. Adding up expected consumer surplus and firms' expected profits,

[30] Indeed, the second-order condition for a maximum point is satisfied if and only if

$$-1908K - (2\sigma + \mu(2I - 1))(10\sigma + \mu(10I - 19)) < 0,$$

which, for $I = 1$, at $K = K_1^\star(1)$ rewrites as

$$(\mu + 2\sigma)(10\sigma - 9\mu) + \frac{1}{477}\left(-178\mu + 155\sigma + 7\sqrt{559\mu^2 - 1204\mu\sigma + 685\sigma^2}\right)^2 > 0,$$

which is violated for all $\sigma \in (\underline{\sigma}, \frac{9\mu}{10})$.

[31] Indeed, the second-order condition is satisfied as

$$(\mu + 2\sigma)(9\mu - 10\sigma) - \frac{1}{477}\left(178\mu - 155\sigma + 7\sqrt{559\mu^2 - 1204\mu\sigma + 685\sigma^2}\right)^2 < 0,$$

for all admissible σ.

we find that expected total welfare under a strict merger policy is given by

$$TW(y=s) \triangleq \frac{3\mu^2 \Pr\left[\theta \leq \theta^\star(I,0)\right]}{8} + \\ + \int_{\theta^\star(I,0)}^{\sigma} [(\mu I+\theta)x_0^\star(\theta,I,0) + \mu x_1^\star(I,0) - \frac{1}{2}(x_0^\star(\theta,I,0)+\mu x_1^\star(I,0))^2]\frac{d\theta}{2\sigma} - \psi I,$$

with $I=1$ if $\psi \leq \Delta\pi_0(s)$ and $I=0$ otherwise.

By contrast, from the results of Proposition 5, we have that, if $\psi \leq \Delta\pi_0(l)$ (so that $I^\star = 1$),

$$TW(y=l) \triangleq \frac{3\mu^2 \Pr\left[\theta \leq \theta^\star(1,K^e(1))\right]}{8} + \\ + \int_{\theta^\star(1,K^e(1))}^{\sigma} [(\mu+\theta)x_0^\star(\theta,1,K^e(1)) + \mu x_1^\star(1,K^e(1)) - \frac{1}{2}(x_0^\star(\theta,1,K^e(1))+\mu x_1^\star(1,K^e(1)))^2]\frac{d\theta}{2\sigma} - \psi,$$

whereas $TW(y=l) = TW(y=s)|_{I=0}$ otherwise — i.e., the merger policy is welfare neutral when the challenger does not invest as, in this case, the incumbent has no incentives to acquire it.

For $\psi \leq \Delta\pi_0(s) < \Delta\pi_0(l)$ (so that $I^\star = 1$ under both policies), comparing the relevant expression yields that $TW(y=s) < TW(y=l)$ for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$.

By contrast, for $\psi \in (\Delta\pi_0(s), \Delta\pi_0(l)]$, there is a threshold

$$\psi^\star \triangleq \frac{1}{936509364\sigma}\Big(- 686\left(76\mu^2 - 1159\mu\sigma + 79\sigma^2\right)\sqrt{559\mu^2 - 1204\mu\sigma + 685\sigma^2} + \\ - 55730721\mu^2\sigma + 19310707\mu^3 + 225689094\mu\sigma^2 + 2964910\sigma^3\Big),$$

such that $TW(y=s) < TW(y=l)$ if and only if $\psi < \psi^\star$, where $\psi^\star > \Delta\pi_0(s)$ for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, and $\psi^\star < \Delta\pi_0(l)$ if and only if $\sigma < \sigma^{TW} \simeq \mu \in [\underline{\sigma}, \overline{\sigma}]$. □

Proof of Proposition 8. To start with, notice that

$$\pi_0^\star(I,K) = \frac{540K^{3/2} + 453\sigma K + 288\mu(1-2I)K + 12(2\sigma - \mu - 2\mu I)^2\sqrt{K} + 8(2\sigma - \mu(1-2I))^3}{1029\sigma},$$

is clearly increasing in K for all $I = 0,1$ (and $\sigma > \underline{\sigma}$), thereby $\pi_0^\star(1,K) - \pi_0^\star(0,0)$ is increasing in K as well. Of course, for $\psi < \Delta\pi_0(s)$, as the challenger invests for all $K \geq 0$ and $CS(1,K)$ is decreasing in K, the strict policy is always optimal.

Next, consider $\psi \in (\Delta\pi_0(s), \Delta\pi_0(l)]$. In this case, as shown before, anticipating the equilibrium offers $K^e(I)$, the challenger optimally invests under a lenient merger policy, as $K^e(1)$ is such that $\psi \leq \pi_0^\star(1,K^e(1)) - \pi_0^\star(0,0)$ — i.e., $K^e(1) \geq K^P$, with equality at $\psi = \Delta\pi_0(l)$. However, $CS(1,K)$ being decreasing in K, consumers would be better off if the incumbent were to offer exactly K^P. Thus, imposing a cap K^P on the takeover price to approve the merger is optimal as long as the incumbent finds it optimal to offer K^P rather than not making any offer — i.e., if $\pi_1(1,K^P) > \pi_1(1,0)$. If such condition fails, anticipating the incumbent's incentive to deviate, the challenger would not make the investment, which would result in lower consumer surplus compared to a fully lenient policy (recall that $CS(1,K^e(1)) > CS(0,0)$). Yet, offering K^P is incentive compatible for the incumbent since we proved that $\pi_1^\star(1,K)$ is increasing in K for all $K \in [0,K^e(1)]$ (and offering $K=0$ is equivalent to not making any offer).

Finally, consider $\psi > \Delta\pi_0(l)$. Recall that, in this case, the challenger does not invest under a lenient merger policy as $\psi > \pi_0^\star(1, K^e(1)) - \pi_0^\star(0,0)$, or, equivalently, $K^e(1) < K^P$. However, consumers would still be better off under $I = 1$ provided that K is not too large. Formally, denoting by K^{CS} the unique solution of $CS(1,K) = CS(0,0)$, it holds $K^{CS} > K^e(1)$, since we proved that $CS(1, K^e(1)) > CS(0,0)$. Therefore, a policy which approves all mergers whose takeover price is $K \geq K^P$ is optimal provided that: (i) the incumbent is willing to offer $K = K^P$ — i.e., $\pi_1^\star(1, K^P) > \pi_1^\star(1,0)$ (as otherwise, anticipating its incentive to *deviate*, the challenger would optimally choose $I^\star = 0$); and (ii) consumers still benefit from the investment — i.e., $CS(1, K^P) > CS(0,0)$ or, equivalently, $K^P < K^{CS}$. Indeed,

$$\pi_1^\star(1,K) > \pi_1^\star(1,0) \quad \Leftrightarrow \quad K \leq K^M \triangleq \left(\frac{178\mu - 155\sigma + \sqrt{25960\mu^2 - 60268\mu\sigma + 36745\sigma^2}}{636} \right)^2,$$

with $K^M \in (K^e(1), \widetilde{K}(1))$. Therefore, imposing a floor K^P is always optimal provided that $CS(1, K^M) > CS(0,0)$ — i.e., $K^M < K^{CS}$,[32] which is satisfied for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. □

Proof of Lemma 8. Throughout the analysis we assume:

$$\psi \leq \overline{\psi} \triangleq \frac{(2\sigma - \mu)^2}{9}.$$

Proceeding as in the baseline model, we find that firm 0 exits the market if and only if

$$\theta < \theta^\star(I, \alpha) \triangleq \frac{12\sqrt{\alpha\psi I} - (4\mu(2I-1) + \sigma)}{7} \in (-\sigma, \sigma),$$

which is increasing in I and α. For $I = 1$, we have

$$\frac{\partial \pi_0^\star(1,\alpha)}{\partial \alpha} = \frac{2\psi(99\alpha\psi + (155\sigma - 94\mu)\sqrt{\alpha\psi} + 4(\mu + 2\sigma)^2)}{343\sqrt{\alpha\psi}} > 0,$$

for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, whereas $\pi_0^\star(I, \alpha)$ does not depend on α. It then immediately follows that firm 0 invests if and only if $\psi \leq \Delta\pi_0(\alpha)$, this threshold being increasing in α. □

Proof of Proposition 9. For $I = 1$, we have

$$\frac{\partial CS(1,\alpha)}{\partial \alpha} = -\psi \frac{36\alpha\psi + 8(5\mu - 2\sigma)\sqrt{\alpha\psi} + 4\sigma^2 - \mu^2}{392\sigma\sqrt{\alpha\psi}} < 0,$$

for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, whereas consumer surplus for $I = 0$ does not depend on α.

Therefore, a sufficient condition in order for the optimal policy to be the smallest value of α such that $I^\star = 1$ — i.e., for $\alpha^\star = \Delta\pi_0^{-1}(\psi)$ — is $CS(1,1) > CS(0,\alpha)$. Indeed, $CS(1,1) - CS(0,\alpha)$ is decreasing in ψ, and

$$[CS(1,1) - CS(0,\alpha)]\big|_{\psi=\overline{\psi}} = \frac{20\mu\sigma^2 + 214\mu^2\sigma - 13\mu^3 - 24\sigma^3}{3} > 0,$$

for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. □

[32] Of course, when ψ is too large, namely $\psi > \pi_0^\star(1, K^M) - \pi_0^\star(0,0)$, the merger policy is welfare neutral as $I^\star = 0$ in all cases.

Online Appendix to "Optimal Exit Policy with Uncertain Demand"

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A Multiple incumbents

Preliminaries. Analogously to the baseline model, we impose the following assumptions:

$$\underline{\sigma}_N \triangleq \frac{2}{N+2}\mu \leq \sigma \leq \overline{\sigma}_N \triangleq \frac{6+5N}{4+3N}\mu,$$

and

$$K \leq \overline{K}_N \triangleq \left(\frac{\sigma + N(\sigma-\mu)}{N+2}\right)^2.$$

Following firm 0's entry — i.e., for $\theta \geq \theta_N^\star(I,K)$ — the equilibrium is determined solving the system of the best-response functions

$$x_0^\star(\theta, I, K) = \frac{\mu I + \theta - N x_1^\star(I,K)}{2}$$

and

$$x_i^\star(I,K) = x^C - \frac{1}{2(N+1)}\left(\frac{1}{2}(\sigma + \theta_N^\star(I,K)) + \mu I - N x_N^\star(I,K)\right), \quad i = 1, \ldots N,$$

with $x^C \triangleq \frac{\mu}{1+N}$ being the equilibrium output of the symmetric N-firm Cournot oligopoly, which realizes if firm 0 exits. Equilibrium outputs are

$$x_0^\star(\theta, I, K) = \frac{\theta}{2} + \frac{\mu I}{N+2} + N\frac{4\mu(I-1) + \theta_N^\star(I,K) + \sigma}{4(N+2)},$$

and

$$x_i^\star(I,K) = \frac{\mu}{N+2} - \frac{2\mu(I-1) + \theta_N^\star(I,K) + \sigma}{2(N+2)}, \quad i = 1, \ldots N.$$

Then, imposing that firm 0 stays in the market whenever $\pi_0^\star(I,K) = x_0^\star(\theta,I,K)^2 \geq K$ yields that it takes the exit value K if and only if

$$\theta < \theta_N^\star(I,K) = \frac{4(2+N)\sqrt{K} - N\sigma - 4\mu((N+1)I - N)}{4+3N} \in (-\sigma, +\sigma)$$

Differentiating $\theta_N^\star(I,K)$ with respect to N gives

$$\frac{\partial \theta_N^\star(\cdot)}{\partial N} = -\frac{4(2\sqrt{K} + \sigma - \mu(I-4))}{(4+3N)^2}.$$

Hence, a sufficient condition in order for $\theta_N^\star(\cdot)$ to be increasing in N is

$$2\sqrt{K} + \sigma - 4\mu < 0 \quad \Leftrightarrow \quad -\frac{(4+3N)(2\mu-\sigma)}{2+N} < 0,$$

which is verified for all $\sigma \in [\underline{\sigma}_N, \overline{\sigma}_N]$. Finally, the difference

$$\Delta\theta_N^\star \triangleq \theta_N^\star(0,K) - \theta_N^\star(1,K) = \frac{4\mu(N+1)}{3N+4},$$

is clearly increasing in N.

Proof of Lemma 9. Substituting $\theta_N^\star(I,K)$ into the equilibrium quantities yields

$$x_0^\star(\theta, I, K) = \frac{4\theta + N(2\sqrt{K} - 4\mu + \sigma + 3\theta) + 4\mu(1+N)I}{8+6N}$$

and

$$x_i^\star(I,K) = \frac{\mu(4-I) - \sigma - 2\sqrt{K}}{4+3N}, \quad i = 1, \ldots, N.$$

We then have

$$\Delta\pi_0(K,N) = \frac{4\mu(N+1)}{3(4+3N)^3\sigma}\Big(2\mu^2(1+N(N-1)) - 6\mu(N^2-1)\sigma + 6(1+N)^2\sigma^2 +$$

$$+3\sqrt{K}N(2(N+1)\sigma - \mu(N-1)) - 12K(1+N)(2+N)\Big).$$

Taking the first-order condition immediately yields

$$K_N^\star = N^2\left(\frac{2(N+1)\sigma - \mu(N-1)}{8(N+2)(N+1)}\right)^2,$$

which is a maximum point if and only if

$$\frac{\partial^2\Delta\pi_0(\cdot)}{\partial K} = \frac{\mu N(N-1)(\mu(N-1) - 2(N+1)\sigma)}{K^{3/2}(4+3N)^3\sigma} < 0 \quad \Leftrightarrow \quad \sigma > \sigma_N^\star,$$

with $\sigma_N^\star < \overline{\sigma}_N$ for all N and $\sigma_N^\star > \underline{\sigma}_N$ if and only if $N \geq 5$. Hence, for $N \leq 4$, $\Delta\pi_0(\cdot)$ is maximized at $K = K_N^\star$ for all $\sigma \in [\underline{\sigma}_N, \overline{\sigma}_N]$, whereas for all $N \geq 5$ the same result holds only for $\sigma \in [\sigma_N^\star, \overline{\sigma}_N]$. By contrast, for $\sigma \in [\underline{\sigma}_N, \sigma_N^\star)$, $\Delta\pi_0(\cdot)$ is maximized at $K = 0$. To see this, it is sufficient to show that $\Delta\pi_0(0) > \Delta\pi_0(\overline{K}_N)$. After simple algebra, we find that this is the case for all

$$\sigma < \frac{N(5+3N)}{2(1+N)(2+N)}\mu,$$

this threshold being larger than $\sigma_N^\star$.

Finally, for $\sigma > \sigma_N^\star$:

$$\frac{\partial K_N^\star}{\partial N} > 0 \quad \Leftrightarrow \quad \sigma > \frac{-1+2N(1+N)}{2(1+N)^2}\mu.$$

As this threshold is increasing in N, it follows that for low (high) values of σ, $K_N^\star$ is always decreasing (increasing) in N, whereas for intermediate values of σ, $K_N^\star$ is inverted U-shaped in N. The other comparative statics results follow from simple inspection. $\square$

Proof of Proposition 10. For $I = 1$, the first-order condition $\frac{\partial CS_N(1,K)}{\partial K} = 0$ admits solutions

$$\sqrt{K} = \frac{N(N^2+5N+8)\sigma - \mu N(11N+24) \pm \sqrt{\Gamma_{1,N}}}{(N+2)(N(N+4)+16)},$$

with

$$\Gamma_{1,N} \triangleq \frac{N(3N+4)^2\left(\mu^2(14N^3+52N^2+47N-4) - 2\mu(N+4)(N+1)^3\sigma - (N+4)(N+1)^2\sigma^2\right)}{(N+1)^2}$$

being positive if and only if

$$\sigma \leq \hat{\sigma}_{1,N} \triangleq \left(\frac{(N+2)\sqrt{N(N+4)(16+4N+N^2)}}{(N+1)(N+4)} - (1+N)\right)\mu.$$

However, under this condition the smaller root of the first-order condition is always negative and can therefore be neglected, whereas the larger one is positive if and only if

$$\sigma \geq \tilde{\sigma}_{1,N} \triangleq \frac{24+11N}{8+5N+N^2}\mu,$$

where $\tilde{\sigma}_{1,N} > \hat{\sigma}_{1,N}$, which implies that there are no interior solutions. Moreover, we have:

- for $\sigma > \frac{N\mu}{N+1}$:

$$\left.\frac{\partial CS_N(1,K)}{\partial K}\right|_{K=\overline{K}_N} < 0$$

  if and only if

$$\sigma > \frac{\sqrt{N(N+1)^2(3N+4)^2\left(8N^5+52N^4+126N^3+132N^2+47N-4\right)} - 3N(N+1)^3(3N+4)}{\left(3N^2+7N+4\right)^2}\mu,$$

  this threshold being lower than $\frac{N\mu}{N+1}$;

- for $\sigma < \frac{N\mu}{N+1}$:

$$\left.\frac{\partial CS_N(1,K)}{\partial K}\right|_{K=\overline{K}_N} < 0 \quad \Leftrightarrow \quad \mu^2 N(4+58N+140N^2+112N^3+28N^4+N^5)+$$

$$-2\mu N(1+N)^2(36+N(5+N)(13+2N))\sigma+(1+N)^2(16+56N+61N^2+24N^3+4N^4)\sigma^2 > 0,$$

  which is satisfied for all σ.

Since $CS_N(1,K)$ is decreasing at $K = \overline{K}_N$ and has no stationary points in $[0, \overline{K}_N]$, we can conclude that it is a decreasing function of K for all $\sigma \in [\underline{\sigma}_N, \overline{\sigma}_N]$, thereby the CS-optimal K, conditional on $I = 1$, is $K^{\star\star}(1,N) = 0$.

For $I = 0$, the first-order condition $\frac{\partial CS_N(0,K)}{\partial K} = 0$ admits solutions

$$\sqrt{K} = \frac{N(N^2+5N+8)\sigma - \mu N(N^2+16N+32) \pm \sqrt{\Gamma_{0,N}}}{(N+2)(N^2+4N+16)},$$

with

$$\Gamma_{0,N} \triangleq \frac{N(3N+4)^2\left(\mu^2 N(N(N+4)(2N+19)+64) - 2\mu N(N+1)^2(N+4)\sigma - (N+1)^2(N+4)\sigma^2\right)}{(N+1)^2}$$

being positive if and only if

$$\sigma \leq \hat{\sigma}_{0,N} \triangleq \left(\frac{\sqrt{N(N+1)^2(N+2)^2(N+4)(16+4N+N^2)}}{(N+1)^2(N+4)} - N\right)\mu.$$

However, under this condition the smallest root of the first-order condition is always negative and can therefore be neglected. Denoting by $K_N^{\star\star}$ the only admissible solution of the first-order condition, it can be easily shown that

$$K_N^{\star\star} \geq 0 \quad \Leftrightarrow \quad \sigma \in \left[\frac{N}{N+1}\mu, \frac{N(10+10N+N^2)}{(N+1)(2+4N+N^2)}\mu\right],$$

with $\frac{N}{N+1}\mu > \underline{\sigma}_N$ for all $N \geq 2$. In this region of parameters, $K_N^{\star\star} \leq \overline{K}_N$. Moreover, for all $\sigma \in [\frac{N}{N+1}\mu, \overline{\sigma}_N]$,

$$\left.\frac{\partial CS_N(0,K)}{\partial K}\right|_{K=\overline{K}_N} = -\frac{\sigma + N(\mu(2N+3)+\sigma)}{2(N+1)^2(3N+4)\sigma} < 0,$$

whereas, for all $\sigma \in \left[\underline{\sigma}_N, \frac{N}{N+1}\mu\right)$,

$$\left.\frac{\partial CS_N(0,K)}{\partial K}\right|_{K=\overline{K}_N} > 0 \quad \Leftrightarrow \quad \sigma > \frac{80+216N+185N^2+54N^3+4N^4}{(1+N)(16+56N+61N^2+24N^3+4N^4)}\mu > \frac{N}{N+1}\mu.$$

Hence, since the objective function is continuous and everywhere differentiable over $K \in [0, \overline{K}_N]$, we can conclude that $K^{\star\star}(0) = K_N^{\star\star}$ if and only if $K_N^{\star\star} > 0$, and $K^{\star\star}(0) = 0$ otherwise.

Notice that, for $\sigma > \sigma_N^\star$, we have

$$K_N^{\star\star} < K_N^\star \quad \Leftrightarrow \quad \mu^2 N(49N^4+514N^3+1433N^2+1508N+528)+$$

$$-4\mu N(N+1)(N+4)(3N+4)(7N+9)\sigma + 4(1+N)^2(4+N)(4+3N)^2\sigma^2 > 0,$$

which is always satisfied.

Next, from Lemma 9, we know that, for all $\sigma \in [\sigma_N^\star, \overline{\sigma}_N]$, $\Delta\pi_0(K,N)$ is positive and strictly increasing in K for all $K \in [0, K_N^\star]$. It then immediately follows that $\widehat{K}_N \in [0, K_N^\star]$ is the smaller root of $\psi = \Delta\pi_0(K,N)$.

To establish that for $\sigma \in (\sigma_N^\star, \overline{\sigma}_N]$ the optimal policy is always $K = \widehat{K}_N$,[1] it is sufficient to show that $CS_N(1,K^\star) > CS_N(0,K^{\star\star}(0))$. As the expressions of consumer surplus are very cumbersome, this result is shown graphically in Figures 1 and 2 for the cases in which $K^{\star\star}(0) = 0$

[1] For $\sigma \in [\underline{\sigma}, \sigma_N^\star]$, the optimal policy is clearly $K = 0$, as $CS_N(1,0) > CS_N(0,0)$.

and $K^{\star\star}(0) = K_N^{\star\star}$, respectively.[2] $\square$

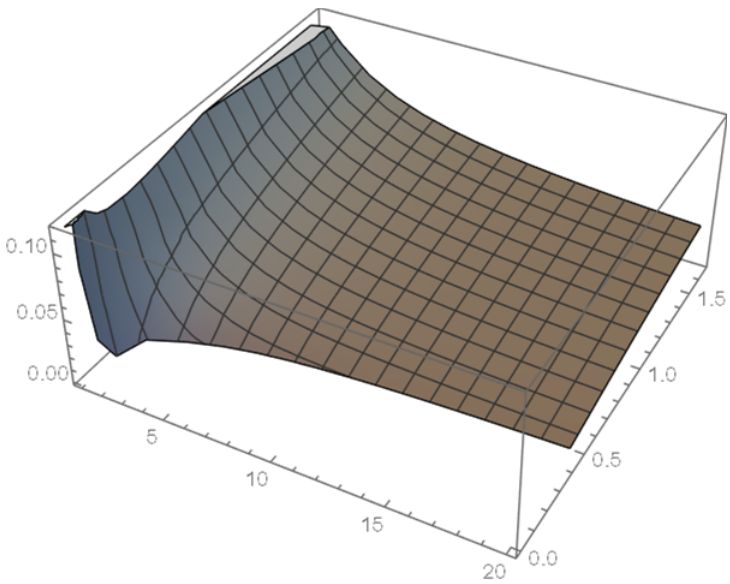


Figure 1: Difference $CS_N(1, K_N^\star) - CS_N(0, 0)$ as function of $N \in [1, 20]$ and $\sigma \in [\sigma_N^\star, \overline{\sigma}_N]$.

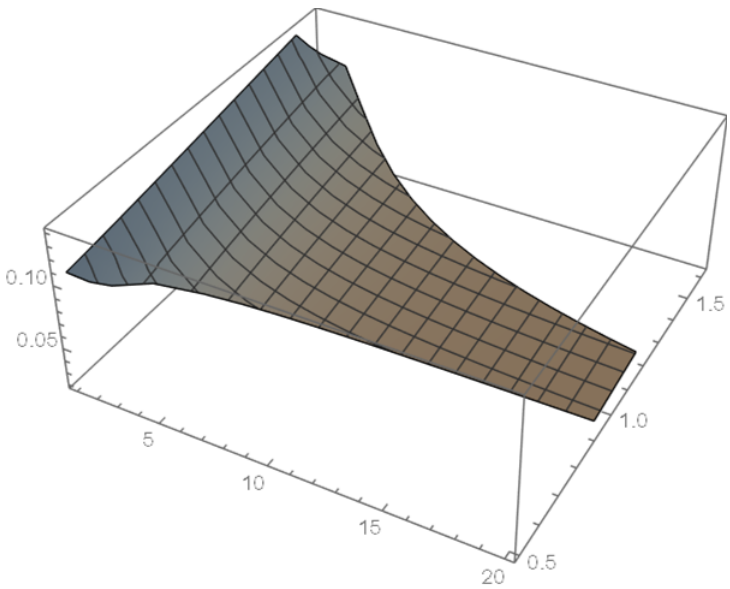


Figure 2: Difference $CS_N(1, K_N^\star) - CS_N(0, K_N^{\star\star})$ as function of $N \in [1, 20]$ and $\sigma \in [\frac{N}{N+1}, \frac{N(10+10N+N^2)}{(N+1)(2+4N+N^2)}]$.

[2]These graphs are obtained setting $\mu = 1$, but this normalization plays no role in the comparison, as μ acts just as a multiplicative factor. Moreover, for the sake of clarity, the graphs show the comparison for $N \leq 20$, even though we checked that the same results apply to larger values of N as well.

B Leapfrogging

Preliminaries. As in the baseline model, we assume that $K \leq \overline{K}$ and that

$$\underline{\sigma} \triangleq \frac{2(2\nu - \mu)}{3} < \sigma < \overline{\sigma} \triangleq \frac{14\mu - 3\nu}{7},$$

with $\frac{\nu}{\mu} < \frac{56}{37}$ so that the above region of parameters is non-empty.

By the same steps of the baseline analysis, for any I and K the Bayes-Cournot equilibrium when the challenger stays in the market is obtained from the system of the best response functions

$$x_0^\star(\theta, I, K) = \frac{\nu I + \theta - x_1^\star(I, K)}{2}$$

and

$$x_1^\star(I, K) = x^M - \frac{1}{2}\mathrm{E}\left[x_0^\star(\theta, I, K) \,|\theta \geq \theta^\star(I, K)\right],$$

yielding

$$x_0^\star(\theta, I, K) = \frac{2\nu I - \mu}{3} + \frac{\theta^\star(I, K) + \sigma}{12} + \frac{\theta}{2},$$

and

$$x_1^\star(I, K) = \frac{2\mu - \nu I}{3} - \frac{\theta^\star(I, K) + \sigma}{6}.$$

Imposing the indifference condition $x_0^\star(\theta^\star, I, K)^2 = K$ we obtain that firm 0 exits the market if and only if

$$\theta < \theta^\star(I, K) \triangleq \frac{12\sqrt{K} - \sigma + 4(\mu - 2\nu I)}{7}.$$

We next compute

$$\Delta\pi_0(K) = \frac{16\nu}{1029\sigma}\left(3\mu^2 - 6\mu(\nu + 2\sigma) + 4(\nu^2 + 3\nu\sigma + 3\sigma^2) + 3\sqrt{K}(\nu - \mu + 2\sigma) - 36K\right),$$

which is positive by the same decomposition of the baseline model. Solving the first-order condition with respect to K yields

$$\sqrt{K^\star} = \frac{\nu - \mu + 2\sigma}{24} \in (0, \sqrt{\overline{K}}),$$

for all $\nu > \mu$ and $\sigma > \frac{\mu}{2}$ (i.e., for all $\sigma > \underline{\sigma}$). This is indeed a maximum point as

$$\frac{\partial^2 \Delta\pi_0(\cdot)}{\partial K^2} = -\frac{4\nu(\nu - \mu + 2\sigma)}{343K^{3/2}\sigma} < 0.$$

The comparative statics results follow from simple inspection.

Proof of Proposition 11. Consumer surplus is given by

$$CS(I, K) = \frac{294\mu^2(6\sqrt{K} + 2\mu - 4\nu I + 3\sigma) - 8(5\sqrt{K} + 4\mu - \nu I - \sigma)^3 + (4\mu + 6(\sigma + \nu I) - 2\sqrt{K})^3}{16464\sigma},$$

with

$$\Delta CS\left(K\right)=\frac{\nu}{294\sigma}\left(12K-9\mu^2+6\mu\nu+4\nu^2+6\sqrt{K}(2\mu-\nu-2\sigma)+12(\mu+\nu)\sigma+12\sigma^2)\right)>0$$

for all admissible K.

Solving the first-order condition $\frac{\partial CS(I,K)}{\partial K}=0$ gives

$$\sqrt{K(I)}=\frac{1}{18}\left(4\sigma+4\nu I-14\mu\pm\sqrt{169\mu^2-40\mu(\sigma+\nu I)-20(\sigma+\nu I)^2}\right).$$

It is easy to see that, for all $\sigma<\overline{\sigma}$, the smaller root is negative for all $I=0,1$. As for the larger root, it is positive if and only if

$$\sigma<\sigma^\star(I)\triangleq\frac{3\mu}{2}-\nu I,$$

with $\sigma^\star(1)<\underline{\sigma}<\sigma^\star(0)$ for all $1<\nu<\frac{56}{37}$, and $\sigma^\star(0)<\overline{\sigma}$ if and only if $\nu<\frac{7}{6}\mu$. Moreover,

$$\left.\frac{\partial CS(I,K)}{\partial K}\right|_{K=\overline{K}}=-\frac{7(2\sigma-\mu)(2\sigma+5\mu)-8\nu(\mu+\sigma)I+12\nu^2I^2}{392(2\sigma-\mu)\sigma}<0,$$

for all $I=0,1$.[3] Thus, for $I=1$, the function $CS(1,K)$ is maximized at $K^{\star\star}(1)=0$, whereas $CS(0,K)$ is maximized at

$$K^{\star\star}(0)=\begin{cases}0 & \text{if } \nu<\frac{7}{6}\mu \text{ and } \sigma>\frac{3\mu}{2}\\ \frac{1}{324}\left(4\sigma-14\mu+\sqrt{169\mu^2-40\mu\sigma-20\sigma^2}\right)^2\in(0,\overline{K}) & \text{otherwise}\end{cases}$$

with $K^{\star\star}(0)<K^\star$ if and only if

$$140\sigma^2-20(7\mu+\nu)\sigma+(35\mu+\nu)(\mu+3\nu)>0,$$

which is always satisfied.

As $CS(1,K)$ is decreasing in K, whereas $\Delta\pi_0(K)$ is increasing in K for all $K\in[0,K^\star]$, a sufficient condition in order for $\widehat{K}\triangleq\Delta\pi_0^{-1}\left(\psi\right)\in[0,K^\star]$ to be the optimal policy in the interesting region of parameters in which $\psi\in\Psi\triangleq\left(\Delta\pi_0\left(0\right),\Delta\pi_0\left(K^\star\right)\right]$ is $CS(1,K^\star)>CS(0,K^{\star\star}\left(0\right))$. From the baseline analysis, we know that this condition is fulfilled whenever $\nu=\mu$. To prove that it holds *a fortiori* for larger values of ν, since $CS(0,K^{\star\star}\left(0\right))$ is of course independent on ν, it is sufficient to show that $CS(1,K^\star)$ is increasing in ν. Indeed, we have

$$\frac{\partial CS(1,K^\star)}{\partial\nu}=\frac{8685\nu^2-7315\mu^2+17044\nu\sigma+8372\sigma^2+2\mu(5099\nu+5222\sigma)}{225792\sigma}>0,$$

for all $\nu>\mu$. Finally,

$$\Delta\pi_0(K^\star)-\Delta\pi_0(0)=\frac{\nu(\nu-\mu+2\sigma)^2}{1029\sigma},$$

from which the comparative statics results immediately follow. □

[3] This can be immediately seen for $I=0$. The easiest way to see that this inequality holds for $I=1$ as well is not notice that $\frac{\partial CS(1,K)}{\partial K}|_{K=\overline{K}}<0$ if and only if the largest root of the first-order condition $\frac{\partial CS(1,K)}{\partial K}=0$ is lower than $\overline{K}$, which is of course always satisfied.

C Uncertain investment return

Preliminaries. When $I = 0$, the duopoly game would feature the standard equilibrium Cournot outcome — i.e.,

$$x_0^\star = \frac{1}{3}(2\nu - \mu), \qquad x_1^\star = \frac{1}{3}(2\mu - \nu).$$

Hence, exit occurs when

$$\sqrt{K} > \frac{1}{3}(2\nu - \mu).$$

Of course, this outcome realizes if and only if $\nu > \frac{\mu}{2}$. By contrast, for $\nu \leq \frac{\mu}{2}$, the challenger always quits when it does not invest.

If the challenger does not leave the industry, the Bayes-Cournot equilibrium of the game for $I = 1$ is obtained from the best-response functions

$$x_0^\star(\theta, K) = \frac{\nu + \theta - x_1^\star(K)}{2}$$

and

$$x_1^\star(K) = x^M - \frac{1}{2}\mathrm{E}\left[x_0^\star(\theta, K) \,|\theta \geq \theta^\star(K)\right],$$

yielding

$$x_0^\star(\theta, K) = \frac{1}{3}(2\nu - \mu) + \frac{1}{2}\theta + \frac{1}{12}(\theta^\star(K) + \sigma),$$

and

$$x_1^\star(K) = \frac{1}{3}(2\mu - \nu) - \frac{1}{6}(\theta^\star(K) + \sigma).$$

Imposing the indifference condition $x_0^\star(\theta^\star, K)^2 = K$ then yields that firm 0 exits the market if and only if

$$\theta < \theta^\star(K) \triangleq \frac{12\sqrt{K} + 4(\mu - 2\nu) - \sigma}{7}.$$

To make things interesting, we consider the following parametric restrictions:

$$K \leq \overline{K} \triangleq \left(\frac{2\sigma + (2\nu - \mu)}{3}\right)^2,$$

and

$$\underline{\sigma} \triangleq \frac{\mu}{2} - \nu \leq \sigma \leq \overline{\sigma} \triangleq \min(4(\mu - 2\nu), 2\mu - \nu),$$

with $\nu < \frac{\mu}{2}$.

In this range of the parameters, we have $\pi_0^\star(0, K) = K$ (as the challenger always exits when it does not invest) and

$$\pi_0^\star(1, K) \triangleq \int_0^\sigma \max\left\{x_0^\star(\theta, K)^2, K\right\} \frac{d\theta}{\sigma} = \underbrace{\int_0^{\theta^\star(K)} K \frac{d\theta}{\sigma}}_{\text{Exit value}} + \underbrace{\int_{\theta^\star(K)}^\sigma x_0^\star(\theta, K)^2 \frac{d\theta}{\sigma}}_{\text{Market value}},$$

from which it immediately follows

$$\Delta\pi_0(K) = \frac{8(3\sqrt{K} + \mu - 2(\nu + \sigma))^2(15\sqrt{K} - 2\mu + 4(\nu + \sigma))}{1029\sigma} > 0.$$

Maximizing this function with respect to K then yields

$$K^\star = \left(\frac{2\nu - \mu + 2\sigma}{45}\right)^2,$$

with $K^\star \in (0, \overline{K})$ for all $\sigma > \underline{\sigma}$.[4] The comparative statics results are straightforward.

Proof of Proposition 12. We have

$$CS\,(1, K) = \frac{1}{8232\sigma}\left(147\mu^2(12\sqrt{K} + 4\mu - 8\nu - \sigma) + 8(\nu + \sigma - 5\sqrt{K} - 4\mu)^3 + (4\mu + 6(\nu + \sigma) - 2\sqrt{K})^3\right).$$

Solving the first-order condition $\frac{\partial CS(1,K)}{\partial K} = 0$ yields

$$\sqrt{K} = \frac{1}{18}\left(4(\nu + \sigma) - 14\mu \pm \sqrt{169\mu^2 - 40\mu(\nu + \sigma) - 20(\nu + \sigma)^2}\right).$$

The smallest solution is negative for all $\sigma > \underline{\sigma}$ and can therefore be neglected. Under this restriction, $CS\,(1, K)$ is decreasing at $K = \overline{K}$, and the largest root is lower than $\overline{K}$. Finally, the largest root is positive for all $\sigma < \frac{3}{2}\mu - \nu$. Therefore, we can conclude:

$$K^{\star\star} = \begin{cases} \frac{1}{324}\left(4(\nu + \sigma) - 14\mu + \sqrt{169\mu^2 - 40\mu(\nu + \sigma) - 20(\nu + \sigma)^2}\right)^2 & \text{if } \sigma < \frac{3}{2}\mu - \nu \\ 0 & \text{otherwise} \end{cases}$$

Actually, an interior solution obtains for all admissible σ whenever $\nu > \frac{5}{14}\mu$.

As, for all admissible σ,

$$\Delta CS\,(K) = \frac{(2(\nu + \sigma) - \mu - 3\sqrt{K})(12K + 24\mu\sqrt{K} - 5\mu^2 + 8\mu(\nu + \sigma) + 4(\nu + \sigma)^2)}{294\sigma} \geq 0,$$

for all $K \in [0, \overline{K}]$, with equality at $K = \overline{K}$ only, it immediately follows that the optimal policy always induces $I = 1$, whenever possible:[5] that is, for all $\psi \leq \Delta\pi_0\,(K^\star)$ the optimal policy is

$$K^R = \arg\max_K CS(1, K) \qquad \text{s.t.} \quad \psi \leq \Delta\pi_0\,(K)\,.$$

Notably,

$$K^{\star\star} > K^\star \quad \Leftrightarrow \quad \sigma < \frac{19}{18}\mu - \nu,$$

which implies that securing the investment may require a lower exit value than the one which would maximize consumer surplus conditional on the investment being sunk. □

[4] This is indeed the maximum point, as the second-order condition is satisfied:

$$\left.\frac{\partial^2 \Delta\pi_0\,(\cdot)}{\partial K^2}\right|_{K=K^\star} = \frac{270K^\star - 2(2(\nu + \sigma) - \mu)^2}{343(K^\star)^{3/2}\sigma} = -\frac{24300}{49\sigma(2\nu - \mu + 2\sigma)} < 0.$$

[5] Of course, if $\psi > \Delta\pi_0\,(K^\star)$ the regulator's problem has a trivial solution: there exists no value of K at which firm 0 is willing to invest. Hence, for any K, $I^\star = 0$ and the monopoly outcome realizes.

D Continuous investment technology

Preliminaries. Besides assumptions **A1** and **A2** considered in the baseline model, we impose throughout that the cost of investing is not too low so that the challenger never exits the market, namely:

$$\psi > \underline{\psi} \triangleq \frac{8\mu^2\sigma}{7(2\mu+3\sigma)}.$$

The quantity-setting subgames and the challenger's exit decision are of course as in the base model, the only difference being that now I is a continuous variable.[6]. Therefore, firm 0's expected revenues are also as in the baseline model, and the optimal investment level is obtained maximizing $\pi_0^\star(I,K) - \psi I^2$, which yields

$$I^\star(K) = \frac{1}{2} - \frac{1}{64\mu^3}\left(16\mu^2\sqrt{K} + 64\mu^2\sigma - 7\sigma\left(49\psi - \Phi(K)\right)\right),$$

with

$$\Phi(K) \triangleq \sqrt{7\psi(343\psi - 128\mu^2) + \frac{256\mu^4 K}{\sigma^2} - \frac{224(\sqrt{K}-2\mu)\mu^2\psi}{\sigma}} > 0.$$

Proof of Proposition 13. Maximizing the optimal investment level with respect to K yields

$$K^\star = \frac{7\psi\sigma}{384\mu^4}\left(2\mu^3 - 4\mu^2\sigma + 21\psi\sigma - \sqrt{21\psi\sigma(4\mu^3 - 8\mu^2\sigma + 21\psi\sigma)}\right),$$

whose comparative statics is as follows:

$$\frac{\partial K^\star}{\partial\psi} = \frac{7\sigma}{384\mu^4}\left(2\mu^3 - 4\mu^2\sigma + 42\psi\sigma - \frac{6\sqrt{21}\psi^2\sigma^2(\mu^3 - 2\mu^2\sigma + 7\psi\sigma)}{\psi\sigma\sqrt{\psi\sigma(4\mu^3 - 8\mu^2\sigma + 21\psi\sigma)}}\right) < 0,$$

$$\frac{\partial K^\star}{\partial\sigma} = \frac{7\psi}{384\mu^2}\left(2\mu^3 - 4\mu^2\sigma + 21\psi\sigma + (21\psi - 4\mu^2)\sigma - \frac{2\sqrt{21}\psi^2\sigma^2(3\mu^3 - 8\mu^2\sigma + 21\psi\sigma)}{\psi\sigma\sqrt{\psi\sigma(4\mu^3 - 8\mu^2\sigma + 21\psi\sigma)}}\right) > 0,$$

for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ and $\psi > \overline{\psi}$.

Expected consumer surplus is

$$CS(K) \triangleq \Pr[\theta < \theta^\star(K)]CS^M + \int_{\theta^\star(K)}^{\sigma} \frac{1}{2}\left(x_0^*(\cdot) + x_1^*(\cdot)\right)^2 \frac{d\theta}{2\sigma} =$$

$$\frac{\mu^2(\sqrt{K}+\sigma)}{8\sigma} - \frac{49\psi - \Phi(K)}{128} - \frac{(48\mu^2\sqrt{K} + 32\mu^3 - \sigma(49\psi - \Phi(K)))^3 + (16(\sqrt{K}-2\mu)\mu^2 - 3\sigma(49\psi - \Phi(K)))^3}{1572864\mu^6\sigma}.$$

To see that it is maximized at a positive value of K, notice that

$$\frac{\partial CS(\cdot)}{\partial K} = \frac{N(K)}{2048\mu^4\sigma^2\Phi(K)\sqrt{K}},$$

whose denominator is strictly positive for all $K \in [0, \overline{K}]$, and whose numerator, $N(K)$, is such

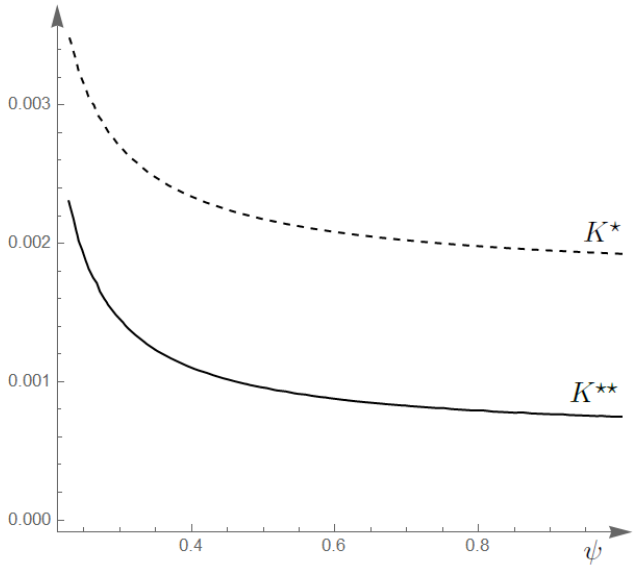
[6] Thus, $\theta^\star(I,K)$ is continuously decreasing in I.

that

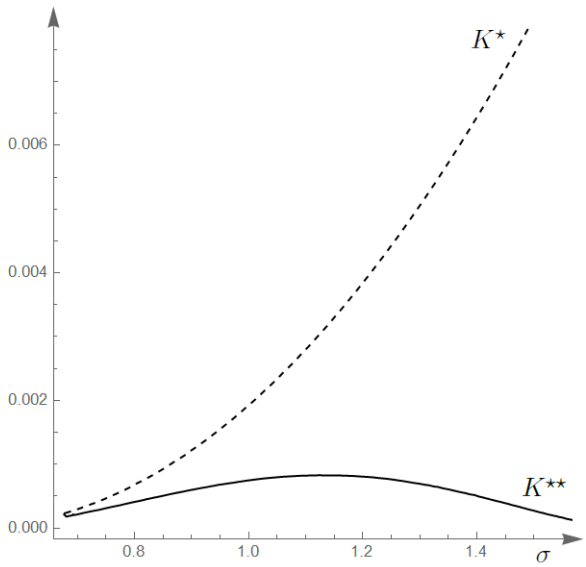
$$N(0) = 7\psi\sigma^2(8\mu^3(441\psi - 5\Phi(0)) + 343\psi\sigma(49\psi - \Phi(0)) - 16\mu^2\sigma(343\psi + 3\Phi(0))) > 0,$$

for all $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ and $\psi > \overline{\psi}$. Hence, we can conclude that $CS(\cdot)$ is strictly increasing at $K = 0$, which of course implies $K^{\star\star} > 0$.

Since it is not possible to obtain the expression for $K^{\star\star}$ analytically, the comparison between $K^{\star}$ and $K^{\star\star}$ and the comparative statics with respect to ψ and σ is illustrated in Figure 3. □



(a)



(b)

Figure 3: Thresholds $K^{\star\star}$ (continuous lines) and $K^{\star}$ (dashed lines) as functions of $\psi \in [\underline{\psi}, 1]$ (for $\sigma = \mu = 1$, panel a) and $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ (for $\psi = \mu = 1$, panel b).